



Manual for the GMAT*Exam

version 8.0

All rights reserved. No part of this manual may be reproduced for distribution to a third party in any form or by any means, electronic or mechanical, including photocopying, recording, or any information retrieval system, without the prior consent of the publisher, The Princeton Review.

This Manual is for the exclusive use of Princeton Review course students and is not legal for resale.

GMAT is a registered trademark of the Graduate Management Admission Council.

The Princeton Review is not affiliated with Princeton University or the Graduate Management Admission Council.

Permission to reprint this material does not constitute review or endorsement by the Educational Testing Service or the Graduate Management Admission Council of this publication as a whole or of any other sample questions or testing information it may contain.

Copyright © 2003 by Princeton Review Management, L.L.C. All Rights Reserved.

800.2Review/ www.princetonreview.com

ACKNOWLEDGMENTS

Thanks to the following for their many contributions to this course manual:

Tariq Ahmed, Kristen Azzara, Shon Bayer, John Bergdahl, Marie Dente, Russ Dombrow, Tricia Dublin, Dan Edmonds, Julian Fleisher, Paul Foglino, Alex Freer, John Fulmer, Joel Haber, Effie Hadjiioannou, Sarah Kruchko, Mary Juliano, Jeff Leistner, Sue Lim, Michael Lopez, Stephanie Martin, Chas Mastin, Elizabeth Miller, Colin Mysliwicz, Magda Pecsénye, Dave Ragsdale, “GMAT” Jack Schieffer, Cathryn Still, Fritz Stewart, Rob Tallia, Tim Wheeler, Stephen White, and the staff and students of The Princeton Review.

Special thanks to Adam Robinson, who conceived of many of the techniques in this manual.

TABLE OF CONTENTS

INTRODUCTION

Welcome

- The TPR Plan
- Structure of the Course
- Make the Commitment

The GMAT

How a CAT Works

PRE-CLASS ASSIGNMENTS

Assignment 1

Subjects, Verbs, and Pronouns

- Sentences
- Phrases and Clauses
- Subject-Verb Agreement
- Pronouns
- Verb Tenses
- Summary
- Answers and Explanations

Math Fundamentals

- Math Vocabulary
- Solving Equations and Inequalities
- Translating from English to Math
- Summary
- Drill
- Answers and Explanations
- Admissions Insight

Assignment 2

Modifiers, Parallel Construction, and Idioms

- Modifiers
- Parallel Construction
- Idioms
- Summary
- Answers and Explanations

Part-Whole Relationships

- Fractions
- Decimals
- Percents
- Probability
- Interest
- Data Sufficiency Ranges
- Summary
- Drill
- Answers and Explanations

Assignment 3

Arguments

- Parts of an Argument
- Common Flaws
- Summary
- Drill
- Answers and Explanations

Ratios and Statistics

- Ratios
- Averages
- Rates
- Median, Mode, and Range
- Standard Deviation
- Data Sufficiency Tricks and Traps
- Summary
- Drill
- Answers and Explanations

Admissions Insight

Assignment 4

Reading Comprehension

- Passage Topics
- Use Your Scratch Paper
- Work the Passage
- Translate the Question
- Paraphrase the Answer
- Summary
- Drill
- Answers and Explanations

Exponents, Roots, and Factors

- Distribution

Exponents
Roots
Quadratics
Factorials
Summary
Drill
Answers and Explanations
Admissions Insight

Assignment 5

Inferences and Paradoxes

Inference Questions
Resolve/Explain Questions
Summary
Drill
Answers and Explanations

Geometry

Can You Trust the Diagram?
Lines and Angles
Triangles
Quadrilaterals
Circles
Coordinate Geometry
Solid Geometry
Summary
Drill
Answers and Explanations

AWA

Scoring
The Basic Approach
The Argument Task
Sample

Admissions Insight

Assignment 6

Sentence Correction Revisited

Beyond the Big 6
Harder Verbal Problems
Harder Sentence Correction Questions
Answers and Explanations

Math Miscellaneous

Groups, Functions, Sequences, and Summation
Harder Permutations and Combinations
Summary
Drill
Answers and Explanations

Analysis of an Issue Essay

Basic Approach
Issue Task
Sample

Admissions Insight

Assignment 7

Harder Verbal Questions

- Harder Arguments Questions
- Harder Reading Comprehension

Harder Math

- Complexity
- Difficult Phrasing
- Tricks and Traps
- Answers and Explanations

Parting Words

- Mental Preparation
- Count Down to the GMAT
- At the Test Center

Admissions Insight

IN-CLASS LESSONS

Lesson 1

Welcome

- Understanding Your Score
- Set Your Goals
- Introduction to Pacing

Verbal Introduction

Sentence Correction 1

- The Format
- Basic Approach
- The Big Six
- Subject/Verb Agreement
- Verb Tense
- Pronouns
- Red Pencil Fever

Math Fundamentals

- Math Introduction
- Data Sufficiency
- Algebra vs. Arithmetic
- Plugging In
- Hidden Plug Ins
- Plugging In the Answers

Homework Review

Practice

- Answers and Explanations

Lesson 2

Sentence Correction 2

- Misplaced Modifiers
- Parallel Construction
- Idioms

Math 2

Keep Plugging Away
Yes/No Data Sufficiency
Percent Change
Probability

Test Smarts**Homework Review****Practice**

Answers and Explanations

Lesson 3**Critical Reasoning 1**

Basic Approach
Assumption Questions
Weaken Questions
Strengthen Questions
Identify-the-Reasoning Questions

Math 3

Ratios
Averages
Rates
Standard Deviation
Pieces of the Puzzle
Simultaneous Equations

Test Smarts

Pacing: The Beginning
Test 2 Goals
Test Analysis

Homework Review**Practice**

Answers and Explanations

Lesson 4**Reading Comprehension**

The Basic Approach
Question Types
Specific Questions
General Questions
Pace Yourself Wisely

Math 4

More Roots and Exponents
More Factors and Factorials
More Quadratics
More Yes/No Data Sufficiency
Permutations and Combinations

Test Smarts**Homework Review****Practice**

Answers and Explanations

Lesson 5

Critical Reasoning 2

- Inference Questions
- Resolve/Explain Questions
- Minor Question Types

Math 5

- Right Triangles
- Coordinate Geometry
- Solid Geometry
- Drill

Analysis of an Argument

- The Template

Test Smarts

- Pacing: The Middle
- Test 3 Goals
- Test Analysis

Homework Review

Practice

- Answers and Explanations

Lesson 6

Sentence Correction Revisited

- Beyond the Big Six
- Harder Sentence Correction Questions
- Sentence Correction Review

Math 6

- Functions
- Sequences
- Groups
- Harder Probability Questions
- Harder Permutations and Combinations

Analysis of an Issue

- The Template

Test Smarts

- Pacing: The End
- Test 4 Goals
- Test Analysis

Homework Review

Practice

- Answers and Explanations

Lesson 7

Arguments and RC Revisited

- Critical Reasoning Review
- Harder Arguments Questions
- Harder Reading Comprehension

Math 7

- Complex Problems

Practice

- Answers and Explanations

INTRODUCTION

WELCOME

THE TPR PLAN

Welcome to The Princeton Review! We're delighted that you chose us to help you prepare for the GMAT. This course provides two important benefits:

- A review of the subjects tested by the GMAT
- Test-taking strategies to help you maximize your performance

The Math section of the GMAT tests concepts from high school arithmetic, algebra, and geometry. The Verbal section measures your reading, grammar, and logical reasoning skills. The Analytical Writing Assessment essays test your organization and writing skills.

You have seen most of these topics previously, in high school or elsewhere. However, you probably haven't used these skills in quite some time. This course will review these areas and introduce you to a few topics you may not have seen before.

In addition to reviewing the material tested by the GMAT, you will learn test-taking strategies to help you best use your knowledge within the format of the test. You will learn the traps the test writers set for you and learn how to avoid them. You will learn how to use the multiple-choice format to your advantage. We'll cover everything you need to know to meet the specific challenges posed by the GMAT.

STRUCTURE OF THE COURSE

The Princeton Review GMAT course consists of six parts: pre-class assignments, class time, homework, diagnostic tests, the online student center (OSC), and help sessions. You'll get the most from the course if you take full advantage of each component.

Use the multiple-choice format to your advantage.

Pre-class Assignments

Prior to each class, your teacher will assign a pre-class section from the manual. These sections review core concepts and skills that your teacher will build on in class. Be sure to complete your reading assignment before every class.

Classes

Bring your materials to every class.

We will show you how to approach every type of question in every section and how to construct a pacing plan. Each week you will learn new skills and concepts and have a chance to review the homework and material from the previous class. Be sure to bring your materials (the course manual, the *Official Guide* and any other handouts) to every class.

Your GMAT class will have no more than eight students. Take advantage of the intimate setting by asking questions, getting involved in the class discussions, and letting your instructor know about your particular needs.

Homework

In addition to your pre-class assignment, your instructor will assign homework each week. You'll work practice problems from the manual and the *Official Guide* and you'll also do online practice each week.

Follow the guidelines below to get the most from your homework.

Use the techniques you learn in class.

Practice using the methods you learn in class.

The techniques we teach work, but some may feel awkward at first. It is extremely important to get comfortable *using* the techniques and methods on questions as opposed to merely *understanding* the concepts.

Keep up with the homework.

Your teacher will expose you to ideas in a logical order, and you will miss out if you fall behind.

Develop an awareness of your timing.

Be conscious of time from the very beginning. Make the small but important investment in a digital timer that can both count down and count up. Your local Radio Shack is a good source for timers. Set it to count up from zero when you begin work on a set of problems. Note how long it takes you to complete a set of ten questions. The purpose will not be to hit a particular target, but rather to make you conscious of how long it takes you to do different types of problems. As you progress through the course, knowing your capabilities will make setting your ultimate pacing strategy that much easier. Resist the temptation to check the answer after each question. Instead, complete at least ten questions before you check answers.

Develop your ability to concentrate.

The GMAT requires intense concentration for extended periods; use your homework sessions to develop this ability. On the actual exam, you'll need to be able to work for 75 minutes at a time without a break. Try scheduling your practice time in blocks of 75 minutes. If you develop the habit of working steadily for the entire period, you'll have the stamina needed for the exam.

Track your progress.

Simply doing problems will not result in significant score improvements. You need to learn from the problems you do as you go along, whether or not you get them right on the first try. Review your work to determine *why* you got questions right or wrong, and look for patterns in your performance. Based on what you observe, adjust your strategy on the next set of questions. Make a list of the problems you want to discuss and bring that list to class. Your teacher can help you out, but he or she is not a mind reader.

Leave your calculator at the door.

The first thing to get used to about the Math portion of the GMAT is that calculators are not permitted. Most of us depend on calculators for basic mathematical computations such as balancing a checkbook. A sure way to increase your math skills is to get accustomed to life without a calculator. Work out everything on scratch paper. Soon, working math without a calculator will seem like second nature.

Knock the rust off your math skills!

Practice Tests

Practice tests are an extremely important component of your GMAT course. In addition to your first practice exam, you will take several Computer Adaptive GMATs. Your instructor will tell you when to take these tests.

These practice tests serve a couple of important functions. First, they give you the opportunity to become familiar with the structure and format of the test. Pacing is essential, and you need a chance to develop a sense of timing. Taking a standardized test is a skill just like any other; it requires practice.

Second, these tests allow you and your instructor to monitor your progress and target areas that need improvement. Please be aware that ETS has not disclosed its exact scoring method. Scores from our practice tests (and all other simulated GMATs) should be viewed as approximate predictions of your GMAT score.

After you take a test, spend some time reviewing your performance. Look at the questions you missed and use the explanations to help you understand the correct answer. Print copies of your score report and any questions you want to discuss and bring them to class. Your instructor will either address them in class or set up an extra-help session to go over them.

You will receive a users' guide that explains the details of using the tester program. If you have any technical problems with the practice tests, please call the technical support phone number in the users' guide or contact us at websupport@review.com.

Online Student Center (OSC)

The Princeton Review's Online Student Center (OSC) has much more than online practice tests. The Your Course section contains online lessons aligned with the classroom lessons. Use these lessons to review and reinforce what you learn in class. The Practice section has targeted drills that allow you to practice working specific types of problems. The more problems you work on the computer, the more comfortable you'll feel at your exam. The *Guide to the Online Student Center* explains how to access all the OSC features.

Extra Help

If after doing the pre-class assignments, attending all classes, and doing the homework, you are still having trouble with some portion of the GMAT, speak to your instructor. He or she can set up a help session outside of class time.

MAKE THE COMMITMENT

Bottom Line: We will show you proven techniques for cracking the GMAT. It is up to you to learn and practice them until you are comfortable using them under the pressure of taking an exam.

We'd love to wave a magic wand and raise your GMAT score. Unfortunately, it doesn't work that way. Improving your GMAT score requires a lot of hard work. Your teacher will be there to support and guide you, but it's up to you to make preparing for the GMAT a priority.

This class involves a substantial amount of work outside the scheduled class sessions. Plan to spend seven to ten hours per week outside of class on pre-class assignments and homework. You'll need to schedule additional time during weeks in which practice tests are assigned. Reserve practice time in your schedule. If you schedule several blocks of time throughout the week, you'll get the work done and make the most progress. If you tell yourself, "I'll work on it when I have some free time," you are likely to fall behind quickly. We've worked with tens of thousands of GMAT students over the years, and we know that students who practice regularly get the best results.

THE GMAT

Business schools use the GMAT (Graduate Management Admission Test) to predict the performance of students applying for MBA programs. The admissions staff will consider your GMAT score, undergraduate GPA, work experience, recommendation letters, and application essays in making admissions decisions.

Who Writes the GMAT?

As you may already know, ETS—the same folks who ruined your high school years with PSATs, SATs, and Achievement Tests (SAT IIs)—is responsible for the GMAT. The folks at ETS write most of the other exams for graduate study, including the GRE (for graduate schools), as well as exams for CIA agents, barbers, golf pros, and travel agents. ETS is a private, nonprofit corporation (though it does have highly profitable for-profit divisions). It is not supervised by the government. It is not supervised by anyone, at any level. What gives ETS the right to administer this test? The fact that it gives this test.

Structure of the GMAT

The GMAT consists of three sections: an essay section (Analytical Writing Assessment or AWA), a multiple-choice Math section, and a multiple-choice Verbal section. Your test session will look something like this:

| Section | Time | # Questions | Type of Questions |
|-----------------------|------------|--------------|-------------------|
| AWA Essay #1 | 30 Minutes | 1 Topic | Essay (typed) |
| AWA Essay #2 | 30 Minutes | 1 Topic | Essay (typed) |
| <i>5-minute break</i> | | | |
| Math | 75 Minutes | 37 Questions | Multiple-choice |
| <i>5-minute break</i> | | | |
| Verbal | 75 Minutes | 41 Questions | Multiple-choice |

On each GMAT exam, you'll be given two essays — Analysis of an Issue (the issue essay) and Analysis of an Argument (the argument essay). Each of the AWA essays contains one question, and you will have 30 minutes to answer it by typing an essay into the computer. The word processor is rudimentary, with only cut, paste, and delete functions. Typing speed is really not much of a factor because your essay will only be three to five paragraphs in length. However, if you do not type at all, you should spend some time getting comfortable with using a keyboard.

The Math section contains two types of questions: problem solving and data sufficiency. Problem solving questions are the typical multiple-choice math questions that you know from the SAT and other standardized tests. The data sufficiency questions are less familiar. They test the same topics (arithmetic, algebra, and geometry), but the format is different. You will learn about this format and how to approach it systematically in the first class. You can expect that 50 to 60 percent of the math questions will be problem solving, and 40 to 50 percent will be data sufficiency.

The Verbal section includes three types of questions: sentence correction, critical reasoning, and reading comprehension. Sentence correction questions test your ability to spot grammatical mistakes. Critical reasoning questions test your ability to understand and analyze arguments. Reading comprehension questions test your ability to find information in a long passage.

Experimental Questions

The Math section includes nine experimental questions, and the Verbal section includes eleven experimental questions. These questions do not count toward your score. Why do you have to answer questions that don't affect your score?

The test writers need to try out new questions to ensure that they are well-written and produce the desired results. In other words, you pay for the privilege of serving as a research subject every time you take a GMAT.

There is no way to identify which questions are experimental. They are sprinkled randomly throughout each section. Don't waste time during the test guessing whether a question is experimental, but keep in mind that approximately one-fourth of the questions are unscored.

Scratch Paper

Avoid mistakes. Write out your work on scratch paper.

Because this test is presented on a computer screen, you will not be able to write on the problem to label diagrams, scratch off answers, circle key words, and so forth. Instead, you are required to do all of your work on the scratch paper provided at the test center. Do not try to work out problems in your head! That is a sure-fire way to make careless mistakes.

You will be provided with six sheets of scratch paper, stapled together, when you begin the test. While it is possible to request more, you may lose time in getting the proctor's attention in order to exchange your old scratch paper for new paper. Instead, try to fit all of your work for at least one section on the sheets provided. During the break between sections, you can exchange your used sheets for a fresh set if necessary.

Start practicing using your scratch paper now. When you work homework problems in this manual, do not circle words or write notes directly on the problem. Instead, write everything off to the side, as if you were using separate scratch paper. Get a notebook to use when you work problems from the *Official Guide* or do online drills and tests. Set up each page just as you would your scratch paper during an exam, and do all your work in the notebook. Be sure to label the problems so that you can review them easily once you have finished.

Scores

The questions and presentation of your GMAT are carefully designed to produce results which, when analyzed, produce a bell curve. In other words, very few people get a perfect score, and equally few people get every question wrong. The majority of us wind up in the middle, somewhere between 200 and 800. Since ETS attains these results each time it administers the test, it claims that the GMAT is an accurate gauge of our abilities. As you'll soon see, that's pretty far from the truth.

Your GMAT score actually consists of several different numbers, each of which covers a part of your performance on the GMAT. The most familiar number is the overall, or composite, score. This is the number you have seen in all the business school rankings and similar literature. It ranges from 200 to 800 in 10-point increments. Your composite score is determined from a combination of your scores on the Math and Verbal sections of the test. A score of 540 ranks in the 50th percentile, meaning half of all examinees score above that level and half score below that level. GMAT scores are valid for up to five years.

You also receive separate Verbal and Math subscores, which theoretically range from 0 to 60 for each section. In practice, scores below 10 or above 50 are rare. The mean (average) Verbal score is 27, and the mean Math score is 35. These are scaled scores, which means that if two people each score a 34, they show

comparable ability, even if they had totally different questions on their respective exams. Most schools are not very interested in your Verbal and Math subscores unless there is a significant gap between the two.

Your Analytic Writing Assessment (AWA) essays are graded on a scale of 0–6. There are two essays, and two readers (one human, one computer) look at each one. ETS then averages all four scores and rounds the result to the nearest half point. For example, if you get a 4 and a 5 on your argument essay and two 5's on your issue essay, your final AWA score is 5.0. The average AWA score is 4.0, and about 75 percent of all students score between 3 and 5. This score does not factor into your composite score.

The AWA score is not very important for most applicants. Focus on the more important areas.

Every score—overall, Verbal, Math, and AWA—is also accompanied by a percentile, so you can determine how well you compare with other test takers. An overall score with a percentile of 76 means that 76 percent of all people who have taken this test in the past four years or so did worse than you did. It also means that 24 percent did better.

Your results from the test will look something like this:

| Math | % | Verbal | % | Overall | % | AWA | % |
|------|----|--------|----|---------|----|-----|----|
| 35 | 48 | 30 | 59 | 550 | 54 | 4.5 | 62 |

HOW A CAT WORKS

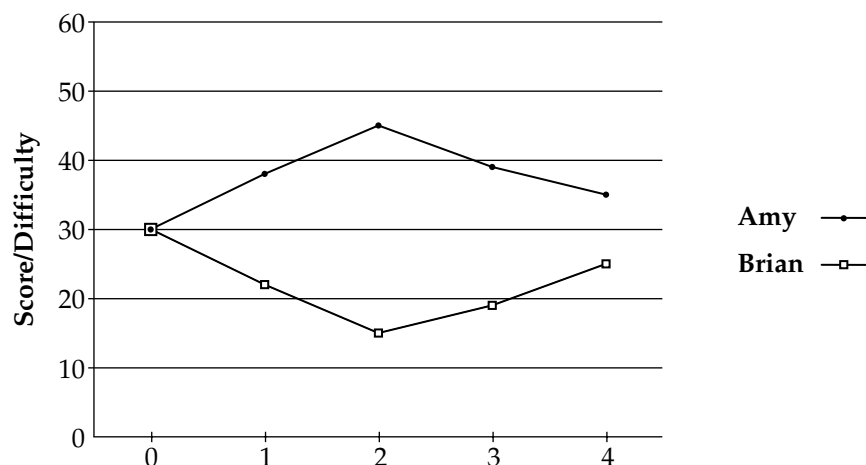
How does the computer determine your score? The GMAT is a CAT, or *computer adaptive test*. The operative word is *adaptive*. The level of difficulty of the test questions adapts to match your performance. In other words, when you answer a question correctly, the next question will be harder. When you answer a question incorrectly, the next question will be easier.

A CAT looks at several things to calculate your score for a section:

- Number of questions you answer correctly
- Difficulty of the questions you answer
- Number of questions you complete

When a section starts, the computer doesn't know anything about you, so it estimates that you have medium ability. You start with a medium score, and the first question is of medium difficulty. Every time you answer a question correctly, the computer raises your score and gives you a more difficult question. Every time you answer a question incorrectly, the computer lowers your score and gives you an easier question.

The computer recalculates your score after every question. The difficulty level of the next question generally matches your current score. However, the computer also has to meet certain requirements for the types of questions in a section. For example, the Math section has to have a balance of problem solving and data sufficiency questions and also has to have the proper mix of arithmetic, algebra, and geometry. Thus, the difficulty of a particular question may not exactly match your current score because these factors also affect the choice of question. In general, though, the question difficulty tracks your performance.



Look at the example in the graph above. Amy and Brian both take a four-question CAT section. Amy starts off well. She answers the first question correctly and her score increases. The difficulty of the questions is also increasing at this point. She gets the second question right, and her score and level of difficulty, increase again. The third question is even more difficult, and Amy gets the question wrong. What happens here is very important. Her next question will be easier than the last one, but not easier than the first one, because she has already “earned” a certain level of difficulty. For the same reason, her final score will drop from where it was, but will not drop from where she started. Amy gets the fourth question wrong. Her score will again decrease, as will her level of difficulty, but it will not drop below that of the first question.

Brian, however, starts by getting questions wrong. He answers the first question incorrectly, thereby lowering his score and decreasing the level of difficulty of the next question. He gets the second question wrong and continues to decrease the level of difficulty and his final score. The third question that Brian sees is easier than the questions he saw before, and he gets the question right. Again, what happens at this point is very important. His next question will be harder than the last one, but not harder than the first one. His score will increase from where he was, but not from where he started. Brian gets the fourth question right. His score will again increase, as will his level of difficulty, but it will not surpass that of the first question.

Amy and Brian each answer two questions correctly and two questions incorrectly. However, who ends up with the higher score? Amy does. Why? Except for the first question, each of the questions she answered was harder than those that Brian answered. Even though their performances might lead you to believe that their scores would end up the same, Amy will finish with a much higher score.

Because the level of difficulty of the questions that you answer is a factor in computing your final score, it is important that you achieve the highest level of difficulty you can early in the test. In effect, you must do well at the beginning of the test.

Earlier questions count more than later questions do.

No Skipping Allowed

On a paper-and-pencil test, you can skip a problem and return to it later. On a CAT, however, that's not possible. The CAT requires you to answer each question before moving on to the next one. Also, you cannot go back to a question once you have answered it.

Pacing

Knowing how a CAT works is the first step in developing a pacing plan. If you divide the time that you have to take the test by the number of questions on the exam, you will find that you have about 1.8 minutes per verbal question and 2 minutes per math question. However, because *all questions are not created equal* you should not spend the same amount of time on each question.

Proper pacing is essential to success on the GMAT. This course will cover many ways to help you pace the test in order to maximize your performance. Learning how to pace yourself is just as important as learning all of the material. The most important guidelines to remember are these:

1. Start slowly and carefully. Eliminate careless mistakes.
2. Gradually pick up speed so that you can finish the section.
3. Don't waste time on killer questions. Guess and move on.

As demonstrated in the example with Amy and Brian, how you perform at the beginning of the section greatly affects your final score. Your score can fluctuate dramatically depending on how many questions you get right or wrong. By the end of each section, however, the computer has already determined the possible range for your final score. Your score will fluctuate only within a narrow range. What does this mean in practical terms? The earlier questions are the most important, so slow down and do your best on them.

While the early questions carry the most weight, that doesn't mean you should focus solely on them. If it were the case that you could work the first half of the questions, guess on the second half, and get a great score, everybody would take that approach. To separate examinees into scoring levels, the GMAT confronts you with more questions than most people can comfortably complete in the time allowed. It rewards those who not only complete the early questions correctly, but also those who complete a greater number correctly. Expect to feel pressed for time when you take a test. Know that this is normal, and stay calm.

The test penalizes you if you do not answer every question in a section. Remember that the total number of questions answered affects your score. If you leave a question unanswered, you did not complete it, nor did you get it right. Therefore, the penalty for leaving any question blank is quite severe. Finish the section, even if it means that you have to guess randomly at the end of the section. Granted, if you randomly answer questions, your score will not be as high as that of someone who correctly answers those questions, but it will still be higher than if you left those same questions blank. You can (and many do) score in the 700s and still guess on a few questions.

If you answer questions correctly, the subsequent questions get increasingly harder. You are virtually guaranteed to see a few questions that you will not be able to solve. For these killer questions, don't waste your valuable time sitting and staring. Take your best guess and move on. Spend your time on questions that you can solve.

Do not leave anything blank.

Don't get stuck on killer questions!

You'll learn much more about constructing a pacing plan in class, and you'll refine your pacing strategy as you take practice tests.

Register for the GMAT

Register for the GMAT soon, since seats at the preferred times tend to fill up quickly. While individual needs vary, plan to take your exam no more than one month after your final Princeton Review class session. To register for the GMAT, call GMAC at 1-800-GMAT-NOW (1-800-462-8669) or register online at www.mba.com.

PRE-CLASS ASSIGNMENTS

ASSIGNMENT 1

SUBJECTS, VERBS, AND PRONOUNS

The test writers claim that GMAT sentence correction questions test your knowledge of grammar. They don't, at least not in any truly substantive way. Remember your grammar handbook from middle school? It was huge. The number of rules that the GMAT could possibly test is infinite. The good news is that the GMAT tests only a limited number of grammar rules.

We won't overwhelm you with unnecessary grammar terminology or make you spend hours diagramming sentences. We'll concentrate only on the grammar rules tested on the GMAT. In class, your instructor will discuss how the GMAT tests each rule and show you the best way to attack the questions.

In order to discuss the rules of grammar, you need to be familiar with a few concepts. This section reviews the building blocks of sentences and introduces some grammar rules related to subjects, verbs, and pronouns.

The GMAT tests only a few basic grammar rules.

SENTENCES

A **sentence** is a group of words that expresses a grammatically complete thought. The basic parts of the sentence are the main subject and main verb. Look at an example of a very simple sentence:

Julian reads.

The main subject is *Julian*, and the main verb is *reads*.

Subjects

The **main subject** is the noun that performs the main action. The subject can be a single word or a group of words. The examples below illustrate different types of subjects.

| | |
|-------------|--|
| Single noun | The <i>cloak</i> was made of velvet. |
| Gerund | <i>Sleeping</i> for eight hours a night is important to me. |
| Infinitive | <i>To learn</i> how to read music requires diligence. |
| Noun clause | <i>That the dog had bitten her</i> gave her reason to fear it. |

Verbs

Verbs express action, condition, or state of being. The **main verb** in a sentence is the word that expresses the main action.

PARTS OF SPEECH

Consider a more complex version of the first example:

Because he wants to improve his GMAT score, Julian very dutifully reads the extremely important pre-class assignments in this manual.

This example tells us much more about who Julian is, what he reads, how he reads, and why he reads. We'll use it to review the parts of speech.

Nouns

Nouns are people, places, and things. **Subject nouns**, like *Julian*, perform an action. **Object nouns**, like *assignments*, receive the action or are objects of prepositions (see below). *Assignments* is the object of the verb *reads*. *Manual* is the object of the preposition *in*. We will discuss the various kinds of nouns in more detail later.

Pronouns

Pronouns take the place of nouns and are used to avoid repetition. Like the nouns they replace, pronouns can function as either subjects or objects. In the sentence above, *he* is the subject of *wants*.

Modifiers

Modifiers describe, or modify, other words in a sentence. **Adjectives** describe nouns. In the sentence above, *important* and *pre-class* modify *assignments*. **Adverbs** modify verbs, adjectives, and other adverbs. They usually provide information about where, when, or how something happens. In the sentence above, *dutifully* modifies *reads*, *very* modifies *dutifully*, and *extremely* modifies *important*.

Prepositions

Prepositions are the little words that show relationships between other words or phrases. They usually create prepositional phrases that act as modifiers. In the sentence above, *in* is a preposition, and *in this manual* is a prepositional phrase that modifies *assignments*. *Manual* is the **object of the preposition**.

Conjunctions

Conjunctions connect words or parts of sentences. In the sentence above, *because* connects *he wants to improve his GMAT score* with *Julian very dutifully reads the extremely important pre-class assignments in this manual*.

PHRASES AND CLAUSES

Phrases and clauses are groups of words that act as modifiers or nouns. Let's examine the distinction between phrases and clauses.

Phrases

A **phrase** is a group of words that acts as a part of speech, not a complete sentence. **Modifying phrases** take on the role of adjectives or adverbs. The prepositional phrase *in this manual* acts as a modifier in our earlier example. It describes the location of the pre-class assignments. **Noun phrases** can be the subject or object of a sentence, as illustrated in the examples below.

Preparing for the GMAT requires hard work.

Allen wanted *to leave*.

In the first example, the phrase *preparing for the GMAT* is the subject. *To leave* is the object of the second example, and it tells us what Allen wanted.

Clauses

A **clause** is a group of words that has a subject and a verb. **Main clauses** can stand alone as complete sentences. **Dependent clauses** cannot stand alone as complete sentences. In the example above, *Julian very dutifully reads the extremely important pre-class assignments in this manual* is the main clause, and *Because he wants to improve his GMAT score* is the dependent clause. Clauses can also act as subjects, objects, or modifiers.

Conjunctions such as *and*, *but*, and *or* often link two independent clauses to create one sentence:

I went to the movies, and she went to the library.

Notice that each of the clauses expresses a complete thought and could stand alone as a complete sentence.

On the other hand, a dependent clause does not express a complete thought, even though it has a subject and a verb. A dependent clause functions as a noun, an adjective, or an adverb.

She is the woman *who was wearing the leopard-skin coat* last night.

I don't know *why you bothered to come to class* without your homework.

Notice that these clauses cannot stand alone as sentences; they depend on independent clauses to form grammatically complete thoughts. The first dependent clause functions as an adjective modifying *woman*. In the second example, the clause is the object of the verb *know*. Dependent clauses begin with *that*, *whether*, *if*, *because*, or the words we use to begin questions (*who*, *whom*, *whose*, *what*, *where*, *when*, *why*, *which*, *how*).

Quick Quiz: Subjects and Verbs

In the following sentences, circle the main subjects and underline the main verbs.

1. What you see is what you get.
2. The unexamined life is not worth living.
3. Commuting by bicycle helps people enjoy the benefits of fresh air and exercise.
4. His courage as a pilot of a U2 spy plane earned Gary Powers a posthumous citation.
5. Made from a single log, a dugout canoe draws very little water.
6. Felicia and Tim went to the same high school.

SUBJECT-VERB AGREEMENT

Subject-verb agreement is one grammar concept tested on the GMAT. The basic rule for subject-verb agreement is straightforward.

Singular subjects take singular verbs, and plural subjects take plural verbs.

Don't Be Fooled

Deciding whether a subject is singular or plural can sometimes be challenging.

Abstract Nouns

Some nouns describe a quality, idea, or state of being. These abstract nouns, such as *sadness*, *truth*, *laughter*, *poverty*, and *knowledge*, represent a single thing.

Abstract nouns are singular.

Justice always prevails.

Wealth is nice, but *happiness* is better.

Collective Nouns

Collective nouns name a group of things, animals, or people. The group has individual members, but it's a single entity.

Collective nouns are singular.

The *committee* votes on the budget this week.

The *school* of fish swims around the reef.

The *family* that just moved in next door is nice.

Verb Forms as Nouns

The *-ing* form of a verb (also known as the present participle) can be used as a noun, and in such cases it is called a **gerund**. When the *to* form of a verb acts like a noun, we call it an **infinitive noun**.

Gerunds and infinitive nouns are singular.

Walking is great exercise.

To err is human; *to forgive* is divine.

Nouns That End in -s

Most of the time, a noun that ends in -s is plural, but some singular nouns end in -s. If you're unsure whether a noun is singular or plural, ask yourself if it represents one thing or several things. (Hint: country names are always singular.)

That *species* has a number of interesting habits.

Economics is one of my favorite subjects.

The Netherlands is a country in Europe.

Not all nouns that end in -s are plural.

Singular Pronouns

Some singular pronouns, such as *everybody* or *no one*, are easy to mistake for plural pronouns. Even though we often treat these as plural words in everyday speech, the GMAT writers use more formal rules and define them as singular.

Each of the witnesses was questioned by the police.

Everyone in the senior class is sick with the flu.

Either of the restaurants is fine with me.

Any pronoun that ends in -body, -thing, or -one is singular.

In the first example, the subject is *Each*, not *witnesses*. *Witnesses* is the object of the preposition. Similarly, *Either* is the subject of the third example, not *restaurants*. When the subject of a sentence is a pronoun followed by a prepositional phrase, the pronoun is the main subject. Make sure the pronoun, not the object of the preposition, agrees with the verb. *Either* and *neither* are singular when they serve as the subject of a sentence.

These pronouns are singular and take singular verbs:

| | | |
|----------|-----------|------------|
| no one | nobody | nothing |
| someone | somebody | something |
| everyone | everybody | everything |
| anyone | anybody | anything |
| none | each | |

Compound Subjects

When a subject includes more than one noun, we call it a **compound subject**.

My best friend and her sister are very similar in personality.

When *and* joins two subjects, you must use a plural verb. However, compound subjects joined by *or*, *either* . . . *or*, and *neither* . . . *nor* follow a different rule. In these cases, the verb agrees with the noun closest to it.

Neither the bride nor *the groom* was able to remember the names of all the guests.

Neither Joe nor *his cousins* were happy on the first day of school.

The Number and A Number

When you refer to *a number* of things, you're talking about many things, and you need a plural verb. When you refer to *the number* of things, you're talking about one particular number, and you need a singular verb.

The number of bad movies showing this summer *is* unbelievable.

A number of my friends *are going* to the beach this weekend.

Quick Quiz: Subject-Verb Agreement

Circle the appropriate verb in the parentheses below.

1. Gloria and Calvin (*are, is*) no longer friends.
2. The number of times I have told you I do not want to go to the concert with you (*amaze, amazes*) me.
3. Samantha, in addition to Carrie, Charlotte, and Miranda, (*is, are*) going to the beach on Saturday.
4. Neither Mark nor his neighbors (*is, are*) able to open the doors to the patio.
5. Next month, Jack and Chrissy, along with Janet, (*is, are*) moving to the larger apartment upstairs.
6. Tom's family (*is, are*) considering whether there (*are, is*) any affordable places to go on vacation in Europe.
7. Each of the boys (*is, are*) overwhelmed by the amount of work to be done.
8. Every one of the golf balls (*has, have*) been hit into the sand trap.
9. This sandwich is the only one of all the sandwiches made at the deli that (*is, are*) inedible.
10. Skiing is an example of a sport that (*is, are*) best learned as a child.

Pronouns

Pronouns are words used in place of nouns, and they are usually used to avoid repetition. In the previous sentence, the word *they* replaces the word *pronouns*. Some pronouns function as subjects, while others function as objects. Another set of pronouns indicates possession.

| Subject Pronouns | Object Pronouns | Possessive Pronouns |
|------------------|-----------------|---------------------|
| I | me | my |
| you | you | your |
| he/she/it | him/her/it | his/hers/its |
| we | us | our |
| they | them | their |
| who | whom | whose |

Pronoun Agreement

Read the following paragraph and underline all the pronouns:

This weekend, Matt is throwing a party to celebrate his birthday. He has invited many friends and family. Matt's sister, Teresa, is bringing the cake. She has promised him that it will be chocolate.

Each pronoun agrees with the noun it replaces. *He*, *his*, and *him* all refer to *Matt*. *She* refers to *Teresa*. *It* replaces *cake*. Just as subjects and verbs must agree, pronouns must agree in number with the nouns they replace. The noun a pronoun replaces is called the **antecedent**.

Use a singular pronoun to replace a singular noun, and use a plural pronoun to replace a plural noun.

The same types of nouns that make subject-verb agreement tricky can cause problems for pronoun agreement. Look at the following examples:

Everyone should do *their* homework.

The golden retriever is one of the smartest breeds of dogs, but *they* cannot do your GMAT homework for you.

In both examples, the use of *their* or *they* is incorrect. Since *everyone* is singular, it must be paired with a singular pronoun, such as *his* or *her*. Either *his* or *her* would be considered correct in this case since we have no information about gender. In the second example, *they* refers to *the golden retriever*. Since the antecedent *retriever* is singular, the pronoun *it* should be used in the second clause.

Pronoun Ambiguity

In addition to agreeing with the noun it replaces, a pronoun must clearly refer to only one noun. If the pronoun could conceivably refer to more than one noun, you face the problem of pronoun ambiguity.

Pronouns must unambiguously refer to a single noun.

Look at the following example:

Lisa Marie was supposed to meet Jen at the museum at eleven, but *she* was late.

It is unclear to whom *she* refers, Lisa Marie or Jen. The sentence should be rewritten to clear up the confusion. If Jen were the one who arrived late, the corrected sentence would read:

Lisa Marie was supposed to meet Jen at the museum at eleven, but Jen was late.

Pronoun Consistency

Pronoun usage must also be consistent. Look at the following example:

One should watch *your* purse on a crowded subway.

The sentence seems to be saying that some unknown person should be on the lookout for your purse on a crowded subway. For the purposes of the GMAT, sentences that refer to an undefined person can use either *you* or *one*, as long as a single pronoun is used consistently. The example above could be rewritten two different ways:

One should watch *one's* purse on a crowded subway.

You should watch *your* purse on a crowded subway.

Quick Quiz: Pronoun Agreement

Correct the pronoun agreement in the following sentences. Not all sentences have an error.

1. Each of the chefs makes their own special dish.
2. I still keep my diary and scrapbooks from childhood because they remind me of my youth.
3. A student must see their advisor before turning in his thesis.
4. The person who stole my bicycle is a thief.
5. One should always look where he is going when you cross the street.
6. In 1980, the Netherlands agreed to limit fishing in certain Atlantic Ocean beds, but in 1981, they terminated the agreement.
7. The flock of seagulls flew overhead before it swooped down and settled on the water.

Who vs. Whom

These two words cause lots of trouble, but they shouldn't. If you're confronted with a choice between *who* and *whom*, look at the role played by the pronoun in the sentence. If the pronoun is performing an action, it's the subject, and you should use *who*. If the pronoun is acted upon by the verb or follows a preposition, it's an object, and you should use *whom*.

The detective knows *who* committed the murder.

To *whom* should I speak about the matter?

If you find yourself struggling with the choice between *who* and *whom*, try replacing the pronoun with *she* or *her*. If a sentence should use *she*, use *who*. If a sentence should use *her*, choose *whom*. For example, it would be correct to say, "The detective knows *she* committed the murder," because *she* is a subject pronoun. Similarly, it would be correct to say, "I should speak to *her* about the matter." The first example needs a subject pronoun, and the second example needs an object pronoun.

VERB TENSES

Verb tense places an action in time, and the basic tenses are past, present, and future. The examples below illustrate the tenses you'll encounter in GMAT sentences.

| | |
|----------------|--|
| Present | <i>I study, I am studying, I have studied</i> |
| Past | <i>I studied, I had studied, I was studying</i> |
| Future | <i>I will study, I will be studying, I will have studied</i> |

The variations within the basic categories of past, present, and future allow us to express ideas more precisely. If a tense uses a helper verb, such as a form of *to be* or *to have*, use the helper verb to determine the tense. For example, *I was walking* is in the past tense because *was* denotes the past. *I am walking* is in the present because *am* denotes the present. Let's look at the variations in more detail. Knowing the names of the tenses isn't necessary, but you need to be able to classify them as past, present, or future.

Present

The **simple present** expresses a habitual action, a fact, or something that is happening now.

Beth *runs* three miles every morning.

Both baseball games *are* on television right now.

When you want to describe something that's in progress right now, use the **present progressive** tense. (It's sometimes called the present continuous tense.) Present progressive uses a form of "to be" followed by the *-ing* form of the verb (also known as the present participle).

The kids on the playground *are laughing* loudly.

The **present perfect** tense describes an action that started at an indefinite time in the past and either continues into the present or has just been completed. This tense uses the helping verb *has* or *have* followed by the past participle.

Astrid *has read* a book a week since she was twelve.

I *have never been* to Spain.

Past

The **simple past** indicates a completed action or condition.

I *wrote* my final paper over the weekend.

Use **past progressive** to describe an action that was ongoing in the past.

We *were sleeping* when the fire alarm went off.

Use the **past perfect** when you want to make it clear that one action in the past happened before another. This tense requires the helping verb *had*.

Before she began college last fall, she *had never been* more than twenty miles from home.

In the example above, the past perfect action was cut off by an intervening event in the more recent past. The past perfect cannot stand alone as the only verb in a sentence.

Future

Simple future, as you would expect, describes an action that will take place in the future. This tense requires the use of the helper verb *will*.

I *will clean* my room tomorrow.

Use **future progressive** to describe an ongoing action that takes place in the future. The tense is formed by using the future form of the helping verb *to be* plus the *-ing* form of a verb.

I *will be cleaning* my room when you arrive.

Use **future perfect** to indicate an action that will be completed by a specified time in the future.

We *will not have finished* dinner by the time you arrive.

Quick Quiz: Verb Tense

Choose the correct verb tense in the parentheses for each sentence below.

1. Yesterday afternoon, clouds rolled in, the sky grew ominous, and thunder (*was, is*) heard in the distance.
2. Before the union leadership even began salary negotiations, it (*had made, made*) up its mind to stand firm in its position.
3. The Boy Scouts (*love, loved*) their new clubhouse, which they built last summer.
4. My new co-workers (*had been, were*) very friendly to me until they learned my salary was considerably higher than theirs.
5. Roberts already (*finished, had finished*) the experiments by the time Fuller made the discovery in his own laboratory.
6. Since 1980, several economies in developed nations (*are experiencing, have experienced*) declines and recoveries.
7. The belief in vampires (*was first recorded, had first been recorded*) in the early fifteenth century.
8. By the time Spanish explorers first encountered them, the Aztecs (*have developed, had developed*) the calendar.
9. Unlike the brown sparrow, the passenger pigeon (*was slaughtered indiscriminately, had been slaughtered indiscriminately*) and became extinct in 1914.

SUMMARY

Subjects and Verbs

- Singular subjects take singular verbs, and plural subjects take plural verbs.
- Abstract nouns, collective nouns, and verb forms acting as nouns are singular.
- Pronouns that end in *-body*, *-one*, or *-thing* are singular.
- *The number* is singular. *A number* is plural.

Pronouns

- Pronouns must agree in number with the nouns they replace.
- Pronouns must unambiguously refer to only one noun.
- *Who* is a subject pronoun. *Whom* is an object pronoun.

Tense

- The basic tenses are past, present, and future.
- Sentences should stay in one tense unless the action takes place at two different times.

ANSWERS AND EXPLANATIONS

Quick Quiz: Subjects and Verbs

The subjects are in italicized text, and the verbs are underlined.

1. *What you see* is what you get.
2. The unexamined *life* is not worth living.
3. *Commuting* by bicycle helps people enjoy the benefits of fresh air and exercise.
4. His *courage* as a pilot of a U2 spy plane earned Gary Powers a posthumous citation.
5. Made from a single log, a dugout *canoe* draws very little water.
6. *Felicia and Tim* went to the same high school.

Quick Quiz: Subject-Verb Agreement

The subjects are in italicized text, and the verbs are underlined.

1. *Gloria and Calvin* are no longer friends.
2. *The number* of times I have told you I do not want to go to the concert with you amazes me.
3. *Samantha*, in addition to Carrie, Charlotte, and Miranda, is going to the beach on Saturday.
4. Neither Mark nor his *neighbors* are able to open the doors to the patio.
5. Next month, *Jack and Chrissy*, along with Janet, are moving to the larger apartment upstairs.
6. Tom's *family* is considering whether there are any affordable *places* to go on vacation in Europe.
7. *Each* of the boys is overwhelmed by the amount of work to be done.
8. Every *one* of the golf balls has been hit into the sand trap.
9. This *sandwich* is the only one of all the sandwiches made at the deli that is inedible.
10. Skiing is an example of a *sport* that is best learned as a child.

Quick Quiz: Pronoun Agreement

If a correction was needed, the original pronoun has been crossed out and replaced with the correct pronoun.

1. Each of the chefs makes ~~their~~ *his* own special dish. The pronoun here refers back to the noun *each*. You could also use *her*.
2. I still keep my diary and scrapbooks from childhood because *they* remind me of my youth. No error. *They* replaces both *diary and scrapbooks*.
3. A student must see ~~their~~ *his* advisor before turning in his thesis. The student must be male, because it is *his* thesis, therefore it must also be *his* advisor.

4. The person *who* stole my bicycle is a thief. No error. *Who* is the subject of the clause *who stole my bicycle* and is used correctly.
5. One should always look where ~~he~~ *one* is going when ~~you~~ *one* crosses the street. Be consistent.
6. In 1980, the Netherlands agreed to limit fishing in certain Atlantic Ocean beds, but in 1981, ~~they~~ *it* terminated the agreement. The Netherlands is a single country.
7. The flock of seagulls flew overhead before *it* swooped down and settled on the water. No error. *It* agrees with the noun *flock*.

Quick Quiz: Verb Tense

The correct verb is underlined.

1. Yesterday afternoon clouds rolled in, the sky grew ominous, and thunder was heard in the distance. There is no reason to switch verb tense, and all the other verbs (*rolled*, *grew*) are in the past tense.
2. Before the union leadership even began salary negotiations, it had made up its mind to stand firm in its position. Past perfect is the correct tense here because, while both actions occurred in the past, one action (*had made*) occurred before the other.
3. The Boy Scouts love their new clubhouse, which they built themselves last summer. Presumably they still love their clubhouse, so it's okay to switch from the past tense to the present tense.
4. My new co-workers had been friendly to me until they learned my salary was considerably higher than theirs. Past perfect is the best tense here because both events happened in the past, but one happened before the other.
5. Roberts already had finished the experiments by the time Fuller made the discovery in his own laboratory. Past perfect is the best tense here because both events happened in the past, but one happened before the other.
6. Since 1980, several economies in developed nations have experienced declines and recoveries. Present perfect is the best tense because the declines began in the past and continue into the present.
7. The belief in vampires was first recorded in the early fifteenth century. The simple past is best here because the sentence describes an action in the past that has been completed.
8. By the time Spanish explorers first encountered them, the Aztecs had developed the calendar. Past perfect is the best tense here because both events happened in the past, but one happened before the other.
9. Unlike the brown sparrow, the passenger pigeon was slaughtered indiscriminately and became extinct in 1914. The simple past is best here because the sentence describes an action in the past that has been completed.

MATH FUNDAMENTALS

The math portion of the GMAT tests nothing but “basic” math skills, which sounds easy enough until you realize that this is exactly how ETS hopes to get you—that is, by testing you on terms and concepts that you haven’t dealt with since high school.

MATH VOCABULARY

In order to beat ETS at its own game, you need to make sure that you understand several core concepts and terms. Many questions on the GMAT are unanswerable unless you know what these terms mean.

Do you know these terms?

Fill in as much of the chart as you can before turning the page to check the answers.

| Term | Definition | Examples |
|----------------|------------|----------|
| Integer | | |
| Positive | | |
| Negative | | |
| Even | | |
| Odd | | |
| Sum | | |
| Difference | | |
| Product | | |
| Divisor | | |
| Dividend | | |
| Quotient | | |
| Prime | | |
| Consecutive | | |
| Digits | | |
| Distinct | | |
| Absolute Value | | |

| Term | Definition | Examples |
|----------------|---|--|
| Integer | A whole number that does not contain decimals, fractions, or radicals. Integers can be negative, positive, or 0. | -500, 0, 1, 28 |
| Positive | Greater than 0 | 0.5, 25, $\frac{5}{3}$ |
| Negative | Less than 0 | -72.3, $-\frac{7}{4}$, -2 |
| Even | An integer that is divisible by 2 | -40, 0, 2 |
| Odd | An integer that is not divisible by 2 | -41, 1, 3 |
| Sum | The result of addition | The sum of 3 and 4 is 7. |
| Difference | The result of subtraction | The difference of 7 and 2 is 5. |
| Product | The result of multiplication | The product of 2 and 7 is 14. |
| Divisor | The number you are dividing by | $8 \div 2 = 4$ (2 is the divisor.) |
| Dividend | The number you are dividing into | $8 \div 2 = 4$ (8 is the dividend.) |
| Quotient | The result of division | $8 \div 2 = 4$ (4 is the quotient.) |
| Prime | A number that is divisible only by itself and 1. Negative numbers, 0, and 1 are NOT prime. | 2, 3, 5, 7, 11 |
| Consecutive | In order, not necessarily ascending | -1, 0, 1 or 10, 9, 8 |
| Digits | 0-9; the numbers on the phone pad | 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 |
| Distinct | Different | 2 and 3 are distinct; 4 and 4 are not distinct |
| Absolute Value | The distance from 0 on a number line. The absolute value is always positive. the symbol " $ $ " means absolute value. | $ 4 = 4$; $ -4 = 4$ |

Use the definitions to solve the problem below.

1. If x and y are distinct negative integers greater than -10 , what is the greatest possible product of x and y ?

- ☐ 2
☐ 4
☐ 72
☐ 81
☐ 90

First you have to figure out which numbers fit the definitions for *distinct*, *negative*, *integer*, and *greater than -10* . The only numbers that work are $-9, -8, -7, -6, -5, -4, -3, -2$, and -1 . The question asks for the product, the result of multiplying x and y . A negative times a negative is positive, so any two of these numbers multiplied together will be positive. That means that the greatest possible value of xy is -9×-8 , or 72. The answer is (C).

Quick Quiz: Positive/Negative and Even/Odd

Do you know the rules of positive/negative and even/odd? (Answers to all questions are found at the end of each chapter.) Circle one:

1. Negative \times or \div negative = positive/negative
2. Positive \times or \div positive = positive/negative
3. Negative \times or \div positive = positive/negative
4. Even \times even = even/odd
5. Odd \times odd = even/odd
6. Even \times odd = even/odd
7. Even $+$ or $-$ even = even/odd
8. Odd $+$ or $-$ odd = even/odd
9. Even $+$ or $-$ odd = even/odd

Factors and Multiples

A **factor** is a positive integer that divides evenly into another positive integer. The factors of 12 are 1 and 12, 2 and 6, and 3 and 4. You can also think of factors as the numbers you multiply together to get a product.

A **multiple** is the product of some positive integer and any other positive integer. For example, the multiples of 12 are 12, 24, 36, 48, 60. . .

The largest factor of any number, and the smallest multiple of any number, is always the number itself.

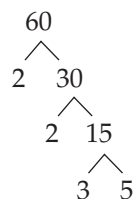
Let's try another question:

2. What is the sum of the distinct, prime factors of 60?

- ☐ 9
☐ 10
☐ 11
☐ 12
☐ 30

A number has *few factors*,
but *many multiples*.

A **prime factor** is a factor that is also a prime number. The factor tree is a great tool for figuring out the **prime factorization** of a number.



Thus, $60 = 2 \cdot 2 \cdot 3 \cdot 5$, the distinct prime factors of 60 are 2, 3, and 5, and (B) is the answer.

Rules of Divisibility

Sometimes it's helpful to know whether one number is divisible by (i.e., a factor or divisor of) another. Learning these rules will save you precious time on the test.

| A number is divisible by | Rule | Center |
|--------------------------|--|---|
| 2 | It's even (i.e., its last digit is even) | 1,57 <u>6</u> ✓ |
| 3 | Its digits add up to a multiple of 3 | 8,532 $8 + 5 + 3 + 2 = 18$ ✓ |
| 4 | Its last two digits are divisible by 4 | 121, <u>532</u> $32 \div 4 = 8$ ✓ |
| 5 | Its last digit is 5 or 0 | 568,74 <u>5</u> ✓ 32 <u>0</u> ✓ |
| 6 | Apply the rules of 2 and 3 | 55,740 It's even and $5 + 5 + 7 + 4 + 0 = 21$ ✓✓ |
| 9 | Its digits add up to a multiple of 9 | 235,692 $2 + 3 + 5 + 6 + 9 + 2 = 27$ ✓ |
| 10 | Its last digit is zero | 11,13 <u>0</u> ✓ |
| 12 | Apply the rules of 3 and 4 | 3,552 $3 + 5 + 5 + 2 = 15$ and $52 \div 4 = 13$ ✓✓ |

Let's try a problem.

3. If x is an integer divisible by 15 but not divisible by 20, then x CANNOT be divisible by which of the following?

- ☐ 6
☐ 10
☐ 12
☐ 30
☐ 150

If x is divisible by (i.e., a multiple of) 15 but not 20, then x could be, for instance, 15, 30, or 45, but not 60. If a number is divisible by 15, then it is also divisible by factors of 15, i.e. 3 and 5. Just looking at the rule of divisibility for 3, (A), (B), and (D) are out. Let's look at answer choice (E): 150 is divisible by 15 but not 20; thus x could be 150 and (E) is out. So by POE, the answer must be (C).

Quick Quiz: Math Vocabulary

Fill in the ovals for all the terms that must be true.

| | Integer | Even | Odd | Positive | Negative | Prime |
|--|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| -1 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 0 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 1 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 2 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 3 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 0.5 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $\sqrt{3}$ | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 0.001 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 51 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Even Positive \times Odd Negative = | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Even Negative \times Even Negative = | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Odd Negative \times Even Positive = | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Odd Positive \times Odd Positive = | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

Order of Operations

Sometimes you're given a long, ugly arithmetic equation to solve, and you have to know the correct order of operations. Does "Please Excuse My Dear Aunt Sally" ring a bell? What exactly does PEMDAS stand for anyway?

PEMDAS

P stands for *parentheses*. Solve expressions in parentheses first.

E stands for *exponents*. Solve expressions with exponents next.

M stands for *multiplication*, and **D** stands for *division*.

Do all the multiplication and division together in the same step, going from left to right.

A stands for *addition*, and **S** stands for *subtraction*.

Do all the addition and subtraction together in the same step, going from left to right.

Quick Quiz: Order of Operations

Solve the following:

1. $17 - 11 \times 3 + 9 =$
2. $(17 - 11) \times (3 + 9) =$
3. $-5(3 - 7)^2 + 94 =$
4. $54 \div 6 \times 3 - 3(5 - 13) =$
5. $54 \div (6 \times 3) - 3(5 - 13)^2 =$

SOLVING EQUATIONS AND INEQUALITIES

You'll learn a lot about how to avoid most algebra in the coming weeks, but you should still know a little about variables and equations. A variable is a letter, such as x , that represents an unknown amount.

Equations with One Variable

The simplest equations have no exponents and are called **linear equations**. Let's try an example.

$$\text{If } 3x + \frac{x}{2} + 7 = 21, \text{ then } x =$$

To solve this equation, we need to get the variable by itself on one side of the equation. Since the fraction is awkward, let's begin by multiplying each term on both sides of the equation by 2:

$$2(3x + \frac{x}{2} + 7) = (21)2$$

That gives us:

$$6x + x + 14 = 42$$

When we combine the x 's, we get:

$$7x + 14 = 42$$

To isolate the variable, we need to subtract 14 from each side:

$$7x + 14 - 14 = 42 - 14$$

That leaves:

$$7x = 28$$

Finally, we can divide both sides by 7 to solve for x :

$$\frac{7x}{7} = \frac{28}{7}$$

Now we know that $x = 4$.

Do the same thing to both sides of an equation.

To solve any linear equation with one variable, isolate the variable on one side of the equation and the numbers on the other side. The process may require multiplication, division, subtraction, or addition of different numbers.

Try this one on your own before you read the explanation.

1. If $\frac{5}{3} = \frac{a}{2}$ and $b = \frac{7}{3}$, then $a + 2b =$

☐ $\frac{10}{3}$

☐ $\frac{14}{3}$

☐ 8

☐ 9

☐ 14

In this question we're asked to find the value of $a + 2b$. The question gives us the value of b . However, we have to solve the first equation in order to find the value of a . Solve by cross multiplying. Simply multiply opposing numerators and denominators, and set them equal to one another:

$$3 \times a = 5 \times 2$$

$$3a = 10$$

$$a = \frac{10}{3}$$

Now, we have enough information to answer the question. All we have to do is perform a substitution. Simply substitute the values of a and b into $a + 2b$. Since $\frac{10}{3} + 2(\frac{7}{3}) = \frac{24}{3} = 8$, the answer is (C).

Inequalities

Let's review symbols that describe inequalities:

- \neq means is not equal to
- $>$ means is greater than
- $<$ means is less than
- \geq means is greater than or equal to
- \leq means is less than or equal to

Flip the inequality sign when you multiply or divide by a negative number.

Solve single-variable inequalities in exactly the same way as you do single-variable linear equations, with one additional rule: If you multiply or divide an inequality by a negative number, you must flip the inequality sign.

Let's work an example:

$$25 < -7x + 4 \leq 60$$

To find the possible values for x , we need to isolate the variable. Begin by subtracting 4 from each part of the inequality:

$$25 - 4 < -7x + 4 - 4 \leq 60 - 4$$

That gives us:

$$21 < -7x \leq 56$$

To isolate x , divide each part of the inequality by -7 :

$$\frac{21}{-7} < \frac{-7x}{-7} \leq \frac{56}{-7}$$

Because we're dividing by a negative number, we need to flip the inequality signs, which leaves us with:

$$-3 > x \geq -8$$

The solution tells us that x is greater than or equal to -8 and less than -3 .

Simultaneous Equations

Some GMAT problems involve sets of equations with more than one variable. These are known as simultaneous equations.

To solve multiple equations, add or subtract the equations so that one of the variables (preferably the one you don't need) cancels out, leaving you with a one-variable equation. Let's look at an example:

If $2x - 3y = 14$ and $x + 3y = 4$, what is the value of y ?

Begin by stacking the equations. If we add the equations, the quantity $3y$ drops out of the sum.

$$\begin{array}{r} 2x - 3y = 14 \\ x + 3y = 4 \\ \hline 3x = 18 \\ x = 6 \end{array}$$

Now plug $x = 6$ into either equation to get $y = -\frac{2}{3}$.

You must have at least as many distinct linear equations as you have variables in order to solve for all the variables.

The rule above does not by itself tell you how to solve for the variables, but it does tell you whether you have enough information to solve for each of the variables, which is very useful for working data sufficiency questions. Consider the equations below:

$$\begin{array}{l} x + 7y = 24 \\ 3x + 3y = 18 \end{array}$$

Do you have enough information to solve for x and y ? Yes. We have two equations with the same two variables. Consider another set of equations:

$$\begin{array}{l} z + 7y = 24 \\ 3x + 3y = 18 \end{array}$$

Do you have enough information to solve for x and y ? In this case, we don't have enough information to solve for all the variables because there are 3 variables but only 2 equations. Try one more set of equations:

$$\begin{array}{l} x + 3y = 24 \\ 72 - 9y = 3x \end{array}$$

Do you have enough information to solve for x and y ? If you subtract $3y$ from both sides of the first equation, and then multiply both sides by 3, you will get $3x = 72 - 9y$, which is the equivalent of the second equation above. Thus, you don't have two distinct equations and cannot solve for the variables.

Two equations are not distinct if one is a multiple of the other.

TRANSLATING FROM ENGLISH TO MATH

The GMAT tests more than your ability to perform calculations and solve equations. You will frequently encounter word problems that test your ability to translate between English and math. It's easy to become confused by long problems with confusing phrasing. Avoid this trap by translating word problems in bite-sized pieces.

Let's look at a sample expression:

The product of x and y is five less than four times as much as one-third of z .

Taken as a whole, the sentence might make your head spin. Let's take it one piece at a time. A product is the result of multiplication, so *the product of x and y* means xy . *Is* means equals, so now we know:

$xy =$ five less than four times as much as one-third of z .

One-third of z means $\frac{1}{3}z$. *Four times as much as* means multiply by four. Adding the new information, we get:

$$xy = \text{five less than } 4\left(\frac{1}{3}z\right)$$

That leaves only *five less than* to translate. *Less than* tells us to subtract, so we need to subtract five from the last part of the expression. Now we have the equation:

$$xy = 4\left(\frac{1}{3}z\right) - 5$$

Use the table below as a guide to translating some common words and phrases.

| English | Math |
|--|-----------------------------|
| More than, greater than, sum of | Addition (+) |
| Less than, fewer than, difference between | Subtraction (-) |
| Times as many/much as, times more than, of, the product of | Multiplication (\times) |
| Goes into, divided by, quotient of | Division (\div) |
| Is, are, was, were equals, the same as | Equals (=) |

Quick Quiz: Translation

Translate the following expressions into their mathematical equivalents.

1. 5 more than a .
2. x is 18 more than y .
3. Dan has 9 fewer pencils than Jeff does.
4. w is 8 less than p .
5. Half of x is 10.
6. The number of boys in the class is one-third the number of girls.
7. k is three times greater than l .
8. The shirt cost five times as much as the pants.
9. Twice a certain number is equal to that number minus 10.
10. A certain number divided by three is equal to 9 more than that number.

SUMMARY

This chapter reviewed a lot of terms and rules. If you're having any difficulty remembering them, make a set of math flashcards. Work with the cards until you know the rules backward and forward. In the coming weeks, you can add to your flashcards as you encounter additional math concepts.

Math Vocabulary

- Learn the math terms presented in the lesson.
- Know the rules of positive/negative, even/odd, divisibility, and order of operations.
- Use a factor tree to find the prime factorization of a number.

Solving Equations and Inequalities

- Linear equations are equations with no exponents.
- Solve linear equations and inequalities by isolating the variable. Any operations performed on one side of an equation or inequality must also be performed on the other side.
- If you multiply or divide by a negative number, you must flip the inequality sign.
- When you work with simultaneous equations, you must have at least as many distinct equations as you have variables in order to solve for all the variables. Equations that are multiples of one another are not distinct.

Translating from English to Math

- Translate word problems in bite-sized pieces.

DRILL

1. The product of two integers is 36 and their sum is 13. What is the positive difference between the two numbers?

☐ 1
☐ 4
☐ 5
☐ 7
☐ 9

2. If r is the remainder when 15 is divided by 4, what is the remainder when 17 is divided by r ?

☐ 0
☐ 1
☐ 2
☐ 3
☐ 4

3. What is the sum of the distinct prime numbers between 50 and 60?

☐ 104
☐ 108
☐ 110
☐ 112
☐ 116

4. If $\frac{2}{5}$ of x is 8, then what is $\frac{1}{4}$ of x ?

☐ 2
☐ 3.2
☐ 4
☐ 5
☐ 16

5. $a = (b - 2)x + 2$

In the equation above, x is a constant. If $a = 14$ when $b = 5$, what is the value of a when $b = 7$?

☐ -84
☐ -22
☐ 10
☐ 22
☐ 30

6. If $2x$ is 12 less than the sum of $6x$ and $4y$, then $x + y =$

☐ 1
☐ 2
☐ 3
☐ 6
☐ 9

7. If $\frac{7-2y}{3} < -5$, then

☐ $y < -4$
☐ $y < 11$
☐ $y > -4$
☐ $y > 4$
☐ $y > 11$

8. If $4r + 3s = 7$, $2r + s = 1$, and $2r + 2s = t - 4$, what is the value of t ?

☐ 6
☐ 8
☐ 10
☐ 12
☐ It cannot be determined from the information given.

9. How many integers between 1 and 200, inclusive, are divisible by both 3 and 4?

☐ 8
☐ 12
☐ 15
☐ 16
☐ 24

10. How many three-digit integers between 310 and 400, exclusive, are divisible by 3 when the tens digit and the hundreds digit are switched?

☐ 3
☐ 19
☐ 22
☐ 30
☐ 90

ANSWERS AND EXPLANATIONS**Quick Quiz: Positive/Negative and Even/Odd**

(circle one)

1. Negative \times or \div negative = ~~positive~~/negative
2. Positive \times or \div positive = ~~positive~~/negative
3. Negative \times or \div positive = positive/~~negative~~
4. Even \times even = ~~even~~/odd
5. Odd \times odd = even/~~odd~~
6. Even \times odd = ~~even~~/odd
7. Even $+$ or $-$ even = ~~even~~/odd
8. Odd $+$ or $-$ odd = ~~even~~/odd
9. Even $+$ or $-$ odd = even/~~odd~~

Quick Quiz: Math Vocabulary

Fill in the ovals for all the terms that must be true.

| | Integer | Even | Odd | Positive | Negative | Prime |
|--|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| -1 | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> |
| 0 | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 1 | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 2 | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> |
| 3 | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> |
| 0.5 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $\sqrt{3}$ | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 0.001 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 51 | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Even Positive \times Odd Negative = | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> |
| Even Negative \times Even Negative = | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| Odd Negative \times Even Positive = | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> |
| Odd Positive \times Odd Positive = | <input checked="" type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |

Quick Quiz: Order of Operations

Solve the following:

1. $17 - 11 \times 3 + 9 = -7$
2. $(17 - 11) \times (3 + 9) = 72$
3. $-5(3 - 7)^2 + 94 = 14$
4. $54 \div 6 \times 3 - 3(5 - 13) = 51$
5. $54 \div (6 \times 3) - 3(5 - 13)^2 = -189$

Quick Quiz: Translation

1. $a + 5$
2. $x = y + 18$
3. $D = J - 9$
4. $w = p - 8$
5. $\frac{1}{2}x = 10$
6. $b = \frac{1}{3}g$
7. $k = 3l$
8. $s = 5p$
9. $2x = x - 10$
10. $\frac{x}{3} = x + 9$

Drill

1. **C** Let's work through this systematically. "Product" means the result of multiplication and "sum" means the result of addition, so we need to find two numbers that give you 36 when you multiply them and 13 when you add them: $1 \times 36 = 36$, but $1 + 36$ is not 13; $2 \times 18 = 36$, but $2 + 18$ is not 13; $3 \times 12 = 36$, but $3 + 12$ is not 13; $4 \times 9 = 36$, and $4 + 9 = 13$, so 4 and 9 are the numbers we're looking for. "Difference" means the result of subtraction: $9 - 4 = 5$.
2. **C** The remainder is what's left over after you divide: $15 \div 4 = 3$ remainder 3. So $r = 3$, and $17 \div 3 = 5$ remainder 2, so the answer is 2.
3. **D** Every prime number other than 2 must be odd, so let's list the odd numbers between 50 and 60: 51, 53, 55, 57, 59. Which of these numbers are prime? Since a prime number is a number only divisible by 1 and itself, we need to figure out whether any of these numbers are divisible by a number other than 1 and itself. Think about small numbers. Obviously, none of these numbers is divisible by 2, but are any of them divisible by 3? (Remember your divisibility rule for 3: sum of the digits is divisible by 3.) 51 is divisible by 3 because the sum of the digits is 6, and 57 is divisible by 3 because the sum of the digits is 12. Therefore, 51 and 57 are not prime. Looking at the remaining numbers, 55 is divisible by 5 because it ends in 5, so 55 is not prime either. That only leaves 53 and 59, and the sum of those numbers is 112, leaving us with (D).
4. **D** Translate and solve for x . If $\frac{2}{5} \cdot x = 8$, then $x = 8 \cdot \frac{5}{2} = 20$. Thus $\frac{1}{4} \cdot 20 = 5$.
5. **D** Since x is a constant, it always represents the same value. Plug in the values given for the other variables and solve for x . Thus you get $14 = (5 - 2)x + 2$, so $14 - 2 = 3x$, and $x = 4$. Now plug that into your equation for x , and plug in your new value for b and you get $a = (7 - 2)4 + 2$, so $a = 22$.
6. **C** Let's translate the English into math. If $2x$ is 12 less than the sum of $6x$ and $4y$, that means $2x = 6x + 4y - 12$. Putting all our variables on one side, we get $12 = 4x + 4y$, so $x + y = 3$.
7. **E** Multiply both sides by 3, and you get $7 - 2y < -15$. Subtract 7 from both sides to isolate the variable ($-2y < -22$). Finally, divide both sides by -2 , and you have $y > 11$. Don't forget to flip the inequality sign when you multiply or divide by a negative number.
8. **C** To isolate t , we need to get rid of r and s . Stack the equations and multiply every term in the second and third equations by -1 . Then, add the equations. That gives us:

$$\begin{array}{r} 4r + 3s = 7 \\ -2r - s = -1 \\ -2r - 2s = -t + 4 \\ \hline 0 = -t + 10 \end{array}$$
 Thus, t must be 10.
9. **D** You can count out the multiples of 12 (especially if it's early in the section and accuracy is crucial): 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, and 192. Or, divide 200 by 12 (it goes in 16 times with a remainder of 2).
10. **D** This one is somewhat sadistic because that last part of the question is a red herring. Since the sum of the digits dictates whether a number is divisible by 3, the order of the digits doesn't matter. Once you get past that, the only answer choice that comes close is 30 because there are 89 numbers between 310 and 400 and every third one must be a multiple of 3.

ADMISSIONS INSIGHT

The Princeton Review and PrincetonReview.com can help you with the business school admission process.

Finding the Right Schools

If you don't already have a list of potential business schools in mind, it's not too early to start browsing. Start thinking about where you might want to apply before you finish the course so you'll be ready to go when your scores come in.

The first step in finding the best business school fit is to determine the direction you hope to take after graduation. Business school programs vary widely in their offerings and strengths. Pinpointing the exact reason for choosing a career in business and a specific field of interest will help in selecting a school. Look for schools that offer a broad-based curriculum if you haven't decided on a particular field.

Choosing a particular path also affects your candidacy; admissions committees favor applicants who have clear goals and objectives. Moreover, once at school, students who know what they want make the most of their two years. Without a targeted direction, opportunities for career development—such as networking, mentoring, student clubs, and recruiter events—are squandered.

Begin researching business schools now on **PrincetonReview.com**. Using our powerful search tools online, the business school search can be narrowed down to those schools that best meet your priorities and interests. Our **Advanced Business School Search** tool provides a list of schools that match indicated needs and preferences. Use it to find a business school, compare top choice schools, or browse an alphabetical list of b-schools across the U.S. Our **School Match** tool is a chance to actually connect with schools. Once you complete the information section—academic record, work history, priorities in school, etc.—interested schools can contact you. Keep an open mind; a smaller or less familiar school may have just the right atmosphere, specialization, or career network.

Our **School Profiles** produce facts and figures on admissions, academics, student body, and career outlook. And **My Review** lets you manage the entire application process online. Research business schools, then save the schools you wish to target in a personal list. It will also keep track of your applications and their deadlines, store results from our financial aid tools, and more.

Keep in mind:

- Too many applicants rely on the magazine rankings to decide where to apply. But statistics rarely show the whole picture. Don't buy into any "bests." Simply seek out the program that is the best for you.
- Get a feel for the spirit of the student body at prospective schools. Even more so than undergraduates, business students rely on one another both inside and outside the classroom. Employ the same strategies used to research undergraduate colleges: Talk to students, alumni, and teachers; visit campuses; sit in on classes; and meet professors. These initial contacts are even more important for business school. Business school benefits are part educational, part networking. Establishing relationships early on may help your job search down the road.
- Choosing the best MBA program involves a lot more than looking at a school's average GMAT scores. On the one hand, just because a school has an average GMAT score that resembles yours doesn't mean that it would be a good fit. Even the best schools have their own strengths and weaknesses. Plus, GMAT scores and other quantitative measures are only part of what admissions officers consider. If your background and goals don't match up with those of the school, don't count on being admitted, even with impressive numbers.

ASSIGNMENT 2

MODIFIERS, PARALLEL CONSTRUCTION, AND IDIOMS

This lesson reviews important concepts related to idioms, modifiers, and parallel construction.

MODIFIERS

As you learned in the last lesson, modifiers are words that describe, or modify, other words in a sentence. Adjectives modify nouns; adverbs modify verbs, adjectives, and other adverbs. Single words or entire phrases can be modifiers.

Quick Quiz: Modifiers

Underline the modifying words and phrases in the sentences below, and draw arrows to what they modify.

1. Walking down the avenue, I was caught in a torrential downpour.
2. Left in the refrigerator for several weeks, the meat was now spoiled.
3. Michael Jordan, who is now 40 years old, is still considered one of the best players in professional basketball.

Misplaced Modifiers

In the movie *Animal Crackers*, Groucho Marx says, “I once shot an elephant in my pajamas. How he got into my pajamas I’ll never know.” The humor comes from a grammatical error (and you thought grammar was no fun). Though *in my pajamas* is meant to describe *I*, it seems to describe the *elephant*. A **misplaced modifier** is created when a modifier is not adjacent to the thing it’s intended to modify.

Misplaced modifiers on the GMAT most often appear in sentences that begin with an introductory phrase. Consider the following example:

Excommunicated by the Roman Church in 1521, the Protestant Reformation was led by Martin Luther.

The sentence begins with the modifying phrase *Excommunicated by the Roman Church in 1521*. Since the noun that immediately follows the phrase is the *Protestant Reformation*, the sentence implies that the Reformation was excommunicated. To correct the misplaced modifier, we need to rewrite the sentence:

Excommunicated by the Roman Church in 1521, Martin Luther led the Protestant Reformation.

Now the sentence tells us that Martin Luther was excommunicated, and the misplaced modifier has been corrected.

Quick Quiz: Misplaced Modifiers

Decide if the modifying phrase in each of the sentences below is placed correctly or if it is misplaced.

1. Arranged in secret, the discovery of Romeo and Juliet's marriage was made only after their deaths. (*no error, misplaced modifier*)
2. Discovered by Marie Curie and Pierre Curie, polonium and radium were first isolated in 1898. (*no error, misplaced modifier*)
3. Invented by James Hargreaves in 1765, the spinning jenny was capable of spinning eight to eleven threads at one time. (*no error, misplaced modifier*)
4. I overheard him say that he had cheated on the exam while I was standing in the hallway. (*no error, misplaced modifier*)
5. Once a very powerful nation, France's status has declined in recent years. (*no error, misplaced modifier*)

Quantity Words

Quantity words that describe nouns raise another modifier issue. Some nouns refer to concrete things, such as *children*, *tables*, or *dollars*, and are countable. Other nouns refer to abstract ideas or amorphous things, such as *air*, *beauty*, or *money*, and are non-countable. Different quantity words apply to countable and non-countable nouns.

| Countable | Not Countable |
|---------------|---------------------------|
| <i>fewer</i> | <i>less</i> |
| <i>number</i> | <i>amount or quantity</i> |
| <i>many</i> | <i>much</i> |

Here are some samples of the proper uses of these quantity words:

If there were *fewer* cars on the road, there would be *less* traffic.

The *number* of cars on the road contributes to the *amount* of traffic.

There's too *much* traffic on this road because there are too *many* cars.

Another issue involves comparisons. There are two separate situations: comparing two things and comparing three or more things. Memorize these rules:

| Two Things | Three or More Things |
|----------------|----------------------|
| <i>-er</i> | <i>-est</i> |
| <i>more</i> | <i>most</i> |
| <i>between</i> | <i>among</i> |

Examples:

Between you and me, I am *taller*.

Among the four of us, I am the *tallest*.

Quick Quiz: Quantity Words

Circle the correct form of the quantity word in the sentences below.

1. The (*better, best*) you prepare for the GMAT, the (*higher, highest*) your score will be.
2. (*Many, most*) of the population lives in poverty.
3. (*Many, most*) of the people live in poverty.
4. Since I withdrew money from my bank account, the (*number, amount*) of dollars in the account is now (*fewer, less*).
5. Since I withdrew money from my bank account, the (*number, amount*) of money in the account is now (*fewer, less*).
6. Some people consider the Yankees to be the (*greatest, greater*) baseball team ever.

PARALLEL CONSTRUCTION

When a sentence includes a list or comparison, each word or phrase in the list or comparison must have the same grammatical structure. The following examples illustrate **parallel construction**:

A melody is a succession of single tones that vary in *pitch, harmony, and rhythm*.

By the time he was thirteen, Mozart had not only *composed* sonatas, but he had also *performed* before royalty.

Walking briskly can be as aerobically beneficial as *jogging*.

Her novel was praised as an *exciting story*, a *social critique*, and a *philosophical inquiry*.

Quick Quiz: Parallelism

Choose the word that creates parallel construction.

1. On Saturday, David had to work on a project, write an e-mail to his mother, and *(play, to play)* in a softball game.
2. Three of the events in a decathlon are the 100-meter dash, *(pole vaulting, pole vault)*, and long jump.
3. Listening to a recording of your favorite band is not quite the same as *(to listen, listening)* to that band at a live concert.
4. The mainland was visited by explorers much later than *(the outlying islands, were the outlying islands)*.

Idioms

Idioms are fixed expressions, groups of words that are used together. There's really no rule that applies to these expressions—the conventions of English simply demand that they be phrased a certain way. Idiom errors show up frequently in sentence correction questions, often as a secondary error. ETS seems to feel that idioms are critical to determining your ability to pursue a graduate business degree, so you need to be concerned with these expressions.

Quick Quiz: Idioms

Fill in the missing word in each sentence.

1. She is not only beautiful, _____ smart.
2. I can't distinguish day _____ night.
3. I can distinguish between black _____ white.
4. My GMAT teacher defines the conclusion _____ the main point of the argument.
5. If you take the GMAT enough times, you might develop the ability _____ choose the credited responses without reading the questions.
6. Art historians regard the Mona Lisa _____ one of the greatest works of art.
7. Art historians consider the Mona Lisa _____ one of the greatest works of art.
8. He is not so much smart _____ cunning.
9. The mule, _____ the donkey, is a close relative of the horse.
10. Many of my favorite ice cream flavors, _____ chocolate chip and strawberry, are also available as frozen yogurt.
11. Her coat is just _____ mine.
12. He walks to work, just _____ I do.

How did you do? Check the answers at the end of the chapter to see how well you know your idioms. As you may have noticed from the quiz, idioms often involve prepositions, those tiny words that establish the relationship between other words.

The best way to learn idioms is through repetition and memorization. Don't get hung up on the "why." Accept that, as former news anchorman Walter Cronkite used to put it, "that's the way it is." We've made the process a little easier by providing a list of the idioms commonly tested on the GMAT. Review the idiom list below, and note any idioms that give you difficulty. Spend time learning those expressions.

Idiom List

The following list contains the idioms tested most frequently on the GMAT:

ABOUT

Worry...about

If you **worry** too much **about** the GMAT, you'll develop an ulcer.

AS

Define...as

My GMAT teacher **defines** the conclusion **as** the main point of the argument.

Regard...as

Art historians **regard** the Mona Lisa **as** one of the greatest works of art.

Not so...as

He is **not so** much smart **as** cunning.

So...as to be

She is **so** beautiful **as to be** exquisite.

Think of...as

Think of it more **as** a promise than a threat.

See...as

Many people **see** euthanasia **as** an escape from pain.

The same...as

Mom and Dad gave **the same** punishment to me **as** to you.

As...as

Memorizing idioms is not **as** fun **as** playing bingo.

AT

Target...at

The commercials were obviously **targeted at** teenage boys.

FOR

Responsible for

You are **responsible for** the child.

FROM**Prohibit...from**

He was **prohibited from** entering the public library after he accidentally set the dictionary on fire with a magnifying glass.

Different...from

Democrats are not so **different from** Republicans in the United States.

OVER**Dispute over**

The men had a **dispute over** money.

THAT**So... that**

He was **so** late **that** he missed the main course.

Hypothesis...that

The **hypothesis that** aspartame causes brain tumors has not been proven yet.

TO BE**Believe...to be**

His friends do not **believe** the ring he bought at the auction **to be** Jackie O's; they all think he was tricked.

Estimate...to be

The time he has spent impersonating Elvis is **estimated to be** longer than the time Elvis himself spent performing.

TO**Forbid...to**

I **forbid** you **to** call me before noon.

Ability...to

If you took the GMAT enough times, you might develop the **ability to** choose the credited responses without reading the questions.

Attribute...to

Many amusing quips are **attributed to** Dorothy Parker.

Require...to

Before you enter the house you are **required to** take off your hat.

Responsibility to

You have a **responsibility to** take care of the child.

Permit...to

I don't **permit** my children **to** play with knives in the living room.

Superior...to

My pasta sauce is far **superior to** my mother-in-law's.

Try...to

Try to stay awake during the essay section of the test.

WITH**Credit...with**

Many people **credit** Christopher Columbus **with** the discovery of America, but Native Americans were here first.

Associate...with

Most politicians prefer not to be **associated with** the Mafia.

Contrast...with

My father likes to **contrast** my grades **with** my brother's.

NO PREPOSITION**Consider...(nothing)**

Art historians **consider** the Mona Lisa one of the greatest works of art.

MORE THAN ONE PREPOSITION**Distinguish...from**

I can't **distinguish** day **from** night.

Distinguish between...and

I can **distinguish between** black **and** white.

Native (noun)... of

Mel Gibson is a **native of** Australia.

Native (adjective)...to

The kangaroo is **native to** Australia.

COMPARISONS AND LINKS**Not only...but also**

She is **not only** beautiful, **but also** smart.

Not...but

The review was **not** mean-spirited **but** merely flippant.

Either...or

I must have **either** chocolate ice cream **or** carrot cake to complete a great meal.

Neither...nor

Because Jenny was grounded, she could **neither** leave the house **nor** use the telephone.

Both...and

When given the choice, I choose **both** ice cream **and** cake.

More...than; Less...than

The chimpanzee is much **more** intelligent **than** the orangutan.

As vs. like

As is used to compare actions.

Like is used to compare nouns.

He did not vote for the Libertarian Party, **as** I did.

Her coat is just **like** mine.

Like vs. such as

Like means *similar to*.

Such as means *for example*.

The mule, **like** the donkey, is a close relative of the horse.

Many of my favorite ice cream flavors, **such as** chocolate chip and strawberry, are also available as frozen yogurt.

The more...the -er

The more you ignore me, the **closer** I get.

From...to

Scores on the GMAT range **from** 200 **to** 800.

Just as...so too

Just as I crossed over to the dark side, **so too** will you, my son.

MISCELLANEOUS**Each vs. all or both**

Use *each* when you want to emphasize the separateness of the items.

Use *both* (for two things) or *all* (for more than two things) when you want to emphasize the togetherness of the items.

Each of the doctors had his own specialty.

Both of the women went to Bryn Mawr for their undergraduate degrees.

All of the letters received before January 15 went into the drawing for the \$10 million prize.

Whether vs. if

Use *whether* when there are *two possibilities*.

Use *if* in *conditional statements*.

Eduardo wasn't sure **whether** he could make it to the party.

If Eduardo comes to the party, he will bring a bottle of wine.

SUMMARY

Modifiers

- Modifiers describe, or modify, other words in a sentence. Adjectives modify nouns. Adverbs modify verbs, adjectives, and other adverbs.
- Modifiers must go next to what they modify; otherwise, the modifier has been misplaced.
- To choose the correct quantity word, decide if the item is countable (such as pencils, coins, or stock options) or not countable (such as Jell-O, love, or soup.)
- Use *fewer*, *number*, and *many* to describe countable nouns. Use *less*, *amount*, *quantity*, and *much* to describe non-countable nouns.
- Use *between* and *-er* adjectives to compare two things and *among* and *-est* adjectives to compare three or more things.

Parallel Construction

- Parallel construction is required for lists and comparisons. Each item in the list or comparison must have the same grammatical construction.

Idioms

- Idioms are fixed expressions.
- Study the idiom list to learn the idioms tested on the GMAT.

ANSWERS AND EXPLANATIONS

Quick Quiz: Modifiers

1. *Walking down the avenue*, I was caught in a *torrential* downpour. *Walking down the avenue* modifies *I*. *Torrential* modifies *downpour*.
2. *Left in the refrigerator for several weeks*, the meat was now spoiled. *Left in the refrigerator for several weeks* modifies *meat*. *Spoiled* also modifies *meat*.
3. Michael Jordan, *who is now 40 years old*, is still considered one of the *best* players in professional basketball. *Who is now 40 years old* modifies *Michael Jordan*. *Still* modifies *considered*. *Best* modifies *players*. *Professional* modifies *basketball*.

Quick Quiz: Misplaced Modifiers

1. *Arranged in secret*, the discovery of Romeo and Juliet's marriage was made only after their deaths. **Misplaced modifier.** The phrase *arranged in secret* modifies *discovery*, but it was really the marriage that was arranged secretly. The corrected sentence: Arranged in secret, Romeo and Juliet's marriage was discovered only after their deaths.
2. *Discovered by Marie Curie and Pierre Curie*, polonium and radium were first isolated in 1898. **No error.** The phrase *Discovered by Marie Curie and Pierre Curie* correctly modifies *polonium and radium*.
3. *Invented by James Hargreaves in 1765*, the spinning jenny was capable of spinning eight to eleven threads at one time. **No error.** The phrase *Invented by James Hargreaves in 1765* correctly modifies *the spinning jenny*.
4. I overheard him say that he had cheated on the exam *while I was standing in the hallway*. **Misplaced modifier.** Because the clause *while I was standing in the hallway* is at the end of the sentence, it seems to describe when the cheating occurred, not when the conversation was overheard. The corrected sentence: While I was standing in the hallway, I overheard him say that he had cheated on the exam.
5. *Once a very powerful nation*, France's status has declined in recent years. **Misplaced modifier.** The phrase *Once a very powerful nation* modifies *status*, but it should modify *France*. The corrected sentence: Once a very powerful nation, France has declined in status in recent years.

Quick Quiz: Quantity Words

1. The *better* you prepare for the GMAT, the *higher* your score will be.
2. *Most* of the population lives in poverty.
3. *Many* of the people live in poverty.
4. Since I withdrew money from my bank account, the *number* of dollars in the account is now *fewer*.
5. Since I withdrew money from my bank account, the *amount* of money in the account is now *less*.
6. Some people consider the Yankees to be the *greatest* baseball team ever.

Quick Quiz: Parallelism

1. On Saturday, David had to work on a project, write an e-mail to his mother, and *play* in a softball game.
2. Three of the events in a decathlon are the 100-meter dash, *pole vault*, and long jump.
3. Listening to a recording of your favorite band is not quite the same as *listening* to that band at a live concert.
4. The mainland was visited by explorers much later than *were the outlying islands*.

Quick Quiz: Idioms

The idiom is in bold text, and the word that goes in the blank is underlined.

1. She is **not only** beautiful, **but also** smart. The idiom is *not only...but also*.
2. I can't **distinguish** day **from** night. The idiom is *distinguish x from y*.
3. I can **distinguish between** black **and** white. The idiom is *distinguish between x and y*. Note that there are two idioms for distinguish. You can use either one, but you can't mix them up. For example, "I can't distinguish day and night," and "I can't distinguish between day from night," are wrong.
4. My GMAT teacher **defines** the conclusion **as** the main point of the argument. The idiom is *define x as y*.
5. If you take the GMAT enough times, you might develop the **ability to** choose the credited responses without reading the questions. The idiom is *ability to*.
6. Art historians **regard** the Mona Lisa **as** one of the greatest works of art. The idiom is *regard x as y*.
7. Art historians **consider** the Mona Lisa one of the greatest works of art. Trick question! The idiom is *consider*, and it should not be followed by any prepositions. Though we often say "consider to be" in everyday speech, the GMAT writers consider this phrase incorrect.
8. He is **not so much** smart **as** cunning. The idiom is *not so much x as y*.
9. The mule, **like** the donkey, is a close relative of the horse. Use *like* when you mean "is similar to."
10. Many of my favorite ice cream flavors, **such as** chocolate chip and strawberry, are also available as frozen yogurt. Use *such as* when you mean "for example."
11. Her coat is just **like** mine. This sentence compares her coat and my coat. Use *like* to compare nouns.
12. He walks to work, just **as** I do. This sentence compares the action of walking. Use *as* to compare verbs.

PART-TO-WHOLE RELATIONSHIPS

Fractions, decimals, and percents are three ways to express a part of a whole. Many questions on the GMAT require you to perform basic operations (adding, subtracting, multiplying, and dividing) with fractions, decimals, and percents.

FRACTIONS

Here's a quick review of fractions:

- A fraction describes a $\frac{\text{part}}{\text{whole}}$.
- The top is the **numerator**. The bottom is the **denominator**. For example, in the fraction $\frac{5}{7}$, 5 is the numerator and 7 is the denominator.
- The **reciprocal** (also called the inverse) of a fraction is that fraction flipped over. The reciprocal of $\frac{5}{7}$ is $\frac{7}{5}$.
- The fraction bar (the line between the numerator and the denominator) is equivalent to division. For example, $\frac{10}{5}$ means $10 \div 5$, or 2.

Let's take a look at how ETS might test your knowledge of fractions.

$$1. \frac{\left(\frac{3}{2} + \frac{2}{3}\right)}{\left(\frac{1}{4} \times \frac{1}{5}\right)} =$$

☐ $\frac{1}{20}$

☐ $\frac{5}{20}$

☐ $\frac{13}{6}$

☐ 20

☐ $\frac{130}{3}$

First, let's deal with adding $\frac{3}{2}$ and $\frac{2}{3}$.

If fractions have the same denominator, you simply add or subtract the numerators and put the sum over the denominator. Don't change the denominator.

You may recall learning something about finding the lowest common denominator to add or subtract fractions with different denominators. Well, we here at the Princeton Review have something much cooler. We call it the Bowtie. Here's how it works:

Use the Bowtie method to add or subtract fractions.

First, multiply diagonally up (opposing denominators and numerators).

$$\begin{array}{cc} 9 & 4 \\ \frac{3}{2} & + \frac{2}{3} \end{array}$$

Second, carry up the sign (in this case, addition).

$$\begin{array}{ccc} 9 & + & 4 \\ \frac{3}{2} & + & \frac{2}{3} = \end{array}$$

Third, add (or subtract) across the top.

$$\begin{array}{ccc} 9 & + & 4 \\ \frac{3}{2} & + & \frac{2}{3} = \frac{13}{6} \end{array}$$

Finally, multiply across the bottom.

$$\begin{array}{ccc} 9 & + & 4 \\ \frac{3}{2} & + & \frac{2}{3} = \frac{13}{6} \end{array}$$

Now, our original problem is:

$$\left(\frac{\frac{3}{2} + \frac{2}{3}}{\frac{1}{4} \times \frac{1}{5}} \right) = \frac{\frac{13}{6}}{\frac{1}{4} \times \frac{1}{5}}$$

To multiply fractions, multiply straight across the top and the bottom.

$$\frac{1}{4} \times \frac{1}{5} = \frac{1 \times 1}{4 \times 5} = \frac{1}{20}$$

Now, our problem is:

$$\frac{\frac{13}{6}}{\frac{1}{20}}$$

Rewritten, this is $\frac{13}{6} \div \frac{1}{20}$.

To divide fractions, multiply by the reciprocal of the second fraction:

$$\frac{13}{6} \div \frac{1}{20} = \frac{13}{6} \times \frac{20}{1} = \frac{13 \times 20}{6 \times 1} = \frac{260}{6}$$

To reduce a fraction, divide the top and bottom by the same number. Both the top and the bottom can be divided by 2:

$$\frac{260 \div 2}{6 \div 2} = \frac{130}{3}$$

If possible, reduce before
you multiply.

Try to reduce before you multiply. When multiplying fractions, you can divide either numerator or either denominator by the same number to reduce. Do NOT do this with addition or subtraction problems. Let's look at that problem again. We can divide a top number (20) by 2, and a bottom number (6) by 2:

$$\frac{13}{\cancel{6}^2} \times \frac{\cancel{20}_2}{1} = \frac{130}{3}$$

Quick Quiz: Fractions

1. $\frac{3}{4} \div \frac{1}{7} =$

2. $\frac{2}{5} + \frac{1}{13} =$

3. $\frac{4}{3} - \frac{5}{2} =$

4. $\frac{3}{5} \times \frac{15}{2} =$

5. $\frac{5}{6} - \frac{1}{14} =$

Try this example:

2. $\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \frac{2}{81} =$

☐ $\frac{40}{81}$

☐ $\frac{68}{81}$

☐ $\frac{80}{81}$

☐ 1

☐ $\frac{107}{81}$

Would you use the Bowtie in this problem? Well, as handy as the Bowtie is, every once in a while it's easier to come up with the lowest common denominator. Use the lowest common denominator when all the numbers in the denominators are factors or multiples of one another. What number would serve as a common denominator for all of these fractions? _____

To put each of the fractions over the same common denominator, decide what number you need to multiply the denominator by in order to convert it to the lowest common denominator. Then multiply the top and bottom of each fraction by that number. For instance, to convert $\frac{2}{3}$:

$$\frac{2 \times 27}{3 \times 27} = \frac{54}{81}$$

The Bowtie method isn't always the fastest way to add fractions.

Now, put the other fractions over 81 and then add them up

($\frac{54}{81} + \frac{18}{81} + \frac{6}{81} + \frac{2}{81} = \frac{80}{81}$). The correct answer is therefore (C).

Fractions Between Zero and One

Let's consider what happens when you perform the basic operations with numbers between 0 and 1. Fill in the answer, then circle either "bigger" or "smaller."

$$\frac{1}{3} + \frac{1}{2} = \underline{\hspace{1cm}} \quad \text{bigger/smaller}$$

Given any number, if you add to it a number between 0 and 1, the result will be bigger than the original number.

$$\frac{1}{3} - \frac{1}{2} = \underline{\hspace{1cm}} \quad \text{bigger/smaller}$$

Given any number, if you subtract from it a number between 0 and 1, the result will be smaller than the original number.

$$\frac{1}{3} \times \frac{1}{2} = \underline{\hspace{1cm}} \quad \text{bigger/smaller}$$

Given a positive number, if you multiply it by a number between 0 and 1, the result will be smaller than the original number.

$$\frac{1}{3} \div \frac{1}{2} = \underline{\hspace{1cm}} \quad \text{bigger/smaller}$$

Given a positive number, if you divide it by a number between 0 and 1, the result will be bigger than the original number.

DECIMALS

Decimals and fractions go hand in hand. On the GMAT, every decimal can be written as a fraction, and every fraction can be written as a decimal. Do your calculations in the format with which you're most comfortable.

$$0.5 = \frac{5}{10} = \frac{1}{2}$$

$$\frac{3}{5} = 3 \div 5 = 0.6$$

To convert a decimal to a fraction, place the decimal over 1, move the decimal points the same number of places on the top and bottom until you have whole numbers on both the top and the bottom, then reduce.

$$0.25 = \frac{0.25}{1.00} = \frac{25}{100} = \frac{1}{4}$$

Convert these to fractions:

$$0.4 = \frac{\quad}{\quad} \qquad 0.125 = \frac{\quad}{\quad}$$

To convert a fraction to a decimal, divide the bottom into the top.

$$\frac{1}{4} = 4 \overline{)1.00} = 0.25$$

Convert these to decimals:

$$\frac{3}{8} = \frac{\quad}{\quad} \qquad \frac{7}{20} = \frac{\quad}{\quad}$$

Now, let's take a look at a question:

$$1. \quad 3\frac{1}{4} - 3.025 =$$

- ☐ 0
- ☐ 0.025
- ☐ 0.125
- ☐ 0.225
- ☐ 0.25

Before we subtract, we need to convert the fraction to a decimal:

$$\frac{1}{4} = 4 \overline{)1.00} = 0.25, \text{ so } 3\frac{1}{4} = 3.25.$$

Whenever you add or subtract decimals, always line up the decimal points. You can add zeros onto the end of a decimal without changing value.

$$\begin{array}{r} 3.250 \\ -3.025 \\ \hline 0.225 \end{array}$$

The answer is (D).

Line up the decimal points when you add or subtract.

Here's another problem:

2. $(0.2)(0.3)(0.05) =$

- ☐ 0.0003
- ☐ 0.003
- ☐ 0.03
- ☐ 0.3
- ☐ 3.0

Here's a good way to multiply decimals. First, ignore the decimal points and simply multiply the numbers: $2 \times 3 \times 5 = 30$. Second, count the number of decimal places in each of the original decimals: 0.2 and 0.3 each have 1 decimal place and 0.05 has 2 decimal places, which gives a total of 4 decimal places. Third, take your product and move the decimal the same number of places to the left:

0.0030

Therefore, the answer is (B).

Here's another decimal example:

3. Which of the following is the closest to $\frac{5.13}{0.02}$?

- ☐ 2500
- ☐ 250
- ☐ 25
- ☐ 2.5
- ☐ 0.25

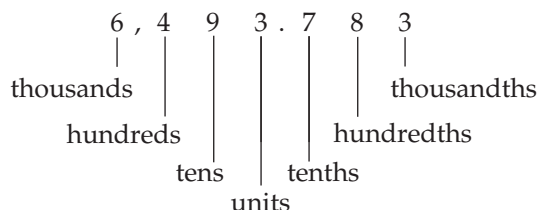
When you divide by a decimal, you can simply change your decimal into a whole number. To do this, move the decimal point in both your numerator and your denominator the same number of places to the right. You can only do this when you *divide* decimals!

$$\frac{5.13}{0.02} = \frac{513}{2}$$

Notice that the question is not asking for the exact solution. You only need to find an approximation. 513 is about 500. 500 divided by 2 is 250. Thus, the answer is (B).

DIGIT PLACES

Some questions on the GMAT involve digit places. Consider the number 6,493.783.



Each place contains a digit, which is a whole number from 0 to 9. For example, 7,542 is a four-digit number.

Round decimals by looking at the digit to the right of the decimal place to which you want to round. For example, to round to the nearest hundredth, look at the thousandths place. If the thousandths place is 5 or more, round the hundredths place up by 1. For example, 0.247 rounded to the nearest hundredth is 0.25. If the thousandths place is less than 5, the hundredths place stays the same. For example, 0.241 rounded to the nearest hundredth is 0.24.

Quick Quiz: Decimals

1. What is 56.7189 rounded to the nearest tenth?
2. What is 1.1119 rounded to the nearest thousandth?
3. $43.25 + 3.9 =$
4. $2.1 \times 0.002 =$
5. $3.02 \div 0.2 =$

PERCENTS

Percents, like fractions and decimals, are just another way of describing a part-to-whole relationship. Any percent can also be written as either a fraction or a decimal.

“Percent” simply means “over one hundred.” Any percent can be expressed as a fraction by putting it over 100, then reducing the fraction. For example,

$$3\% = \frac{3}{100}$$

$$25\% = \frac{25}{100} = \frac{1}{4}$$

To convert a fraction into a percent, you have to know that $\frac{\text{part}}{\text{whole}} = \frac{x}{100}$, where

x is the percentage. For instance, $\frac{3}{5}$ is the same as $\frac{60}{100}$ or 60%.

To convert a percent to a decimal, drop the percent sign and move the decimal point two places to the left:

$$25\% = \frac{25}{100} = 0.25$$

$$5\% = \frac{5}{100} = 0.05$$

$$0.1\% = \frac{.1}{100} = 0.001$$

$$200\% = \frac{200}{100} = 2.0$$

Likewise, to convert a decimal to a percent, move the decimal point two places to the right and add a percent sign:

$$0.18 = \frac{18}{100} = 18\%$$

$$0.7 = \frac{70}{100} = 70\%$$

$$3.5 = \frac{350}{100} = 350\%$$

$$0.002 = \frac{.2}{100} = 0.2\%$$

And now for the trick question: Convert 0.003 into a percent. That's right, it's 0.3%. It is NOT 3%. Note that percents can be less than 1. Be careful with your place holders.

As you may have noticed, ETS likes to phrase percent problems on the GMAT as long word problems. Instead of trying to invent your own equation for the problem, use this table to translate a percent word problem into a mathematical equation.

| English | Mathematical translation |
|---------------|------------------------------------|
| percent | $\frac{\quad}{100}$ |
| of | \times (times) |
| what | $x, y, \text{ or } z$ (a variable) |
| is, are, were | $=$ |

Try using translation on this problem before reading the explanation below it.

1. 6.5 is what percent of 1,300?

- ☐ 0.5%
☐ 1%
☐ 5%
☐ 50%
☐ 500%

6.5 is what percent of 1,300

$$6.5 = \frac{y}{100} \times 1,300$$

Then, solve for y . The correct answer is (A).

Let's try a word problem:

2. A merchant raises the price of a \$100 item by 20%. After finding that she cannot sell the item at the higher price, she discounts it by 20%. What is the final price of the item?

- ☐ \$96
☐ \$97
☐ \$98
☐ \$99
☐ \$100

Take it one sentence at a time. First, we need to increase \$100 by 20%. One way to do this is to find 20% of \$100 and then add that amount. 20% of \$100 is \$20. So, $\$100 + \$20 = \$120$. Be careful. The problem says that the higher price (not the original price) was discounted by 20%. We need to find 20% of \$120.

$\frac{20}{100} \times \$120 = \frac{1}{5} \times \$120 = \$24$. The final price is $\$120 - \$24 = \$96$. The answer is (A).

Percent Change

Here's a slightly different kind of percent problem:

3. The average television prime-time advertising unit of 30 seconds cost \$30,000 in 1973 and \$50,000 in 1977. What was the approximate percent increase in the cost of a unit?

- ☐ 20%
☐ 33%
☐ 40%
☐ 60%
☐ 67%

Many percent questions ask for the **percent increase** or the **percent decrease** from one number to another. For such questions, use this formula:

$$\text{Percent change} = \frac{\text{difference}}{\text{original}} \times 100$$

The *difference* is what you get when you subtract the smaller number from the larger number: $50,000 - 30,000 = 20,000$. The *original* is the starting number. Putting these into our formula, we get:

$$\text{Percent change} = \frac{20,000}{30,000} \times 100$$

$$\frac{2}{3} \times 100 = \frac{200}{3} \% = 66\frac{2}{3} \%, \text{ which is closest to answer choice (E).}$$

Deciding which number counts as the “original” can be confusing. Remember, if the question asks for percent *increase*, the original is the *smaller* number. If the question asks for percent *decrease*, the original is the *larger* number.

Shortcuts to Calculate Percents

Always looking for the shortcut, eh? That’s wise on a timed test. Here’s a shortcut for percent problems:

First, it’s easy to find 10% or 1% of a number: just move the decimal point one or two places to the left. For instance, 10% of 1,024 is 102.4. Once you have 10%, it’s easy to find 5% (by dividing your result by 2) or 30% (by multiplying your result by 3), etc. Thus, 5% of 1,024 is 51.2. 30% of 1,024 is 307.2. You can even use this GMAT shortcut in real life: To calculate a 15% tip on a \$156 dinner tab, you can figure out that $10\% + 5\% = \$15.60 + \$7.80 = \$23.40$. Want a further shortcut? Before you calculate, eyeball the answer choices to see how exact you need to be in your calculations. For example, if the answer choices aren’t terribly close in value, saying 5% of 1,024 is “a bit more than 50” will do just fine.

Comparing Decimals, Fractions, and Percents

It’s worth memorizing this chart. It will save you a lot of time on the GMAT if you can comfortably switch among decimals, fractions, and percents.

| Fraction | Decimal | Percent |
|---------------|--------------------|---------------------|
| $\frac{1}{2}$ | 0.5 | 50% |
| $\frac{1}{3}$ | $0.\overline{33}$ | $33.\overline{3}\%$ |
| $\frac{2}{3}$ | $0.\overline{66}$ | $66.\overline{6}\%$ |
| $\frac{1}{4}$ | 0.25 | 25% |
| $\frac{3}{4}$ | 0.75 | 75% |
| $\frac{1}{5}$ | 0.2 | 20% |
| $\frac{2}{5}$ | 0.4 | 40% |
| $\frac{3}{5}$ | 0.6 | 60% |
| $\frac{4}{5}$ | 0.8 | 80% |
| $\frac{1}{6}$ | $0.1\overline{66}$ | $16.\overline{6}\%$ |
| $\frac{1}{8}$ | 0.125 | 12.5% |

Did you memorize the numbers above, or did you just scan over them? Which ones did you already know? Which ones don't you know YET? Try converting some of the ones you don't know yet, then check the chart. For example: What's

$\frac{1}{6}$ as a decimal? What's 40% as a fraction?

Estimating Percents

As you know, your job on the GMAT isn't to calculate the correct answer. It's to do just enough on the problem to spot which answer choice is correct. Often, you just need to estimate the correct answer. For example, pretend that you just worked a long problem. You've gotten to the end of it and the last thing to do is:

$$\frac{21120}{64000}$$

Yuck. Before you calculate this, look at the answer choices:

- ☐ 22%
- ☐ 33%
- ☐ 44%
- ☐ 66%
- ☐ 77%

Notice that the answer choices are not terribly close in value. Thus, you just need a rough calculation of your fraction:

$$\frac{21120}{64000} \approx \frac{20000}{60000} = \frac{2}{6} = \frac{1}{3} = 33\frac{1}{3}\%.$$

Therefore, the answer is (B).

Quick Quiz: Estimating

Come up with rough estimates for the following:

1. 32% of 6,050 is approximately _____.
2. 9.2% of 41 is approximately _____.
3. $\frac{374}{720}$ is approximately _____%.
4. $\frac{50}{2412}$ is approximately _____%.

PROBABILITY

You've heard probability used in a variety of ways. There's a 30% chance of rain today. There's a one in a million chance of beating Michael Jordan at a game of HORSE. Probabilities are just another type of part-to-whole relationship. They can be expressed as fractions, decimals, or percents.

Probability is always between zero and one, inclusive. If something can absolutely never happen, its probability is 0. If something absolutely will happen, it's probability is 1 (or 100%). This is handy to keep in mind for ballparking. If the situation seems unlikely, then its probability is less than $\frac{1}{2}$. If it seems likely, then its probability is greater than $\frac{1}{2}$.

The formula is:

$$\text{probability} = \frac{\text{number of outcomes you want}}{\text{number of total possible outcomes}}$$

Let's look at an example.

- Jeffrey has a bag of marbles. The bag contains 6 red, 6 yellow, 12 green, and 12 blue marbles. It contains no other marbles. What is the probability that a marble chosen at random will be either red or yellow?

☐ $\frac{1}{6}$

☐ $\frac{1}{3}$

☐ $\frac{1}{2}$

☐ $\frac{2}{3}$

☐ $\frac{3}{4}$

First, find the number of outcomes you want. This means the number of ways that you could get a red or yellow marble: $6 + 6 = 12$. Next, find the number of total possible outcomes. The total outcomes are red, yellow, green, and blue: $6 + 6 + 12 + 12 = 36$. Now, put it in the formula. The probability of getting a red or yellow = $\frac{12}{36} = \frac{1}{3}$. So, the answer is (B).

Quick Quiz: Probability

1. Jose has a box of doughnuts. There are 4 plain doughnuts, 2 jelly doughnuts, and 8 chocolate doughnuts. If he picks one at random, what is the probability that he will pick a chocolate one?
2. A streetlight is green for 15 seconds, yellow for 5 seconds, and red for 30 seconds. What is the probability that the light will be red the moment you arrive at the intersection?
3. Mary made trail mix that has 30 peanuts, 40 raisins, 10 chocolate candies, and 15 banana chips. She picked one item from the bag at random. What is the probability that she picked either a chocolate candy or a banana chip?

INTEREST

GMAT problems may ask you to calculate simple or compound interest. **Simple interest** is as easy as finding a percentage. Here's an example:

1. If Molly puts \$500 in a savings account that pays 4 percent simple annual interest, how much money will be in the account after one year?

- ☐ \$20
- ☐ \$500
- ☐ \$504
- ☐ \$520
- ☐ \$540

This is a basic percentage problem: *What is 4 percent of \$500?*

$$y = \frac{4}{100} \times 500$$

$$y = 20$$

The account earns \$20 over the course of the year, so there would be \$520 in the account. The answer is (D).

Compound interest is a slightly different animal. It means that you earn interest on your interest. Really, you only end up earning slightly more than you would have earned with simple interest. Compound interest problems are fairly rare on the GMAT. When they do show up, you can usually just calculate the simple interest, then look for the answer choice that is slightly more than that. For example:

2. If Molly puts \$500 in a savings account that pays 4 percent annual interest compounded semiannually, how much money will be in the account after one year?

- ☐ \$5.20
- ☐ \$504.00
- ☐ \$520.00
- ☐ \$520.20
- ☐ \$540.00

We calculated that, with simple interest there would have been \$520 in the bank. So, we need the answer choice that is slightly more than \$520. That would be (D), \$520.20.

Dying for the formula? Here it is. You might see a problem in which you need to recognize the formula, but you won't have to calculate it out.

$$\text{principal} + \text{interest} = \text{principal} \times (1 + r)^t$$

r = interest rate *for the compounding period* expressed as a decimal
 t = number of compounding periods

So, for Molly: Use $r = 0.02$ because she earns 2 percent per half year. Use $t = 2$ because one year contains two compounding periods. Thus $\text{principal} + \text{interest} = 500 \times (1 + 0.02)^2 = 520.20$. So, the answer is still (D).

Quick Quiz: Interest

- Kevin has \$1,500 in a bank account that pays 5 percent simple interest annually. How much money will be in the account after one year?
- Sandy has \$2,000 in a bank account that pays 4 percent interest compounded annually. After three years, approximately how much money will be in the account?

DATA SUFFICIENCY RANGES

To answer a question asking the value of something, you need to be able to find a single value. If Statements (1) and (2) each give a list of possible values, and the two lists have *exactly* one value in common, then the correct answer is (C).

Let's look at an example:

- What is the value of the odd integer x ?

(1) $18 < 2x < 30$

(2) $33 < 3x < 75$

If you divide the expression in Statement (1) by 2, the result is $9 < x < 15$. So x (an odd integer) can be 11 or 13. Write BCE. Fact (2) can be divided by 3 to give $11 < x < 25$. So $x = 13, 15, 17, 19, 21$, or 23. Eliminate (B). If you combine (1) and (2), the only number that makes both statements true is 13. Thus, there is a single value for x , and the answer is (C).

- What is the value of x ?

(1) $|x| = 4$

(2) $x^2 - 16 = 0$

Since both 4 and -4 have an absolute value of 4, Statement (1) is insufficient. Write BCE. Similarly, both 4 and -4 work for Statement (2). Eliminate (B). Even when you combine (1) and (2), you still have two possible values. Since you cannot determine a single value for x , the answer is (E).

SUMMARY

Fractions

- Terms: The top is the numerator. The bottom is the denominator. The reciprocal is the fraction flipped over.
- The fraction bar is equivalent to \div .
- To add or subtract fractions, use the Bowtie.
- To multiply fractions, multiply straight across.
- To divide fractions, multiply by the reciprocal of the second fraction.

Decimals

- Adding and subtracting decimals: Always line up the decimal points.
- Multiplying decimals: Ignore the decimal points and multiply the numbers, count the number of decimal places in each of the original decimals, then move the decimal point the same number of places to the left.
- Dividing by a decimal: Move the decimal point in both the numerator and the denominator the same number of places to the right. You can only do this when you *divide* decimals!

Percents

- “Percent” means “over 100.”
- Use translation chart for wordy percent problems.
- Percent change = $\frac{\text{difference}}{\text{original}} \times 100$
- “Percent *increase*” tells you that the original is the *smaller* number. “Percent *decrease*” tells you that the original is the *larger* number.
- Whenever possible, estimate percents.

Converting Fractions, Decimals, and Percents

- To convert a decimal to a fraction, place the decimal over 1, move the decimal points to the right the same number of places on the top and bottom, then reduce.
- To convert a percent to a fraction: Put the percent over 100, and reduce the fraction.
- To convert a fraction into a percent, $\frac{\text{part}}{\text{whole}} = \frac{x}{100}$, where x is the percentage.

- To convert a percent to a decimal, drop the percent sign and move the decimal point two places to the left.
- To convert a decimal to a percent, move the decimal point two places to the right and add a percent sign.

Probability

- Probability is always between zero and one inclusive.
- $\text{probability} = \frac{\text{number of outcomes you want}}{\text{number of total possible outcomes}}$

Interest

- For **simple interest**, find the percent.
- For **compound interest**, find the percent, then look for the answer choice slightly more than this result.

DRILL

1. Franklin's wage increased by what percent?
 - (1) Franklin's wage after the increase was \$45,000.
 - (2) Franklin's wage increased by \$2,000.
2. Which of the following is the greatest?
 - ☐ 0.03
 - ☐ $1 - 0.3$
 - ☐ $\frac{0.3}{3}$
 - ☐ $\frac{1}{0.3}$
 - ☐ 0.3
3. The profits from the three divisions of Company X totaled \$53,000. The profit from Division A was \$26,000. The profit from Division B was 20 percent of the profit that was not from Division A. The rest of the profit came from Division C. What was the amount of profit from Division C?
 - ☐ \$5,400
 - ☐ \$20,800
 - ☐ \$21,600
 - ☐ \$27,000
 - ☐ \$42,400

4. What is the one hundred-thousandths digit in the decimal form of $\frac{3}{11}$?

☐ 0
☐ 1
☐ 2
☐ 6
☐ 7

5.
$$\frac{\left(1\frac{1}{3} + 3\frac{2}{5}\right)}{\left(\frac{2}{5} + 2\frac{1}{3}\right)}$$

☐ $1\frac{30}{41}$
☐ 2
☐ $2\frac{11}{15}$
☐ $4\frac{11}{15}$
☐ $10\frac{46}{225}$

6. Company XYZ had \$498.2 million in profits for the year. The sales department brought in \$302.6 million of the profits. Approximately what percent of the profits were not brought in by the sales department?

☐ 4%
☐ 6%
☐ 25%
☐ 40%
☐ 60%

7. In 1998, Company Q had \$359,000 in profits. In 2002, Company Q's profits were 250 percent greater than they were in 1998. Approximately what were Company Q's profits in 2002?

☐ \$144,000
☐ \$450,000
☐ \$610,000
☐ \$900,000
☐ \$1,260,000

8. A total of 20 interns and 15 full-time employees work in the 35 offices of a company, with one person working in each office. One office is selected at random to get an upgraded desk. What is the probability that the office selected will be that of a female intern?

- (1) Of the interns, 15 of them are female.
 (2) Of the interns, 5 of them are male.

ANSWERS AND EXPLANATIONS

Quick Quiz: Fractions

1. $(\frac{21}{4})$ Flip the fraction, then multiply straight across:

$$\frac{3}{4} \times \frac{7}{1} = \frac{3 \times 7}{4 \times 1} = \frac{21}{4}$$

2. $(\frac{31}{65})$ Use the Bowtie: $\frac{26+5}{5 \times 13} = \frac{31}{65}$

3. $(-\frac{7}{6})$ Use the Bowtie: $\frac{8-15}{3 \times 2} = -\frac{7}{6}$

4. $(\frac{9}{2})$ Reduce the 5 and the 15 by 5: $\frac{3}{5} \times \frac{15}{2} = \frac{3}{1} \times \frac{3}{2}$,
then multiply straight across: $\frac{3 \times 3}{1 \times 2} = \frac{9}{2}$

5. $(\frac{16}{21})$ Use the Bowtie: $\frac{70-6}{6 \times 14} = \frac{64}{84}$.

Then reduce by 4: $\frac{64}{84} = \frac{16}{21}$

Fractions Between Zero and One

$$\frac{1}{3} + \frac{1}{2} = \frac{5}{6} \quad \text{bigger}$$

Given any number, if you add to it a number between 0 and 1, the result will be bigger than the original number.

$$\frac{1}{3} - \frac{1}{2} = -\frac{1}{6} \quad \text{smaller}$$

Given any number, if you subtract from it a number between 0 and 1, the result will be smaller than the original number.

$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6} \quad \text{smaller}$$

Given a positive number, if you multiply it by a number between 0 and 1, the result will be smaller than the original number.

$$\frac{1}{3} \div \frac{1}{2} = \frac{2}{3} \quad \text{bigger}$$

Given a positive number, if you divide it by a number between 0 and 1, the result will be bigger than the original number.

Decimals

Convert these to fractions:

$$0.4 = \frac{2}{5} \qquad 0.125 = \frac{1}{8}$$

Quick Quiz: Decimals

- 56.7
- 1.112
- 47.15
- .0042
- 15.1

Quick Quiz: Estimating

- Approximately 2,000. 32% is close to $33\frac{1}{3}\%$, which is $\frac{1}{3}$. 6,050 is close to 6,000. $\frac{1}{3}$ of 6,000 is 2,000.
- Approximately 4. 9.2% is close to 10%. 41 is close to 40. 10% of 40 is 4.
- Approximately 50%. $\frac{374}{720}$ is close to $\frac{350}{700} = \frac{1}{2} = 50\%$.
- Approximately 2%. $\frac{50}{2412}$ is close to $\frac{50}{2500} = \frac{1}{50} = 2\%$.

Quick Quiz: Probability

- $(\frac{4}{7})$ The "number of outcomes you want" is "8 chocolate doughnuts." Divide this by the total:
 $4 + 2 + 8 = 14$. Thus, $\frac{8}{14}$. Reduce to $\frac{4}{7}$.

2. $(\frac{3}{5})$ The “number of outcomes you want” is “red for 30 seconds.” Divide this by the total: $15 + 5 + 30$. Thus, $\frac{30}{50}$. Reduce to $\frac{3}{5}$.
3. $(\frac{5}{19})$ The “number of outcomes you want” is “10 chocolate candies or 15 banana chips”: $10 \text{ chocolate candies} + 15 \text{ banana chips} = 25$. Divide this by the total: $30 + 40 + 10 + 15 = 95$. Thus, $\frac{25}{95}$. Reduce to $\frac{5}{19}$.

Quick Quiz: Interest

1. \$1,575 Find 5 percent of \$1,500: $\frac{5}{100} \times 1,500 = 75$. Then, add that to the 1,500 that’s in the bank: $\$75 + \$1,500 = \$1,575$.
2. Slightly more than \$2,240. Find the simple interest and ballpark. The simple interest would be $\frac{4}{100} \times \$2,000 = \80 . She made this for three years, so she made $\$80 \times 3 = \240 . Add this to Sandy’s \$2,000 in the bank, and you get \$2,240. She would have \$2,240 if it was simple interest, so you know the compound interest is slightly more than this. If you’re just itching to know, the precise answer (if you used the formula) is \$2,249.73.

Drill

1. **C** In order to know the percent increase, we need to be able to find out the difference and the original. Statement (1) does not give us a way to find the difference or the original. So, cross out (A) and (D). Statement (2) only tells us the difference. Cross out (B). Together, we have the difference (from the second statement) and a way to find the original ($\$45,000 - \$2,000$). Thus, the statements together are sufficient to answer the question.
2. **D** Ballpark the answer choices to see if they’re more than 1 or less than 1. (A), (B), and (E) are less than one. Take a moment with (C) and (D). (C) is 0.1. (D) is more than one, because when you divide by a number between 0 and 1, the number gets bigger. So, only (D) is greater than 1. It must be the greatest.
3. **C** First, find the profit that is “not from Division A”: $\$53,000 - \$26,000 = \$27,000$. Division B is responsible for 20% of this. That means that Division C is responsible for the remaining 80%. Translate: Division C is 80% of the profit not from Division A. $C = \frac{80}{100} \times \$27,000 = \$21,600$.
4. **C** Divide out until you see a pattern: $11 \overline{) 0.27272}$. The one hundred-thousandths place is five places to the right of the decimal point. The order is: tenths, hundredths, thousandths, ten-thousandths, one hundred-thousandths.

5. **A** Make your life easier by converting these into improper fractions. $(1\frac{1}{3} + 3\frac{2}{5}) = (\frac{4}{3} + \frac{17}{5})$. Then bowtie: $\frac{20+51}{3 \times 5} = \frac{71}{15}$. Do the same for the denominator: $(\frac{2}{5} + \frac{7}{3}) = (\frac{6+35}{3 \times 5}) = \frac{41}{15}$. So, the problem is now: $\frac{71}{15} \div \frac{41}{15} = \frac{71}{15} \times \frac{15}{41}$. Take 15 out of the top and bottom: $\frac{71}{1} \times \frac{1}{41} = \frac{71}{41} = 1\frac{30}{41}$.
6. **D** Two important words to notice in the question: “approximately” and “not.” “Approximately” is a tip-off to use nicer numbers, such as \$500 (million) and \$300 (million). The profits NOT brought in from the sales department were: $500 - 300 = 200$. Then, translate “what percent of the profits were not brought in by the sales department”: $\frac{y}{100} \times 500 = 200$, which yields $y = 40$.
7. **E** Notice that it’s “250 percent greater,” NOT “250 percent of.” Also notice the “approximately” that tells you it’s okay to use a nicer number, such as \$360,000. You can use the percent change formula on this one: percent change = $(\text{Difference} \div \text{original}) \times 100$. So, $250\% = (\text{Difference} \div 360,000) \times 100$. Solve for Difference = \$900,000. YOU’RE NOT DONE! That’s the difference between the years. Add this to the original, and you’ll get the profits for 2002 (which is what the question is asking for): $\$360,000 + \$900,000 = \$1,260,000$.
8. **D** To know the probability, we need the “number of outcomes we want,” which is the number of female interns and the total number of possibilities. The question stem gives us the total possibilities: 35 offices. Thus, all we need from the statements is some way to find the number of female interns. Statement (1) gives us that info. Cross out (B), (C), and (E). Statement (2) also gives us that information (assuming the only options are male and female, we can subtract the male interns from the total interns to get the number of female interns).

ADMISSIONS INSIGHT

Application Timeline

Here's the business school game plan:

January–May

- Research schools.

- Take GMAT prep course and study hard.

June–July

- Take the GMAT.

- Request official undergraduate transcripts.

- Consider a business school admissions counseling service.

- Secure recommenders.

July–August

- Complete and submit GMAC Additional Score Reporting (ASR) form if needed.

- Consider applying online.

- Start those essays.

- Follow up with recommenders.

- Update resume.

September

- Fine-tune essays.

- Stay in touch with recommenders; make sure they can meet deadlines.

October–November

- Submit first applications.

- Send thank-you notes to recommenders.

November–December

- Submit more applications.

January–March

- Start planning for fall.

ASSIGNMENT 3

ARGUMENTS

Arguments is The Princeton Review's name for the critical reasoning questions. Arguments questions typically make up about 30 percent of the Verbal section. You can expect to see about twelve arguments questions on your exam. Each question is composed of a short passage, a question about that passage, and five answer choices.

With the Princeton Review's approach to arguments, you will learn how to analyze the reasoning in an argument, identify the different types of arguments questions, and determine the answer by using Process of Elimination (POE). This lesson introduces some key skills needed to solve arguments questions: identifying argument components and recognizing common reasoning errors.

PARTS OF AN ARGUMENT

Becoming an active reader will help you work arguments questions efficiently and accurately. For most questions on the GMAT, begin by breaking the argument down into its parts. Three connected parts make up an argument. The first two, the **conclusion** and **premises**, are stated explicitly in the argument, while the third part, the **assumption**, is unwritten. Let's look at each of the parts in more detail.

Conclusion

The conclusion is the main point or central claim of the argument. Think of an argument as a television commercial. After you read the argument, ask yourself, "What is the author trying to sell me?" The claim the author wants you to accept is the conclusion of the argument.

Often, you will see indicator words that will help you find the conclusion. Common conclusion indicator words include:

- Therefore
- Clearly
- Thus
- So
- Hence
- Consequently
- In conclusion

Premises

Once you have found the conclusion, identify the premises—any reasons, statistics, or evidence—the author provides to support the conclusion. Premises usually sound like facts, rather than opinions. Even if you disagree with the premises provided as support for a conclusion, you must accept them as true for the purposes of the GMAT. Sometimes indicator words can help you find the premises. Look for words like:

- since
- because
- as a result of
- suppose

The Why Test

Indicator words can help you find the conclusion and premises, but not every argument uses them. The most reliable method for identifying these parts is the **Why Test**. Once you have found the conclusion, ask yourself why the author believes the conclusion to be true. The premises should provide the answer to the question. If you try the Why Test and the answer does not make sense, you have probably reversed the conclusion and premises.

Let's break down an example:

Cream cheese contains 50 percent fewer calories per tablespoon than does butter or margarine. Therefore, a bagel with cream cheese is more healthful than is a bagel with butter on it.

First, find the conclusion. The word “Therefore” is a conclusion indicator. The conclusion of this argument is that “a bagel with cream cheese is more healthful than is a bagel with butter on it.” Next, let's use the Why Test to confirm that we have correctly identified the conclusion. Ask, “Why is a bagel with cream cheese more healthful than a bagel with butter?” The answer is, “Cream cheese contains 50 percent fewer calories per tablespoon than does butter or margarine.” Since the information makes sense as support for the conclusion, we know we correctly broke down the argument. Had we reversed the premise and conclusion, the Why Test would have failed. It is not logical to conclude that, “Cream cheese contains 50 percent fewer calories per tablespoon than does butter or margarine” because “Therefore, a bagel with cream cheese is more healthful than a bagel with butter on it.” Always use the Why Test to separate the conclusion and premises.

Use the Why Test to identify the conclusion and premises.

Drill: Conclusions and Premises

Using indicator words and the Why Test, identify the conclusion and premise(s) in the following examples.

1. In a free society people have the right to take risks as long as they do not harm others as a result of taking the risks. Therefore, it should be each person's decision whether or not to wear a seat belt.

Conclusion:

Premise(s):

3. Last year, the city of Melville increased the size of its police force by fifty officers. This year, there was a 10 percent decrease in the number of violent crimes reported in Melville. Clearly, a larger police force discourages criminal activity.

Conclusion:

Premise(s):

2. Companies have found that giving workers the option of flexible hours leads to happier employees. Happier employees are more productive. Company X has flexible hours. Company Z operates on a strict 9 to 5 workday. Company X will certainly beat Company Z in worker productivity.

Conclusion:

Premise(s):

Find the Gap

Now that you are getting the hang of reading for the conclusion and premises, let's move on to the next step in analyzing an argument. You probably found the preceding argument unconvincing. That's because, like most GMAT arguments, they contain faulty reasoning. Whether or not you agree with an argument's conclusion, the reasoning leading to that conclusion likely contains a flaw. The test writers construct arguments with questionable reasoning so that they can test your ability to identify and describe these errors.

Identify the flaws in an argument by looking for a gap between the conclusion and premises. The premises rarely provide enough evidence to lead convincingly to the conclusion. **Find the gap by determining what is mentioned in the conclusion that was not mentioned in the premises.** Once you have found the gap, you can make the argument work by filling in the assumption. **The assumption is the unstated part of the argument that is required to connect the premises to the conclusion.** Though not explicitly stated by the author, the assumption must be true for the argument to be well reasoned. If you want to weaken an argument, widen the gap between the conclusion and premises by attacking the assumption.

Let's examine the cream cheese argument again:

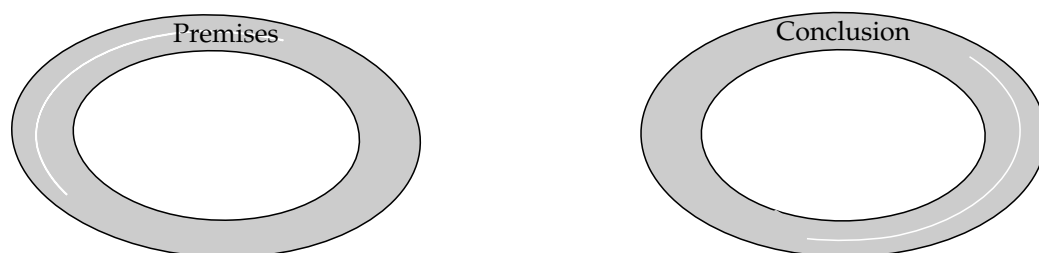
Conclusion: A bagel with cream cheese is more healthful than is a bagel with butter on it.

Premise: Cream cheese contains 50 percent fewer calories per tablespoon than does butter or margarine.

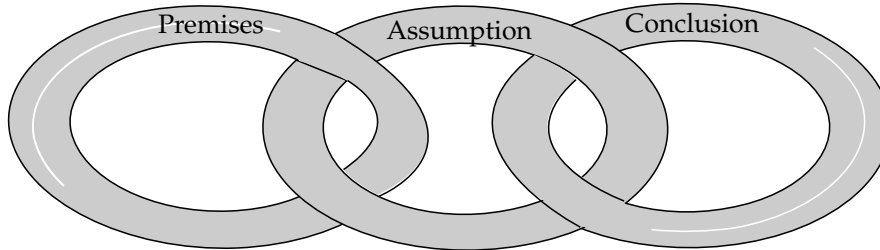
Look for the gap
between the premises
and conclusion.

The author broadened the scope of the argument from calories (in the premise) to the more general statement about health in the conclusion. Thus, there is a gap between "fewer calories" and "more healthful." The argument's assumption must link these two ideas. To fill in the gap, we must accept that having fewer calories per set amount is enough to qualify one food as more healthful than another. We also need to believe that people use similar quantities of butter, margarine, or cream cheese on a bagel, proving that the combination of bagel and cream cheese has fewer calories than the combination of bagel and butter. These two assumptions are necessary to make a valid argument. If we wanted to weaken the argument, we would need to attack the assumptions. For example, we could say that most people use significantly more cream cheese on their bagels than they do butter. Someone who uses more than twice as much cream cheese as butter would actually consume more calories. Even if calories were the only determining factor in healthfulness, the bagel with cream cheese would be less healthful.

Think of GMAT arguments as an incomplete chain of reasoning:



When you state the assumption, you provide the missing link that fills in the gap and completes the chain:



Identifying the gap is the most important step in analyzing arguments. If you have difficulty finding the gap in an argument, slow down and think about the argument. If an argument seems logically sound, you have probably made the same assumption as did the author. Examine your thought process and think about why you believe the conclusion makes sense. Then think about whether any of your beliefs might be false. Ask yourself, “How could the premises remain true, but the conclusion be false?” If you can think of a way to invalidate the conclusion without disputing the premises, you have located the logical gap between the premises and conclusion.

COMMON FLAWS

If you learn to recognize these common flaws, you will find it much easier to work arguments questions.

Causal Flaws

Frequently, an argument’s premises state that two things happened, and the author concludes that one caused the other. Causal arguments are by far the most common type of logical flaw you will encounter on the GMAT. Look at an example:

A study indicated that adults who listen to classical music regularly are less likely to have anxiety disorders. Clearly, classical music helps calm the nerves and lower anxiety.

What’s the gap between the conclusion and premises?

The author concludes that classical music helps calm the nerves and lower anxiety because the study found a correlation between listening to classical music and experiencing a lower likelihood of having anxiety disorders. However, the fact that two things are related does not prove that one caused the other. To make the causal link in this argument, we must assume that listening to classical music was the only factor responsible for lowering anxiety. We must rule out the possibility that any other factors played a role. We must also rule out the idea that having a lower anxiety level causes people to listen to classical music. To break the causal link, we could show that another factor explains the lower rate of anxiety disorders in people who listen to classical music. For example, if we knew that a majority of people who listen to classical music take anti-anxiety drugs, we might conclude that the drugs, not the music, lowered their anxiety. We could also weaken the argument by showing the causality is reversed. If we knew that calmer people were predisposed to enjoy classical music because it reflects their mood, the conclusion would be invalid.

Sampling and Statistical Flaws

When you encounter an argument based on percentages, numbers, or samples, the flaw usually relates to the failure of the data to prove the conclusion. If you look at all evidence skeptically, you will be able to find these arguments more easily. These types of arguments are less common than causal arguments, but they still appear on the GMAT fairly often.

Sampling arguments reach a conclusion based on evidence about a subset of a group. They assume that the subset is typical and reflects the larger group. Arguments about survey results usually fall into this category. Look at an example:

Contrary to popular belief, high school students overwhelmingly approve of the high school administrative staff. We know this to be true because the student council expressed admiration for the high school principal and her staff in the council's editorial for the school paper.

What's the gap between the conclusion and premise?

The author concludes that students approve of the school administration based on the student council's opinion as expressed in the paper. The gap is between the student council and the general student body, and the author draws on a sample population to reach a conclusion about the whole population. To make the link, we must assume that the editorial is an accurate reflection of the feelings of the general student population. We could break the link by proving the student council's view does not represent the views of the rest of the students. For example, maybe the student council is made up of sycophants who want to get favorable college recommendations from members of the administration.

Whenever an author bases a conclusion about a general population on a sample or survey, remain skeptical. The author assumes that the part of the population sampled or surveyed is representative of the entire population. To strengthen a sampling argument, provide a reason why the sample is representative of the whole. To weaken a sampling argument, show that the sample is not necessarily representative of the whole.

Statistical arguments hinge on a questionable interpretation of numerical data. Most often, the author confuses percentages with actual values. Look at an example:

Ninety percent of the population of Prelandia lived in rural areas in 1800. Today, only 20 percent of the population lives in rural areas. Clearly, more people lived in the countryside two centuries ago.

What's the gap between the conclusion and premises?

The author believes that since the percentage of people living in rural areas decreased, the actual number of people living in rural areas must have decreased. Arguments involving percentages are often math problems. Remember that percentages compare a part to a whole. In this case, the percentages tell us:

| |
|-------------------------|
| <u>Rural population</u> |
| Total population |

Whether or not that is true depends on how the past and present populations compare. To make the link, we need to prove that the total population in the past and present are comparable. For example, if the population was 100 people in 1800 and 100 people today, we could say that 90 people lived in rural areas in the past and 20 people live there today. The argument would be valid. To attack the argument, we need to prove that the total population has changed. For example if, the country's population was 100 people in 1800, 90 of them lived in rural areas. If the population is 1,000 people today, then 20 percent would be 200 people. More people actually live in rural areas now, even though their percentage in the population has decreased. The argument is invalid.

Arguments that involve a comparison of percentages assume that the percentages are based on comparable totals. Be wary whenever an author uses information from percentages to draw conclusions about actual values and vice versa. To strengthen statistical arguments, prove that the author's interpretation of the figures is valid. To attack statistical arguments, add information that calls the author's interpretation of the statistics into question. Weakening statistical arguments most often involves demonstrating that the author incorrectly compared two percentages or confused percentages with actual values.

Analogy Flaws

Some arguments use evidence about one thing to reach a conclusion about another. These arguments assume that two things are similar enough to sustain the comparison. Look at an example:

Contrary to opponents' charges that a single-payer health-care system cannot work in a democratic nation such as the United States, an overhaul of the American health-care system is necessary. Opponents of the single-payer system in the United States should remember that Canada, a nation with a strong democratic tradition, has run a viable single-payer health-care program for many years.

What's the gap between the conclusion and premise?

The author concludes that because a single-payer health-care system works in Canada, it will work in the United States. The gap is between the United States and Canada, and we must assume they are similar enough to make the comparison valid. To strengthen this argument, we could add additional reasons why the two nations may be compared. We could weaken the argument by suggesting reasons why the comparison is not valid. For example, we could say that the differences in the populations and economies of the two nations mean that policies that work in one country won't work in the other.

SUMMARY

To analyze an argument:

Find the *conclusion* and *premises*.

Use the *Why Test* to ensure you have correctly identified each part. Identify the conclusion, and ask why it's true. Restate the conclusion and premises in your own words, and write them down in shorthand form on your scratch paper.

Weed out the useless information.

Some arguments contain information that is neither the premise nor a conclusion. Sometimes the test writers provide background information to introduce a topic, or they use extra statements as a device to hide the information that you need. Using the *Why Test* will help you find the important information in the argument.

Find the gap.

Look for words, ideas, or conditions mentioned in the point that were not mentioned in the reasons. The weak point of every argument lies in the gap between the indisputable (for the purposes of the GMAT) reasons and the disputable point.

Common Flaws

Causal Arguments

In a causal argument, the author assumes that an observed result can be explained by a single cause, factor, or reason. Causal assumptions can be phrased in a few different ways. An author might wrongly assume:

- Because two things are related, one caused the other (correlation equals causality).
- There are no other possible causes of or explanations for a result.
- The causality did not occur in reverse.
- To strengthen a causal argument, add a premise that supports the cause cited by the author or rule out other causes. To weaken a causal argument, add a premise that shows another cause of the result, that the cause and effect are reversed, or that one event can occur without the other.

Sampling and Statistical Arguments

When you encounter sampling or statistical arguments, accept the facts provided in the premises as true. Find the gap by questioning the conclusion the author draws using those facts. Strengthen these arguments by showing the sample or statistics are sufficient to prove the point. Weaken these arguments by showing the author misinterpreted the evidence. Show the sample is not representative of the whole or that the percentages are not representative of the total populations.

Analogy Arguments

When an author bases a conclusion on a comparison, he or she assumes that the items are similar and that what is true for one is true for the other. Strengthen these arguments by providing another reason why the items in question are comparable. Weaken these arguments by showing the items are not similar.

DRILL

This drill gives you a chance to put together all you have learned so far about analyzing arguments. For each argument, find the point, reasons, and gap. Then try to either state the questionable assumption or what is wrong with the reasoning. Be on the lookout for the common flaws discussed above.

1. A study of drinking habits shows that the rate of heart disease among those who drink one or two drinks a day (based on one drink = 1 ounce of 80-proof distilled spirits) is significantly lower than it is among those in the population at large. The study also shows that among those who drink excessively (six or more drinks each day), the rate of severe depression is much higher than it is among the general population. It was concluded from this evidence that level of alcohol consumption is a determining factor in the development of certain physical and psychological disorders.
 Conclusion:

 Premise(s):

 Gap:

 Flaw/Assumption(s):
2. Fortunately for the development of astronomy, observations of Mars were not very precise in Kepler's time. If they had been Kepler might not have discovered that the curve described by that planet was an ellipse, and he would not have discovered the laws of planetary motion. There are those who complain that the science of economics is inexact, that economic theories neglect certain details. That is their merit. Theories in economics, like those in astronomy, must be allowed some imprecision.
 Conclusion:

 Premise(s):

 Gap:

 Flaw/Assumption(s):
3. The ancient Egyptian pharaoh Akhenaten, who had a profound effect during his lifetime on Egyptian art and religion, was well loved and highly respected by his subjects. We know this from the fierce loyalty shown to him by his palace guards, as documented in reports written during Akhenaten's reign.
 Conclusion:

 Premise(s):

 Gap:

 Flaw/Assumption(s):
4. Until he was dismissed amid great controversy, Hastings was considered one of the greatest intelligence agents of all time. It is clear that if his dismissal was justified, then Hastings was either incompetent or disloyal. Soon after the dismissal, however, it was shown that he had never been incompetent. Thus, one is forced to conclude that Hastings must have been disloyal.
 Conclusion:

 Premise(s):

 Gap:

 Flaw/Assumption(s):

5. Sixty adults were asked to keep a diary of their meals, including what they consumed, when, and in the company of how many people. It was found that at meals with which they drank alcoholic beverages, they consumed about 175 calories more from nonalcoholic sources than they did at meals with which they did not drink alcoholic beverages. Therefore, those wishing to restrict their caloric intake should refrain from drinking alcoholic beverages with their meals.

Conclusion:

Premise(s):

Gap:

Flaw/Assumption(s):

ANSWERS AND EXPLANATIONS

Drill: Conclusions and Premises

1. Conclusion: It should be each person's decision whether or not to wear a seat belt. Premise(s): In a free society, people have the right to take risks as long as they do not harm others as a result of taking the risks.
2. Conclusion: Company X will certainly beat Company Z in worker productivity. Premise(s): Flexible hours lead to happier employees, who will be more productive. Company X has flexible hours and Company Z does not.
3. Conclusion: A larger police force discourages criminal activity. Premise(s): Melville increased the size of its police force and saw a decrease in reports of violent crime.

Drill

1. Conclusion: Level of alcohol consumption is a determining factor in certain physical and psychological disorders. Premises: In the study, low alcohol consumption correlates with a low rate of heart disease, and high consumption correlates with a high rate of depression. Gap: The new idea in the conclusion is that alcohol was a "determining factor," in other words, the cause of the observed results. This is a causal argument that views the correlation between alcohol and disease found by the study as evidence that alcohol is a causal factor in disease. To make that link, we must assume the only reason the first group had a low rate of heart disease was its low level of alcohol consumption. The only reason the second group had a high rate of depression was its high level of alcohol consumption. We also need to rule out a reverse of cause and effect. We must believe that disease had no effect on drinking habits.
2. Conclusion: Economic theory must be allowed imprecision. Premise: The Kepler example shows it is good that theories in astronomy were not always precise. Gap: The premise deals with astronomy, but the conclusion deals with economics. Flaw/ Assumption: The author quite clearly

compares astronomy and economics, yet she does not explain why it is acceptable to compare the two fields. We must assume that astronomy and economics are similar. This argument also has a causal aspect. The author assumes the Kepler example is sufficient to show that inexact theories have been beneficial to astronomy. She fails to consider that this might not be true in all cases.

3. Conclusion: The ancient Egyptian pharaoh Akhenaten...was well loved and highly respected by his subjects. Premise: According to reports written during Akhenaten's reign, his palace guards were fiercely loyal to him. Gap: The conclusion talks about love and respect from his subjects, but the premise talks about loyalty of the guards. Flaw/Assumption(s): This argument has several flaws. The argument takes a particular fact (the reports of palace guards' feelings toward the pharaoh) and interprets it to have some specific meaning. The argument assumes that the guards were representative of the pharaoh's subjects as a whole. It also assumes that the reports accurately reflect the palace guards' feelings and that loyalty is equivalent to love and respect.
4. Conclusion: Hastings must have been disloyal. Premise: If his dismissal was justified, he must have been either incompetent or disloyal. He was not incompetent. Gap: This one is a little tricky. The premise contains a condition (if his dismissal was justified), but the conclusion doesn't take into account whether the dismissal really was justified. Flaw/ Assumption: The dismissal was justified. This is an example of a causal argument with a condition. When an argument involves a condition, you must assume the condition holds true. If we accept that the dismissal was justified, then only two possible causes remain, and if one cause is ruled out (incompetence), the other must apply (disloyalty).

5. Conclusion: Those wishing to restrict their caloric intake should refrain from drinking alcoholic beverages with their meals.
Premise: According to their diaries, the sixty adults in a sample group consumed more calories from food when they drank with the meal than they did when they didn't drink with the meal. Gap: The premise deals with a sample of sixty people, but the conclusion seems to be aimed more broadly. Also, the correlation between alcohol consumption and calories in the premise is interpreted as a causal relationship in the conclusion. Flaw/ Assumption: Again, as with all survey arguments, this one makes the assumption that the sixty adults in question were representative of those wishing to restrict their caloric intake (the group for whom a recommendation is made in the conclusion). This is, in addition, a causal argument. While it doesn't seem especially plausible that the causation could be reversed—that, in other words, eating more could cause them to drink—this argument does definitely assume that there's no third cause of both the drinking and the increased caloric intake at some meals. It may be, for instance, that the sample group (and people in general) most often drink when celebrating some special occasion over a meal, and that in such cases they also tend to eat more.

RATIOS AND STATISTICS

This lesson focuses on ratios, mean, mode, median, range, and standard deviation.

RATIOS

Earlier we dealt with fractions, decimals, and percents, which express $\frac{\text{part}}{\text{whole}}$ relationships. Ratios also express a relationship between two or more values,

but with one big difference: ratios describe $\frac{\text{part}}{\text{part}}$ relationships.

For example, consider a recipe for punch that calls for 2 parts grape juice to 3 parts orange juice. The relationship, or ratio, of grape juice to orange juice is 2:3. It doesn't matter if you are mixing up one glass of punch or a whole pitcher—the relationship remains constant. Don't confuse ratios with fractions. In this example, the 2 parts grape juice and 3 parts orange juice add up to 5 total parts.

Thus, the fraction of grape juice in the mixture is $\frac{2}{5}$.

All of the following are equivalent ways of writing this ratio:

- the ratio of grape juice to orange juice is 2 to 3
- the ratio of grape juice to orange juice is 2 : 3
- the ratio of grape juice to orange juice is $\frac{2}{3}$

Knowing that the ratio of grape juice to orange juice is 2: 3 does not tell you the actual amount of grape juice or orange juice. We could have a small glass or a large vat of this punch, but if the recipe is followed, then dividing the amount of grape juice by the amount of orange juice will always give $\frac{2}{3}$.

A ratio tells you only the relative amounts of each quantity. It does not, by itself, tell you the actual amounts.

RATIO BOX

If you're given a ratio and want to determine actual amounts, you need more information. Let's look at an example:

1. In a club with 35 members, the ratio of men to women is 2 to 3.
How many men belong to the club?

- ☐ 2
☐ 5
☐ 7
☐ 14
☐ 21

| | MEN | WOMEN | TOTAL |
|---------------|-----------------------|-----------------------|-----------------------|
| Ratio | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ |
| Multiply by | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ |
| Actual Number | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ | $\frac{\quad}{\quad}$ |

We'll use the Ratio Box to keep track of the information. Start by plugging in the ratio numbers from the problem, and add them up to get your *ratio whole*. You have one actual number, namely 35 members, which is the *actual whole*. Fill in that box, and figure out the *multiplier*. The *ratio whole* is 5 ($2 + 3$), and you must multiply 5 times 7 to get the *actual whole*, 35. To keep the ratio intact, the same multiplier must be applied to all parts. Thus, the actual number of men is $2 \times 7 = 14$.

| | MEN | WOMEN | TOTAL |
|---------------|-------------------|-------------------|-------------------|
| Ratio | $\frac{2}{\quad}$ | $\frac{3}{\quad}$ | $\frac{5}{\quad}$ |
| Multiply by | $\frac{7}{\quad}$ | $\frac{7}{\quad}$ | $\frac{7}{\quad}$ |
| Actual Number | 14 | 21 | 35 |

Let's try ratios with data sufficiency.

2. Rachel throws a cocktail party for her friends. At the party, she serves martinis, screwdrivers, and boileermakers. How many martinis did Rachel serve at the party?
 - (1) Rachel served martinis, screwdrivers, and boileermakers in a ratio of 5:7:9 respectively.
 - (2) Rachel served a total of 35 screwdrivers.

Statement (1) gives you a ratio, and nothing else. Remember that a ratio alone tells you nothing about the actual amounts. If you need proof, draw a ratio box and try to find the multiplier. You won't be able to! Narrow it down to BCE. Now look at Statement (2), forgetting all about Statement (1). That Rachel served 35 screwdrivers does not tell us anything about the number of martinis or boileermakers she might have served. Eliminate (B). Now consider the statements together. Can you find a multiplier and fill in your ratio box? Since you know the ratio number for screwdrivers is 7, and the actual number of screwdrivers is 35, the multiplier must be 5 ($35 = 5 \times 7$). The multiplier applies to all parts of the ratio, so you have enough information to determine the number of martinis served. However, don't waste time solving a data sufficiency question. Once you realize that you CAN solve it, you know that the statements together are sufficient. The answer is (C), and you don't need to fill in the whole box.

Here's another type of ratio problem:

3. If $3u = 5v$, then the ratio of $5u$ to v is

- ☐ 1:3
☐ 3:1
☐ 4:3
☐ 15:1
☐ 25:3

Since the problem involves variables, let's Plug In. What are some easy numbers to plug in for u and v ? How about $u = 5$ and $v = 3$? In that case, the ratio of $5u$ to v would be 5×5 to 3, or 25:3, so the best answer is (E).

Proportions

Some problems set two ratios equal to one another. These fixed relationships are called **proportions**. For example, the relationship between hours and minutes is fixed, so we can set up a proportion: $\frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{3 \text{ hours}}{180 \text{ minutes}}$.

The key to doing a proportion problem is to set one ratio equal to another, making sure to keep your units in the same places.

Let's try an example:

4. On a certain map, Washington, D.C., and Montreal are 4 inches apart. If Washington, D.C., and Montreal are actually 500 miles apart, and if the map is drawn to scale, then 1 inch represents how many miles on the map?
- ☐ 125
☐ 150
☐ 250
☐ 375
☐ 500

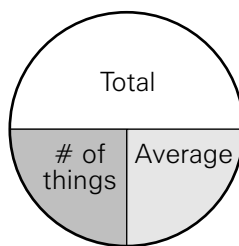
The information tells you that the *ratio* of inches to miles is 4: 500. Those are equivalent units, since 4 inches on the map is the same as 500 miles. Put those equal amounts in a fraction, like this: $\frac{4 \text{ inches}}{500 \text{ miles}}$. Next, set up the other side, keeping units in the same places in the fraction: $\frac{4 \text{ inches}}{500 \text{ miles}} = \frac{1 \text{ inch}}{x \text{ miles}}$. Now solve for x . You can cross multiply to get $4x = 500$, so $x = 125$, which is choice (A).

AVERAGES

On the GMAT, *average* is also called *arithmetic mean*, or simply *mean*. All averages are based on this equation:

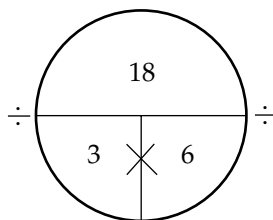
$$\text{Average} = \frac{\text{Total}}{\text{Number of Things}}$$

Drawing an Average Pie will help you organize your information.



Here's how the Average Pie works. The *total* is the sum of the numbers you are averaging. The number of *things* is the number of quantities you are averaging, and the *average* is, of course, the average.

Let's apply the Average Pie to a simple example. Say you wanted to find the average of 3, 7, and 8. You would add up the numbers and then divide by 3 $\left(\frac{3+7+8}{3} = \frac{18}{3} = 6\right)$. Here's how to organize the same information in the average pie:



The horizontal line across the middle means *divide*. If you have the total and the number of things, divide to get the average. If you have the total and the average, divide to solve for the number of things. If you have the average and the number of things, multiply to get the total. As you will see, the key to most average questions is finding the total. You may also need more than one pie if a problem involves multiple averages. Draw a separate pie for every average in a problem.

The Average Pie is a great tool because it:

- Organizes the information clearly.
- Allows you to solve for one piece of the pie when you have the other two.
- Helps you to focus on what else you need to solve the problem, which is great for data sufficiency.

Let's try a problem.

1. The average (arithmetic mean) weight of three people is 160 pounds. If one of these people weighs 200 pounds, what is the average weight, in pounds, of the two remaining people?

☐ $73\frac{1}{3}$

☐ 140

☐ 160

☐ 240

☐ 480

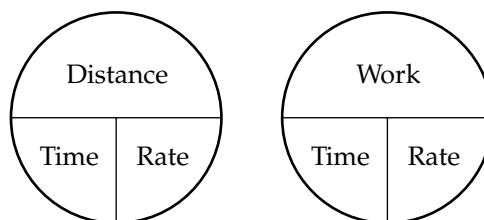
Draw an Average Pie, and fill in the first set of information. You know the average of three things is 160, so can you multiply to get the total (480). Notice that you can cross off (E) as a trap. The question asks for the average weight of the two remaining people, so you're not done yet. Draw another Average Pie for the next average in the problem. There are two remaining people, but you don't know their total weight. However, you can get their total weight by subtracting 200 from the total for all three (480). Since $480 - 200 = 280$, you know the total weight of the other two people is 280. To get the average, just divide your new total by the number of people, $\frac{280 \text{ pounds}}{2 \text{ people}} = 140$. Choice (B) is the answer.

RATES

Rate problems are very similar to average problems. They often ask about average speed or distance traveled. Other rate problems ask about how fast someone works or how long it takes to complete a task. All of these problems involve this important relationship:

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}} \text{ or } \text{Rate} = \frac{\text{Amount of Work}}{\text{Time}}$$

Because this relationship is identical to that in the average formula, you can use the *Rate Pie* to organize your information:



The Rate Pie has the same advantages as the Average Pie. Whenever you have two pieces of the pie, you can solve for the third. Let's put the Rate Pie to work.

- It takes Mike 1 hour and 30 minutes to commute from home to work at an average speed of 40 miles per hour. If Mike returns home along the same route at an average speed of 45 miles per hour, how long does the return trip take?
 - ☐ 1 hour, 15 minutes
 - ☐ 1 hour, 20 minutes
 - ☐ 1 hour, 25 minutes
 - ☐ 1 hour, 30 minutes
 - ☐ 1 hour, 35 minutes

Draw a Rate Pie, and fill in the information you know about the first trip: Mike's rate was 40 mph, and the time it took was 1 hour and 30 minutes, or 1.5 hours. Now you can solve for total distance by multiplying: $40 \times 1.5 = 60$ miles. That's the same distance he'll need to travel on the way home, so make another Rate Pie for the return trip, and fill in that piece. Now you also know the return trip average speed is 45 miles per hour, so go ahead and put that in too. So the time for the return trip can be found by $\frac{60}{45} = \frac{4}{3} = 1\frac{1}{3}$ hours, or one hour and twenty minutes. The answer is (B).

Let's try another one, this time with work.

- A machine at the golf ball factory can produce 16 golf balls in 5 minutes. If several of these machines work independently and each machine performs at the same rate, how many machines are needed to produce 32 golf balls per minute?
 - ☐ 3
 - ☐ 6
 - ☐ 8
 - ☐ 10
 - ☐ 13

Put the information you have into a Rate Pie. You know that one machine produces 16 balls in 5 minutes, so fill in those pieces. That gives you a rate of $\frac{16}{5} = 3.2$ balls per minute. Then set up a proportion, since each machine works at the same rate, $\frac{1 \text{ machine}}{3.2 \text{ balls}} = \frac{x \text{ machines}}{32 \text{ balls}}$. Cross-multiply and you get $x = \frac{32 \times 1}{3.2} = 10$, choice (D).

MEDIAN, MODE, AND RANGE

You're already familiar with the *mean*, which is another word for *average*. Here are some other terms you need to know:

Median is the number in the middle after your set of numbers has been arranged in ascending order. If the set has an even number of elements, the median is the average of the two numbers in the middle.

Mode means the most frequently occurring number (or numbers) in the set.

Range is the difference between the highest and the lowest numbers in the set.

Okay then, in the following set of numbers, can you identify the mean, median, mode, and range?

{2, 25, 10, 6, 13, 50, 6}

Mean =

Median =

Mode =

Range =

To find the **mean**, add the numbers to find the total, and divide by the number of things in the set. The total is $2 + 25 + 10 + 6 + 13 + 50 + 6 = 112$. There are 7 numbers in the set, so the average, or mean, is $112 \div 7 = 16$.

To find the **median**, put the numbers in order from least to greatest. That gives you 2, 6, 6, 10, 13, 25, 50. Because this set has an odd number of elements, the median is just the number in the middle, **10**. If we take out the number 50 from this set, what is the new median? Now there is an even number of things, so take the two numbers in the middle, 6 and 10, and average them: $\frac{6+10}{2} = 8$.

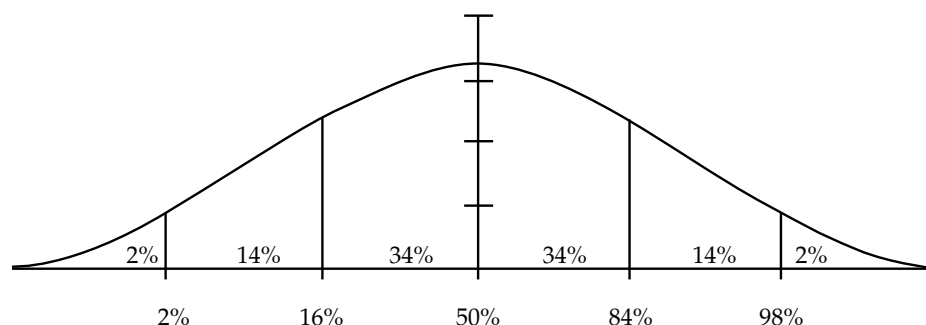
To find the **mode**, look for the number that occurs most often. Since only the number **6** shows up more than once, it's the mode.

To find the **range**, subtract the lowest number from the highest. That's $50 - 2 = 48$.

Fortunately, the problems that involve these terms usually aren't very difficult, as long as you don't confuse their meanings.

STANDARD DEVIATION

Standard deviation is another statistical term like mean, median, range, and mode. If you see a problem that uses the terms *normal distribution* or *standard deviation*, think about ETS's favorite object in the whole wide world: the bell curve.



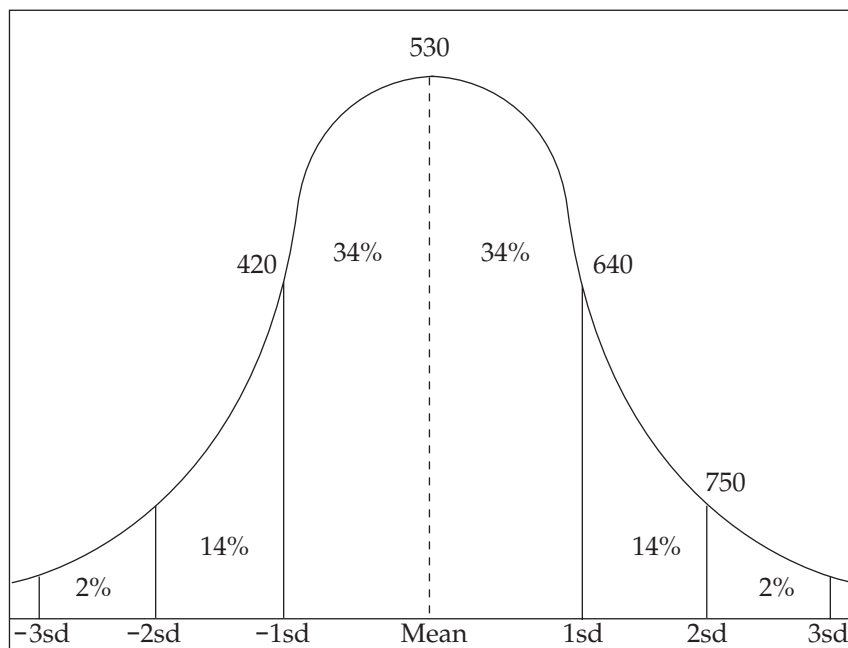
Any *normal distribution* of a set of values can be plotted along this bell curve. A few key parts to our curve:

- The **mean** is indicated by the line down the center of the curve. In questions dealing with standard deviation, the mean will either be given to you, or it will be easy to figure out.
- The **standard deviation** is a statistically derived specified distance from the mean. In the figure above, the standard deviations are represented by the solid lines. You will never have to calculate the standard deviation from the data alone. GMAT problems tell you the standard deviation for a set of data.
- The percentages indicated in the picture represent the portion of the data that fall between each line. These percentages are valid for **any** question involving a normal distribution, and you should memorize them: 34: 14 : 2. The percentages correspond to the 1st, 2nd, and 3rd standard deviations on each side of the mean.

Working with standard deviation on the GMAT is mostly a matter of drawing the curve and filling in the information. Let's take the case of an exam scored on a scale of 200 to 800.

Use the bell curve to organize your information.

If the mean on this exam is 530, and the standard deviation (sd) is 110, what percent of test takers score between 420 and 750?



Use the bell curve to organize your information.

Start with the bell curve. Label what you know: the mean is 530, and each standard deviation is 110 points, so the first sd above the mean is at a score of $530 + 110 = 640$, and the second sd above the mean is $640 + 110 = 750$. You could go and figure out the third one, but the question only asks for scores up to 750, so you should save yourself the effort. You still need lower scores, so the first sd below the mean is $530 - 110 = 420$. We have enough information to answer the question. Add up the percentages in your included portions of the curve, from 420 to 750, we have $34\% + 34\% + 14\% = 82\%$.

As long as you understand how to draw the chart and you memorize the percentages that correspond to each standard deviation, standard deviation problems are very manageable.

DATA SUFFICIENCY TRICKS AND TRAPS

Some data sufficiency questions contain “tricks” designed to fool the unwary test taker. The trick usually involves some assumption that people tend to make or some esoteric math concept that people forget. Beware of the trap answers!

Each of the following questions contains a trap. Try to spot and avoid the trap as you work the question. Don’t read the explanation until you’ve tried the question on your own.

1. What is the value of x ?

- (1) $x > 9$
- (2) $x < 11$

Statement (1) is not enough because x could be any number above 9, so BCE. Statement (2) is not enough because x could be any number below 11, so eliminate B. If you chose (C), you probably assumed that x was 10 because 10 is the only integer greater than 9 but less than 11. However, nothing in the question requires x to be an integer. For example, it could be 10.2 or 10.5. Since there’s no way to determine a single value for x , choose (E).

2. If $xy \neq 0$, what is the value of $\frac{x^4y^2 - (xy)^2}{x^3y^2}$?

- (1) $x = 2$
- (2) $y = 8$

The natural instinct is to go straight for (C) because it provides values for both variables. However, (C) is a trap. This question nicely illustrates why you should go through the step-by-step AD/BCE approach. If you substitute the information from Statement (1) into the expression, you get:

$$\frac{16y^2 - 4y^2}{8y^2} = \frac{12y^2}{8y^2} = \frac{3}{2}.$$

Thus, Statement (1) alone is sufficient to determine the value, and the only possible answers are (A) and (D). If you substitute the information from Statement (2) into the expression, you get:

$$\frac{64x^4 - 64x^2}{64x^3} = \frac{64(x^2 - 1)}{64x^3} = \frac{(x^2 - 1)}{x^3}.$$

Since you haven't solved for a value, Statement (2) is insufficient, and the answer is (A). Another way to work this problem is to factor y^2 out of each part of the expression right at the start. Once you realize that y^2 cancels out of the expression, it's clear that you only need to know the value for x .

3. John is driving from Town A to Town B. What is his average speed over the entire trip?

- (1) He drives the entire 120 miles in 3 hours.
- (2) His maximum speed during the trip was 50 miles per hour and his minimum speed was 30 miles per hour.

Because you have distance and time, Statement (1) is sufficient to find the average speed, or rate ($120/3 = 40$ mph). The only possible answers are (A) and (D). If you chose (D), you probably assumed that Statement (2) also told you the average speed was 40 mph. What you learned from the first statement may have influenced your reading of the second statement. However, you can never average the averages; you must always derive the average from the total and number of things. Choose (A).

4. How far is Town A from Town C?

- (1) Town A is 160 kilometers from Town B.
- (2) Town B is 155 kilometers from Town C.

Statement (1) isn't enough because Town C is not mentioned, so choices (B), (C), and (E) remain. Statement (2) isn't enough because it omits Town A. That leaves (C) and (E). If you chose (C), you probably assumed that towns A, B, and C are arranged in a straight line and that you could add up the distances ($160 + 155 = 215$). However, the actual order could be A—C—B, or the towns could form a triangle. Choose (E).

When you work data sufficiency questions be wary of:

- **The Integer Trap:** Don't assume numbers are integers unless the problem tells you so.
- **The Variables Trap:** Don't assume you need to find values for all the variables in a problem, especially if the question contains an expression with more than one variable. Try manipulating or simplifying the expression before you read the statements.
- **The Statements Trap:** Forget about Statement (1) when you consider Statement (2). Be sure to consider each statement by itself. Combine the information in the two statements only if each statement alone is insufficient and you're down to (C) or (E).
- **The Diagram Trap:** Don't assume a diagram looks a certain way unless the problem tells you so.

SUMMARY

Ratios

- A **ratio** expresses a $\frac{\text{part}}{\text{part}}$ relationship.
- For any problem that gives you a ratio and an actual number, make a **Ratio Box**.
- Plug In on ratio problems with variables.

Proportions

- A **proportion** is an equation that sets two ratios equal to each other.
- For problems that give you 3 out of 4 variables and asks you to solve for the fourth, set up your proportions (equal ratios), making sure to keep your units in the same places, and cross-multiply.

Averages

- For average problems, use the equation:

$$\text{Average} = \frac{\text{Total}}{\text{Number of Things}}$$

- Use the Average Pie to organize your information.
- For data sufficiency, ask yourself, "Do I have enough information to fill in the missing piece of the puzzle?"

Rates

- For problems dealing with rates of work or travel, use the equation:

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}} \text{ or } \text{Rate} = \frac{\text{Amount of Work}}{\text{Time}}$$

- Use the Rate Pie to organize your information.

Median, Mode, and Range

- The **mean** is the average of all the numbers in a set.
- The **median** is the middle number in a set.
- The **mode** is the number (or numbers) that appear(s) most frequently in a set.
- The **range** of a set is the difference between the highest number and the lowest number in the set.

Standard Deviation

- On the GMAT, problems dealing with standard deviations, or normal distributions, are about the bell curve.
- The mean, or average, is the center of the bell curve.
- A standard deviation is a specific distance away from the mean.
- The percentages 34%, 14%, and 2% correspond to the 1st, 2nd, and 3rd standard deviations on each side of the mean.

DRILL

1. If a certain concrete mixture contains gravel, water, and sand in a 1 to 3 to 5 ratio by weight, how many pounds of gravel would be needed to produce 72 pounds of the mixture?

☐ 72
☐ 40
☐ 24
☐ 9
☐ 8

2. A disc jockey plays only hip-hop and country-western records. If the disc jockey plays four country-western records for every seven hip-hop records he plays, what fraction of the records he plays are country-western?

☐ $\frac{4}{5}$
☐ $\frac{7}{11}$
☐ $\frac{4}{7}$
☐ $\frac{4}{11}$
☐ $\frac{1}{28}$

3. It's rumored that in 1976, Elvis consumed three times his body weight in peanut butter and banana sandwiches. If Elvis's body weight in 1976 was 250 pounds, and if a peanut butter and banana sandwich weighs four ounces, then how many such sandwiches did Elvis consume in 1976? (1 pound = 16 ounces)

☐ 750
☐ 1,000
☐ 1,500
☐ 3,000
☐ 4,000

4. What is the ratio of x to y , given that $xy \neq 0$?

(1) $3x = 4y$
 (2) $x = 4$

5. The average (arithmetic mean) of eight numbers is 7. If two numbers are discarded, the average of the remaining numbers is $6\frac{1}{2}$. What is the average of the two discarded numbers?

☐ $\frac{3}{4}$
☐ $1\frac{1}{2}$
☐ $3\frac{1}{2}$
☐ $8\frac{1}{2}$
☐ 17

6. The average (arithmetic mean) of 6, 21, x , and y is 13, where x and y are integers with a product of 100. Which of the following could be x ?

☐ 50
☐ 20
☐ 10
☐ 4
☐ 1

7. If a , b , c , d , and e are consecutive integers in increasing order, what is the average of the five numbers?

(1) $a = 13$
 (2) $a + e = 30$

8. For the set of positive, distinct integers $\{v, w, x, y, z\}$, the median is 10. What is the minimum value of $v + w + x + y + z$?

☐ 25
☐ 32
☐ 36
☐ 40
☐ 50

9. The sum of the numbers in the set $\{5, 28, 10, 50, x, 7\}$ is 120. What is the median of the set?
- ☐ 10
☐ 15
☐ 18
☐ 20
☐ 24
10. For the set of numbers $\{20, 14, 19, 12, 17, 20, 24\}$, let x equal the median, v equal the mean, w equal the mode, and y equal the range. Which of the following is true?
- ☐ $v < w < x < y$
☐ $v < x < w < y$
☐ $y < v < w < x$
☐ $y < v < x < w$
☐ $w < y < v < x$
11. The mean score on a certain test is 80. If the test scores have a normal distribution and the standard deviation is 7, what percent of examinees score between 66 and 94?
- ☐ 50%
☐ 68%
☐ 82%
☐ 96%
☐ 98%
12. A courier van delivers a package from office A to office B. If the van travels the 20 miles between the two offices at 50 miles per hour, how fast must the van travel on the return trip if the round-trip travel time is to be 40 minutes?
- ☐ 55 mph
☐ 60 mph
☐ 65 mph
☐ 70 mph
☐ 75 mph
13. Runners X and Y started an 18-mile race at the same time. Runner X completed the course in 6 hours, and Runner Y finished 2 hours earlier. Runner Y ran an average of how many miles per hour faster than Runner X ?
- ☐ 1
☐ $1\frac{1}{2}$
☐ $2\frac{1}{4}$
☐ 3
☐ $4\frac{1}{2}$
14. The Lewiston Road Race is how many kilometers long?
- (1) It takes 2 more hours to complete the race at an average speed of 40 kilometers per hour than at an average speed of 50 kilometers per hour.
 (2) At an average speed of 20 kilometers per hour, it took Brett 10 hours to complete half the race.
15. Three typists worked to type a single manuscript. What percent of the total number of words was typed by the slowest of the three typists?
- (1) All three typists typed steadily and independently for 4 hours at the rates of 40, 50, and 60 words per minute to complete the manuscript.
 (2) The fastest typist typed 150% as fast as the slowest.

ANSWERS AND EXPLANATIONS

1. E Set up a Ratio Box:

| | Gravel | Water | Sand | Total Concrete |
|---------------|------------|-------|------|----------------|
| Ratio | 1 | 3 | 5 | 9 |
| Multiplier | $\times 8$ | | | $\times 8$ |
| Actual Number | 8 | | | 72 |

The total number of parts is then $1 + 3 + 5 = 9$. The multiplier is 8, because $9 \times 8 = 72$. Use the same multiplier for gravel, and $1 \times 8 = 8$. Note: you could also ballpark out some of the larger answer choices, like (A) and (B).

2. D The ratio of country-western to hip-hop records is 4 to 7, but the question asks, “what fraction of the records” are country-western, so you want the $\frac{\text{part}}{\text{whole}}$, which is $\frac{4}{4+7} = \frac{4}{11}$.
3. D There are a couple of different proportions to set up. First, convert ounces to pounds, $\frac{16 \text{ oz}}{1 \text{ lb}} = \frac{4 \text{ oz}}{x \text{ lbs}}$. Cross-multiply to get $16x = 4$, $x = \frac{1}{4}$ lb, or 0.25 lb per sandwich. Elvis ate a total of three times his body weight of 250 lbs, or 750 lbs, worth of sandwiches. The next proportion is: $\frac{1 \text{ sandwich}}{0.25 \text{ lbs}} = \frac{x \text{ sandwiches}}{750 \text{ lbs}}$. Cross-multiply to get $750 = 0.25x$, so $x = \frac{750}{0.25} = 3,000$ lbs.
4. A Set up AD/BCE. Fact (1) gives you $3x = 4y$. It seems you can’t solve for x or y , but then again, you don’t have to! You are just trying to get the ratio, so rearrange your equation to get $\frac{x}{y} = \frac{4}{3}$, and that’s sufficient! Cross off BCE. Looking at Fact (2), $x = 4$ alone is insufficient without any information about y . Eliminate (D).
5. D Draw some average pies! The first pie is for all 8 numbers, which average 7. This pie is going to be split into two groups, so you need two more pies, one for the remaining 6 numbers, and one for the discarded two. The average for the remaining 6 numbers is 6.5, which means the total is 39. Subtract that from the previous total of 56 from the first pie, 8×7 , to get the remaining total 17. 17 is the total for the two discarded number pie, so the average is $8\frac{1}{2}$.
6. B If the average of the four elements is 13, their total must be $13 \times 4 = 52$, so $6 + 21 + x + y = 52$, and thus $x + y = 52 - 21 - 6 = 25$. So if x and y add up to 25 and their product is 100, then they could be 20 and 5. Since 5 is not an option, the answer is 20.

7. **D** What is the missing piece of the puzzle? To get an average, you need a total, and number of things. We have 5 things, so you just need the total. Fact (1) tells you $a = 13$, and the other numbers are consecutive integers, so $b = 14$, $c = 15$, $d = 16$, $e = 17$, and we know we can add those up. Get rid of BCE. Fact (2) tells you $a + e = 13$, which looks like not enough information, but because you know the numbers are consecutive, e must be 4 numbers bigger than a and you can figure out both, and all the numbers in between. If you are doubtful, plug in values for a and e , and the only ones that work are 14 and 17.
8. **C** The key here is that we're trying to find the *minimum* value of five *distinct, positive* integers where the median is 10. If the middle number is 10, the smaller two numbers must be 1 and 2, and the larger two numbers must be 11 and 12, and $1 + 2 + 10 + 11 + 12 = 36$.
9. **B** They tell us that the numbers in the set $\{5, 28, 10, 50, x, 7\}$ have the sum of 120, so we can figure out that $x = 20$, which also happens to be the mean. But we're looking for the median. Rearranged, the set now looks like $\{5, 7, 10, 20, 28, 50\}$. Since 10 and 20 are the numbers in the middle, we'll split the difference and get 15.
10. **D** It looks like we're going to have to calculate the mean, median, mode, and range for $\{20, 14, 19, 12, 17, 20, 24\}$. Do the easy calculations first: The median $x = 19$, and the only recurring number, 20, is mode w . The range $y = 24 - 12 = 12$. The only answer choice in which $y < x < w$ is (D), so we don't have to calculate the mean after all.
11. **D** The range from 66 to 94 represents 14 points above and below the average. If the standard deviation is 7, then 14 points represents two standard deviations. Two standard deviations in each direction is $14 + 34 + 34 + 14 = 96\%$ of all examinees.
12. **E** Make a Rate Pie! Plug in 20 miles for distance on top, and 50 miles per hour for the rate. The time it takes is then $\frac{20}{50} = \frac{2}{5}$ hour, or $\frac{2}{5}(60) = 24$ minutes. The round-trip time needs to be 40 minutes, so the time we have left for the return trip is $40 - 24 = 16$ minutes, which is $\frac{16}{60} = \frac{4}{15}$ hour. So now we can make another Rate Pie, for the distance of 20 miles, and time of $\frac{4}{15}$ hours, so the rate is $\frac{20}{\frac{4}{15}} = 20 \times \frac{15}{4} = 75$ miles per hour.
13. **B** Make a rate pie for both Runner X and Runner Y. Runner X goes the 18-mile distance in 6 hours, so the speed is $\frac{18}{6} = 3$ miles per hour. Runner Y finished 2 hours earlier, in 4 hours, so the rate is $\frac{18}{4} = 4\frac{1}{2}$ miles per hour. Finally, $4\frac{1}{2} - 3$ is $1\frac{1}{2}$, which is (B).
14. **D** Set up AD/BCE. What is the missing piece of the puzzle? In order to get a distance, you need both rate and time. Fact (1) gives you both rates and times. Prove it to yourself by making 2 Rate Pies, one for a rate of 40 kilometers per hour, and one at a rate of 50. The times are Time + 2 and Time, respectively. Since the two distances are the same, you could say $40(T + 2) = 50T$. One equation, one variable, so it's solvable! Now check out Fact (2). We have Brett's speed, and also the time for half the race, but you can double it and get the full time.
15. **A** Set up AD/BCE. Then figure out the missing piece of the puzzle. To get the percent of total number of words, you'll need the total number, plus some information on how much of the total the slowest typist typed, which you could also get from the rates. Looking at Fact (1), which gives each individual's rate and time, we could figure out each typist's total number of words, and therefore the percentage of the slow one, so eliminate BCE. The second fact tells us the relative rates of two of the typists, but we have no information about the third person, so this is insufficient, and you're left with (A).

ADMISSIONS INSIGHT

When to Apply

The filing period ranges anywhere from six to eight months. Therefore, application timing can be key. While there are no guarantees, early applications often have better chances. This is mainly because there's more space available early on. As the application deadline nears, spaces fill up. As the deadline draws close, the greatest number of applicants compete for the fewest number of spaces. In addition, admissions officers have little clue about how selective they can be when the first applications come in. An early application may be judged more on its own merit than how it stacks up against others. So admissions decisions may be more generous at this time. Finally, early applications show a strong interest. The admissions committee sees those who apply early as especially enthusiastic about their school.

Online Applications

Applying online is increasingly preferred by students and schools alike, especially business schools. **PrincetonReview.com** has online applications to more than a hundred business schools. There are no extra fees to apply through our site, and the process is completely efficient: Once you fill out the application profile, all subsequent applications will be pre-populated with that information.

How Criteria are Weighted

While exact admissions requirements vary from institution to institution, most rely on some combination of the following criteria (not necessarily in this order): GMAT score, undergraduate grade point average (GPA), work experience, essays, letters of recommendation, interviews, and extracurriculars. Of these criteria, the first four are usually the most heavily weighted. As a general rule of thumb, the more competitive the school, the less room there is for weakness in any one of these areas.

Most applicants suspect that the GMAT score or GPA pushes their application into one of three piles: "yes," "no," or "maybe." But unless one or more of your numbers is so low it forces a rejection, the piles are "looks good," "looks bad," "hmmm, interesting," and all variations of "maybe." In MBA admissions, the whole is greater than the sum of the parts. Each of the numbers has an effect but doesn't provide the total picture.

The system promotes fairness by allowing applicants to compensate for problem areas. Even a mediocre GPA can be counter-balanced with a high GMAT score, quantitative work experience, or the completion of an accounting or statistics course.

No one single thing counts more than everything else. Scores, grades, work experience, and essays should give the admissions committee a clear idea of your capabilities, interests, and accomplishments. Any particular weakness can be overcome by a particular strength in another area, so emphasize strengths and downplay weaknesses.

GMAT Scores

Virtually every MBA program in the U.S. requires their applicants to take the GMAT. In fact, one of the notable holdouts—Harvard—started requiring the GMAT again for their Fall 1997 admissions. Great GMAT scores won't necessarily get you into the school of your choice (there are too many other factors), but low scores will almost certainly keep you out.

As a general rule, if your scores are more than 50 points below a school's average, you are probably facing an uphill battle. If this is the case, consider taking the GMAT again. Business schools generally focus on the most recent score.

A common myth is that a GMAT score of 700 or higher is mandatory to be considered a competitive applicant. However, only seven percent of all GMAT examinees score at that level. While some schools report an average GMAT score around 700, this is simply an average, which means a sizeable proportion of the class scored below 700. And even if you are among the 7 percent who break the 700 mark, acceptance to a particular school is not guaranteed. The GMAT score is only one of several admissions factors.

Use PrincetonReview.com to find all the stats on potential business schools. The site contains lists of the average GMAT score and GMAT score range of a school's students, as well as students' average undergrad GPA, average months of work experience, and the percentage of applicants accepted.

The Essay

Admissions committees consider essays the clincher, the swing vote to admit or deny. This is because the essays offer the most substance about who a candidate really is. The GMAT and GPA reveal little about a person. While your work history provides a record of performance and justifies your stated desire to study business, the essays tie all the application pieces together and create a summary of your experiences, skills, background, and beliefs.

On the upside, an essay can reveal positive traits—creativity, witty, open-mindedness, articulateness, maturity. Likewise, an essay can showcase negative characteristics, such as arrogance, sloppiness, or inability to think and write clearly. In essence, the essays are the inside picture.

What Makes a Good Essay?

While there is no proven formula for success, all good essays have certain things in common. First, they are well-written and organized. Second, they contain arguments that are well-supported with specific examples from either personal or professional experience relevant to the essay's topic. Third, they reveal something unique, something that convinces an admissions officer that the writer has something valuable to add to their mix of students.

Several months before the application is due, read the essay questions and begin to formulate responses. Could you describe an ethical dilemma at work? Are you involved in anything outside the office (or classroom)? Use the essay questions as a framework for your personal and professional activities. Look back over your business calendar for specific situations from your work life. Keep your eyes open for an instance that involves questionable ethics. If nothing from work seems to apply, get involved in activities that round out your background and provide thoughtful essay material.

Each school has its own set of essay questions. Although posed differently, all search for the same insights. Take the time to decipher what the admissions committees are really after and craft the essays accordingly.

ASSIGNMENT 4

READING COMPREHENSION

Reading comprehension questions make up roughly one-third of the questions in the Verbal section. You can expect to see four reading passages on your test, and three or four questions will accompany each passage. The passages can occur at any point in the section, but all questions related to a passage will be presented together.

Reading comprehension intimidates many test takers because of the sudden influx of information on the screen. The passages tend to be dense, wordy, and boring. When you know the clock is ticking, it's hard to pay attention to a passage about the migratory habits of the monarch butterfly or an innovative management strategy. The good news is that everything you need to know to answer the questions can be found in the text. Although the passages cover a variety of topics, you do not need, and should not use, any outside knowledge.

The Princeton Review's approach to reading comprehension will help you to:

- Read the passages efficiently and seek out the most important information.
- Understand the different types of questions and what they require you to do.
- Quickly find the credited response by eliminating incorrect answers.

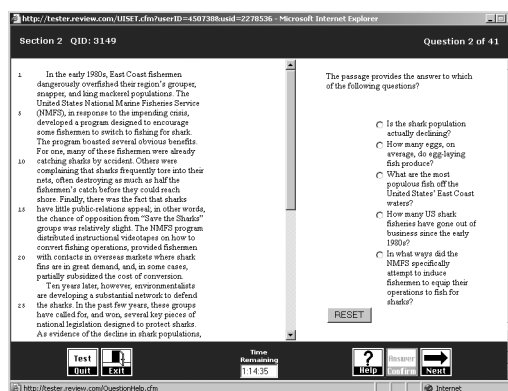
We'll begin by working on the core skills needed to work reading comprehension questions: reading the passage, translating the questions, and paraphrasing the answers.

PASSAGE TOPICS

GMAT passages cover a variety of topics, but they come from three primary areas:

- **Social Science:** social or historical issues, such as the civil rights movement or World War I.
- **Science:** natural science (biology, geology, physics, etc.) topics, such as photosynthesis or black holes.
- **Business:** business-related topics, such as management strategies or the loosening of international trade restrictions.

THE FORMAT



The passage appears on the left side of the screen. In most cases, it's too long to fit in the window, so you must use the scroll bar to read it completely. The questions appear on the right side of the screen, one at a time. After you answer the first question, another appears in its place. The passage remains so you can always refer to it.

USE YOUR SCRATCH PAPER

One factor that makes reading on the GMAT challenging is that you have to read the passage and questions from the screen. You can't underline or make notes on the passage as you would on a paper-and-pencil exam. Since there is no printed passage for you to annotate, use your scratch paper to keep track of the passage's key ideas and structure.

WORK THE PASSAGE

Think about how average test takers react to a reading passage. What do you think they do first? ETS knows that people try to read the passage carefully, understand all the details, and then answer the questions. In fact, the test writers want you to become bogged down in the passage. Since only a few questions accompany each passage, it makes little sense to waste time reading the passage thoroughly. You don't get any points for reading the passage. Time spent on reading useless parts of the passage can be put to better use working the questions.

Read for the Big Idea

To break down arguments, you learned to look for the conclusion and premises. Think of reading comprehension passages as long arguments, and look for similar features. When you read a passage, identify the key parts, and note them on your scratch paper. Your notes will help you to understand the passage's structure and important points the author makes. Ultimately, your reading and notes should lead you to the Big Idea. That is, if someone asked you, "What's the big idea of this passage?" you could respond with a clear, concise sentence that would summarize the main point.

Trigger Words

Certain words tell a lot about the structure of the passage.

Changes of Direction

If a sentence begins, “On the other hand,” you would expect to find a change of direction in the sentence. These structural signposts show an alert reader what’s going to happen next.

Here are some common trigger words and phrases that indicate a change of direction:

- But
- Nonetheless
- Although
- Notwithstanding
- However
- Except
- Yet
- While
- Despite
- Unless
- Nevertheless
- On the other hand

Same as It Ever Was

Some words let you know that there will be no change in the direction of the paragraph or passage. If you see “first of all,” you expect to see another reason that continues in the same vein.

Here are some common trigger words and phrases that indicate the author will continue in the same direction or elaborate on a point:

- In addition
- By the same token
- Likewise
- Similarly
- For example
- Thus

When you work a passage, read for the key ideas. Pay close attention to the first and last sentences of the first paragraph, the first sentence of each body paragraph, and the last sentence of the passage. Skim over the details. Look for and note the following information:

Main Point/Thesis

Look for the author’s main point or thesis statement, which is most often found in the first paragraph or at the end of the passage.

Author’s Tone or Attitude

Look for positive or negative assessments of the subject. Use a “+” to note positive opinions and a “–” to note negative opinions.

Main Idea/Function of Each Paragraph

Central ideas most often occur at the beginning or end of a paragraph. Make a brief note that describes the main idea of each paragraph.

Changes of Direction or Conflicting Opinions/Theories

Look for trigger words that signal a change of direction. Make a note of where the passage contains a change of viewpoint or introduces another theory.

More of the Same

Look for trigger words that signal a continuation of direction or elaboration on a topic.

If you read for the main features, you'll be able to get through a passage in just a few minutes. Let's work a sample passage one paragraph at a time. Use the space provided to jot down a few words that encapsulate the main idea of each paragraph.

What does "however" do to the paragraph?

5 Retail stores and diversified manufacturing companies have operated under a set of traditional assumptions that warrant challenge. The most basic assumption that company managers make is that their companies should provide a high level of service to all their customers. However, acting on this assumption can lead to loss of market share, less value for some customers, and maintenance of unwieldy structures for distributing products, which ensures higher fixed costs.

If we look closely at the first and last sentences, what do we know? Stores and companies make assumptions the author wants to challenge. The word "however" in the last sentence lets us know that the author opposes the idea that companies should provide a high level of service to all customers. Once you have the main idea, it isn't necessary to pay attention to the details in the rest of the paragraph. You may have noticed the end of the last sentence listed specific examples. We don't really care what the examples are, but we know where to look for them.

Try reading the second paragraph in the way we suggested above.

What does "one area" indicate?

10 According to Joseph Fuller, James O'Connor, and Richard Rawlinson, one area that needs particular scrutiny is the way in which companies handle logistics, including transportation costs, handling costs, management of inventory, storage costs, and order processing. Different
15 customers often have entirely different needs: designing a logistical structure that provides every customer with the same level of service is wasteful and inefficient. Fully one-third of a company's product may be stuck in the "pipeline" between manufacturer and customer, where it only
20 drains away money through transportation and storage costs; if a particular customer does not need certain products to be available immediately, a company does not need to spend money to ensure that all its merchandise is on hand. Another problem is traditional averaging, in which
25 products that cost the manufacturer relatively little to produce are given prices similar to those of products that are expensive to produce. While this means that the retailer is able to move more low-volume products out of inventory, high-volume products tend to be overpriced, and more specialized products are not delivered speedily
30 enough and may be underpriced.

How does "another problem" help you understand the structure of the passage?

You should have noticed that this paragraph focuses on two problems, logistics and averaging. If you went into more detail than that, you're working too hard!

Now for the last paragraph:

- Fuller et al. describe a soft drink company's decision to stop in-store promotions and special sales in favor of standard pricing because the inconsistent demand caused
- 35 by the swings in prices necessitated variability in the manufacturing and distribution systems. Many retail managers tend to overlook logistics out of a concern for gross margin; that is, they are swayed by the gross profit made by the sale of a specific item, instead of looking at
- 40 the net profit that remains after logistics costs have been subtracted. Low-volume, high-margin products may not ultimately be as profitable as high-volume, low-margin products that are easy to move around, such as T-shirts or calculators.

What does the first sentence do?

The first sentence gives us a specific example, a soft drink company, to explain one problem with logistics, the "one area," mentioned in the first paragraph. The last sentence tells us that high-margin products may not be as profitable as low-volume products. The author clearly believes that challenging traditional assumptions can have positive results.

Now that we've made it through the entire passage, let's think about its organization. Notice how each paragraph develops concepts from the previous paragraph. The first paragraph says that some traditional assumptions about service need to be challenged. The second paragraph raises two problems that can occur when companies stick to traditional assumptions. The third paragraph uses an example to show that going against traditional assumptions can be a good thing.

Becoming comfortable with this reading technique may take some time, but it will get easier with practice. When you read a GMAT passage, think about how the passage as a whole is linked together. State the main idea of the passage in your own words. Consider the author's tone or attitude. Understanding the general tone of the passage will give you a clearer sense of what to look for when you are eliminating answer choices.

TRANSLATE THE QUESTION

Once you've worked the passage, it's time to move on to the questions. Some questions are pretty straightforward:

Which of the following best describes the organization of the passage?

Others are not:

The author's discussion of the traits that distinguish the economic system of Japan from the economic systems of Western countries is primarily concerned with explaining

If you don't understand what the question is asking, you are much less likely to find the correct answer. Take time to translate wordy, confusing questions into everyday language. The question above could be reworded to read, "What do the differences between Japanese and Western economic systems help explain?" To translate a reading comprehension question:

- Read slowly.
- Don't get bogged down by difficult language.
- Turn it into a simple, straightforward question.

Another example:

Which of the following would be most inconsistent with the author's claim that Peter Cooper did not lack influence within the business community in late nineteenth-century New York City?

When you cut away the extraneous stuff and clarify the wording, this question asks, "Which choice shows that Cooper had limited influence?"

As with the examples above, the key to getting started on reading comprehension is to find clear, effective translations of awkwardly phrased questions.

Quick Quiz: Translating Questions

Translate the following questions.

1. In his discussion of the changes that occurred during the second half of the fifteenth century, the author implies which of the following?

2. The passage suggests that one reason suppliers would benefit most from the reallocation of funds previously earmarked for retailers is that

3. The reference to labor negotiation as an example of third-party intervention is meant to illustrate which of the following theories in the context of the author's discussion of triadic and quadratic models?

4. The passage implies that if the experiments mentioned had not taken place in a vacuum, which of the following could have occurred?

5. Of which of the following hiring policies mentioned in the second paragraph would the company mentioned be most clearly in violation?

6. The passage implies that the sudden decrease in the number of children at homeless shelters was an indirect result of

7. The researcher's conclusion concerning the increase in blue-green algae would be most seriously undermined if it were found that

8. The author's claim that the behavior of certain unusual stars has helped "account for previously unexplained phenomena" (lines 35–36) would be most justified by an astronomer's ability to measure the luminosity and pulsation of which of the following types of stars?

9. The passage implies that the tasks performed by the mice in Group A differed from those performed by the mice in Group B in that the mice in Group A

10. According to the passage, the contract standards for Global Com permitted new subsidiaries to include which of the following in their standards of operations?

PARAPHRASE THE ANSWER

Keep in mind that reading comprehension is an open-book test; all the answers are there for you to find. The trouble, of course, is that the test writers go out of their way to disguise the information. Since the correct answer is almost always a paraphrase of the information in the passage, the more adept you become at paraphrasing the answer, the less painful reading comprehension will be. You'll be better able to recognize the best answer and eliminate wrong answers. Do your best to answer the question in your own words before you go to the answer choices.

Read this sentence from a passage:

Musculoskeletal diseases may also respond to the manipulations of a practitioner of Trager work, a technique that increases functionality by releasing the accumulated tension and blockages associated with accidents, stress, and fear.

Often, the test writers disguise information in complex sentences so that you'll waste time trying to figure out what the sentence means. Don't become intimidated by long sentences or unfamiliar vocabulary. Dissect long sentences into smaller parts, just as you break long math problems into bite-sized pieces. We know the sentence above deals with musculoskeletal diseases and something called Trager work that seems to help by releasing tension.

Quick Quiz: Paraphrase the Passage

Try these examples. Choose the best paraphrase from the three options given.

- Although it is tempting to arrange living species in a series of presumed primitive to advanced stages, series of this sort may be misleading if they are taken to represent evolutionary lineages because each species is specialized for its own ecological niche, and most species combine primitive and advanced features.
 - ☐ Plants and animals can be clearly categorized into advanced or primitive species.
 - ☐ It is a misrepresentation to order different species into set sequences.
 - ☐ Determining the physiological makeup of groups can lead to an understanding of a species' past.
- Since an object moving across a visual field stimulates a set for a short period, it creates onset and offset patterns similar to those of an intermittent light.
 - ☐ When an object travels briefly across your line of vision, it triggers an effect not unlike that of a strobe light.
 - ☐ When visual stimuli quickly enter and exit your sight range, they generate an open/close effect within the eye that stimulates the eye's innermost cells.
 - ☐ Since the cells are stimulated for a short period of time, an object in front of the eye will cause intermittent stimulation.

3. Information about a novelist's life can provide important critical insight into his or her novels, but such information should be subordinated to ideological investigation.
 - ☐ One should always investigate the political ideology of an artist to gain critical insight into that artist's work.
 - ☐ Key events in the life of an author can provide insight into the author's work, but should be of secondary concern to an examination of the author's belief system.
 - ☐ Those who write seem to develop critical insight into the political ideology of others, even as they gain a better understanding of their own beliefs.

Paraphrase the following excerpts. Write down your own understanding of the key idea.

4. Although modern culinary training continues to emphasize the 250 basic competencies that were first codified by the French as fundamental to mastering a classic culinary repertoire, the new influx of fusion and pan-Asian-influenced restaurants in the United States has succeeded in subverting the hold of this classic, and some say anachronistic, methodology.

5. The evidence that stomach ulceration may, in fact, result from bacterial infection by *Helicobacter pylori* has been accumulating since the 1970s, but has only been granted validity by the scientific establishment since 1994.

6. The advent of this new information technology has not been the panacea technophiles had promised; however, electronic collection and analysis of pharmaceutical data has indeed become a routinized procedure, but one with unforeseen pitfalls.

7. Resultant criticism of the book's strong moral stance often took the form of derogatory remarks in local papers. One critic in Kansas City, in fact, criticized the author for using a language "so elevated in tone as to be virtually unintelligible."

SUMMARY**Work the Passage**

- Read for the main idea and organization of the passage. Don't get bogged down by the details.

Translate the Question

- Take a moment to put the question in your own words and think about where to look in the passage for the answer.

Paraphrase the Answer

- Go to the passage to find the information you need to answer the question. Put the information into your own words before you look at the answer choices.

DRILL

Let's return to the passage we previewed earlier. Using what you've learned about translating questions and paraphrasing answers, work the questions below.

Retail stores and diversified manufacturing companies have operated under a set of traditional assumptions that warrant challenge. The most basic assumption that company managers make is that their companies should provide a high level of service to all their customers. However, acting on this assumption can lead to loss of market share, less value for some customers, and maintenance of unwieldy structures for distributing products, which ensures higher fixed costs.

According to Joseph Fuller, James O'Connor, and Richard Rawlinson, one area that needs particular scrutiny is the way in which companies handle logistics, including transportation costs, handling costs, management of inventory, storage costs, and order processing. Different customers often have entirely different needs: designing a logistical structure that provides every customer with the same level of service is wasteful and inefficient. Fully one-third of a company's product may be stuck in the "pipeline" between manufacturer and customer, where it only drains away money through transportation and storage costs; if a particular customer does not need certain products to be available immediately, a company does not need to spend money to ensure that all its merchandise is on hand. Another problem is traditional averaging, in which products that cost the manufacturer relatively little to produce are given prices similar to those of products that are expensive to produce. While this means that the retailer is able to move more low-volume products out of inventory, high-volume products tend to be overpriced, and more specialized products are not delivered speedily enough and may be underpriced.

Fuller et al. describe a soft drink company's decision to stop in-store promotions and special sales in favor of standard pricing because the inconsistent demand caused by the swings in prices necessitated variability in the manufacturing and distribution systems. Many retail managers tend to overlook logistics out of a concern for gross margin; that is, they are swayed by the gross profit made by the sale of a specific item, instead of looking at the net profit that remains after logistics costs have been subtracted. Low-volume, high-margin products may not ultimately be as profitable as high-volume, low margin products that are easy to move around, such as T-shirts or calculators.

- According to the passage, when management assumes that its top priority is customer service, it risks
 - ☐ streamlining its logistical structure to lower fixed costs
 - ☐ losing business to competitors
 - ☐ weighing inventory in favor of high-margin volume products
 - ☐ customer dissatisfaction due to steadily rising prices
 - ☐ having a majority of its merchandise stuck in the "pipeline"
- Which of the following may be inferred from the passage about traditional averaging?
 - ☐ Businesses using this pricing method will face declining market share.
 - ☐ Companies with large inventories are likely to employ this method to reduce stock.
 - ☐ It leads to significant fluctuations in the inventory levels of specialized products.
 - ☐ It can create inconsistencies in the pricing of goods with respect to their manufacturing costs.
 - ☐ The technique is only effective for certain nonspecialized products.
- According to the passage, a soft drink company rejected former marketing methods that included promotions and special sales because
 - ☐ stores complained about having to make special arrangements for storing the extra products
 - ☐ high-volume products such as soda are consistently overpriced
 - ☐ the company realized it had given too much attention to the gross margin of some of its products
 - ☐ the promotions and sales earned the company high profits, but when the promotions ended, the profits dropped
 - ☐ the promotions and sales created uneven demand, which continually forced the company to change its logistical operations

4. It can be inferred from the passage that which of the following might be the most profitable item to sell?

- ☐ cassette tapes
- ☐ big-screen television sets
- ☐ bunk beds
- ☐ rowing machines
- ☐ imported wine

ANSWERS AND EXPLANATIONS**Quick Quiz: Translating Questions**

1. What were the changes during the second half of the fifteenth century?
2. How would suppliers benefit from the money retailers used to receive?
3. Which theory does labor negotiation represent?
4. How does a vacuum affect the experiment?
5. Which hiring policies is the company violating?
6. Why did the number of homeless kids decrease?
7. What would weaken the researcher's conclusion?
8. What star types would account for unexplained phenomena?
9. How did the mice differ in their tasks?
10. What do the contract standards say about operating standards?

Quick Quiz: Paraphrase the Passage

1. **B**
2. **A**
3. **B**
4. Fusion and pan-Asian restaurants in the U.S. are weakening the hold of traditional culinary training.
5. Evidence has been around since the 1970s but has only recently been accepted.
6. Technology has made data collection routine, but the system still has bugs.
7. Some people didn't like the book's moral stance.

Drill

1. **B** Question Translation: What's wrong with having customer service as top priority? Lines 6–10 say that concentrating too heavily on customer service might damage your business and cost you market share. *Losing business to competitors* is a nice paraphrase of *loss of market share*.
2. **D** Question Translation: What do we know about traditional averaging? Lead words *traditional averaging* appear in Lines 27–36. By traditional averaging, high volume products *tend to be overpriced* and more specialized products *may be underpriced*. Another nice paraphrase.
3. **E** Question Translation: Why did the soda company stop doing promotions and special sales? Lead words *soft drink company* appear in Lines 36–42. The company used standard pricing to respond to *inconsistent demand*. *Uneven demand* is a blatant paraphrase.
4. **A** Question Translation: What kinds of products did the passage say might be most profitable? Lines 47–50 dictate that *high-volume, low-margin products* are more profitable. Cassette tapes are the best example of this sort of product.

EXPONENTS, ROOTS, AND FACTORING

In this lesson, we'll review the rules for working with exponents, roots, and quadratic expressions.

DISTRIBUTION

Let's look at a mathematical rule that is useful for working many GMAT problems.

Work the following examples:

$$4 \times 8 =$$

$$4(3 + 5) =$$

$$(4 \times 3) + (4 \times 5) =$$

What did you notice about all three of the problems above? The solution to each one is 32. For the second example, you added 3 and 5, then multiplied the sum by 4. In the third example, you multiplied first and then added the products. These examples illustrate the **distributive rule**. You can use this property when two numbers that are being added or subtracted have common factors. As we'll see in this lesson, the distributive property is especially helpful when you need to simplify because it lets you factor out a common term. The distributive rule states:

$$ab + ac = a(b + c) \quad (2 \times 4) + (2 \times 7) = 2(4 + 7)$$

$$ab - ac = a(b - c) \quad (2 \times 7) - (2 \times 4) = 2(7 - 4)$$

EXPONENTS

Exponents are a shorthand notation for repeated multiplication. Exponents tell us to multiply a number or a variable by itself a certain number of times. For example, $5 \times 5 \times 5$ can also be written as 5^3 .

Basic Rules

There are three basic rules to remember about exponents.

When **multiplying** terms with the same base, **add** the exponents:

$$x^4 \cdot x^3 = x^7$$

$$6^3 \cdot 6^2 = 6^5$$

When **dividing** terms with the same base, **subtract** the exponents:

$$\frac{x^8}{x^5} = x^3 \quad \text{and} \quad \frac{(\sqrt{3})^7}{(\sqrt{3})^3} = (\sqrt{3})^4$$

When an expression with an exponent is **raised to another exponent**, **multiply** the exponents:

$$(x^3)^2 = x^6 \text{ and } (6^4)^3 = 6^{12}$$

When in doubt, expand it out.

What happens if you forget one of the rules? You can bet that ETS will fill the answer choices with every wrong thing that you could have done. There's a way around that, however. Take a look at the last example. You could rewrite it as:

$$(x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = x^6$$

Let's try a few problems.

1. If $y \neq 0$, then $\frac{8(y^2)^3 y^6}{2y^3 y^2} =$
- ☐ $4y^{30}$
 - ☐ $8y^7$
 - ☐ $4y^6$
 - ☐ $4y^7$
 - ☐ $8y^{30}$

Let's start with the numerator. First, multiply the exponents raised to a higher power, $(y^2)^3 = y^6$. That makes the top $8y^6 y^6$. Now, add the exponents to get $8y^{12}$. For the bottom, add the exponents to get $2y^5$. Now we have $\frac{8y^{12}}{2y^5}$. Since we're dividing, we need to subtract the exponents and reduce the numbers, which leaves $4y^7$. The answer is (D).

2. The number 70^6 is how many times the number 35^6 ?

- ☐ 2
- ☐ 6
- ☐ 9
- ☐ 35
- ☐ 64

The values in this problem make it impractical to multiply out the terms. Exponent problems like this may seem difficult, but they become easier if you rearrange the information. Translating the problem into an equation gives us $70^6 = 35^6 x$. When you see a problem like this, rewrite the numbers in comparable terms. Since 70 is the same as 2×35 , we can rewrite the equation as $(2 \times 35)^6 = 35^6 x$. Remember the distributive property we discussed earlier? It works for exponents, so we distribute the exponent to get $2^6 \times 35^6 = 35^6 x$. This version of the equation tells us that $2^6 = x$. Since 2^6 is 64, the answer is (E).

Negative Exponents

What if you had to simplify an expression such as $\frac{x^6}{x^8}$? Subtracting the exponents gives:

$$\frac{x^6}{x^8} = x^{6-8} = x^{-2} = \frac{1}{x^2}$$

A negative sign in the exponent means reciprocal. If you need to work with a negative exponent, put the whole expression under 1 (as above) and change the negative exponent to a positive one.

In this case, x^2 and x^{-2} are reciprocal expressions because $\frac{1}{x^2} \cdot x^2 = 1$

Try this one:

$$3^3 \cdot 3^{-3} =$$

We could rewrite the problem as $3^3 \cdot \frac{1}{3^3} = 27 \cdot \frac{1}{27} = 1$. We could also use the rule that allows us to add exponents that have the same base. That gives us $x^{3-3} = x^0 = 1$. In fact, anything to the zero power is 1. That's a pretty special and important fact to remember about exponents. Here are some other important exponent facts:

Any nonzero number to the zero power equals 1. ($5^0 = 1$)

Any number to the first power equals itself. ($5^1 = 5$)

One to any power equals 1. ($1^{37} = 1$)

Zero to any nonzero power equals 0. ($0^{37} = 0$)

By the way, zero to the zero power is undefined. That's why we had to word a few of those rules carefully. That's one fact, however, that's not tested on the GMAT.

Try the problem below.

3. If $x \neq 0$ and p is an integer, then what is the value of x^p ?

(1) $p = p^2$

(2) $p < 1$

Statement (1) tells us that either $p = 0$ or $p = 1$. Those are the only values that could make the statement true. Since we have two possible values, BCE. Statement (2) by itself doesn't really tell us much. The value of p could be 0 but it could also be lots of other numbers. Cross off (B). When we put the statements together, we know that $p = 0$ and hence the value of x^p must be 1, so the answer is (C).

Most of us tend to think of exponents as operations that make numbers bigger. However, note that this is not true for all numbers. When you raise a number between 0 and 1 to a positive power, the number gets smaller. ETS is hoping that you'll forget this important fact.

For example:

$$\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}, \text{ and } \frac{1}{4} \text{ is smaller than } \frac{1}{2}.$$

4. Is $x > 0$?

(1) $x^2 > 0$

(2) $x^5 < 0$

Statement (1) doesn't help us because both positive and negative numbers become positive when they are squared. For example, x could be 2 or -2 . So, BCE. Statement (2), on the other hand, tells us that x must be negative. For example, x could be -2 or -3 . That tells us that the answer to the question is "no." Hence, the answer is (B).

It's also important to remember how negative numbers work when raising them to powers.

- A negative number raised to an **odd** power is **negative**.
- A negative number raised to an **even** power is **positive**.

Quick Quiz: Exponents

What happens to each of the following values when raised to the indicated power?

| | <u>Gets bigger</u> | <u>Gets smaller</u> | <u>Stays the same</u> |
|-------------|-----------------------|-----------------------|-----------------------|
| $(-26)^2$ | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $(-4)^5$ | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $(-0.14)^4$ | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $(0)^{100}$ | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $(1)^0$ | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $(1)^{-2}$ | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $(0.52)^3$ | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

Scientific Notation

Scientific notation is a way to write very big or very small numbers using powers of 10. When dealing with scientific notation, the exponent for the power of 10 can be either positive or negative.

A **positive** exponent means to move the decimal to the right by a number of places represented by the exponent.

$$7.8 \times 10^5 = 780,000$$

A **negative** exponent means to move the decimal to the left by a number of places represented by the exponent.

$$2.3 \times 10^{-4} = .00023$$

When multiplying or dividing numbers expressed in scientific notation, work the exponent part separately from the number part.

$$5. \quad \frac{1.872 \times 10^2}{5.2 \times 10^6} =$$

- ☐ 3.06×10^{-5}
- ☐ 3.6×10^{-5}
- ☐ 3.6×10^{-3}
- ☐ 3.6×10^4
- ☐ 3.06×10^4

First, notice that you can use POE. Using the exponent rules for division, it is clear that the exponent will be negative ($\frac{10^2}{10^6} = 10^{-4}$). Eliminate (D) and (E). Since 1.872 divided by 5.2 gives .36, simply rewrite $.36 \times 10^{-4}$ as 3.6×10^{-5} . (Since the decimal point was moved one place to the right, we needed to increase the exponent from -4 to -5 .) The answer is (B).

Roots

Square roots are the flip side of exponents. For example, $\sqrt{9}$ indicates that some number times itself equals 9. Since we know that $3^2 = 9$, $\sqrt{9} = 3$.

Here's how a typical GMAT problem might test your understanding of roots:

- What is the value of x ?

$$(1) \quad x^2 + 1 = 17$$

$$(2) \quad x = \sqrt{16}$$

Statement (1) tells us that x could be either 4 or -4 , so BCE. Statement (2) tells us that x must be 4, so the correct answer is (B). **A square root is defined as the positive root only.**

There are a few important rules to keep in mind when working with expressions that contain roots.

You can add expressions with roots if they have the same number under the radical sign:

$$2\sqrt{7} + 4\sqrt{7} = 6\sqrt{7}$$

Essentially, you are adding like terms, just as if you were adding $2x + 4x$. However, just as you can't add $x + y$, you can't add values with different numbers under the radical sign.

To multiply roots, first multiply the numbers outside the radical sign, and then multiply the numbers under the radical sign:

$$3\sqrt{6} \cdot 2\sqrt{3} = (3 \times 2)\sqrt{6 \times 3} = 6\sqrt{18}$$

Finally, simplify the expression, if possible. To simplify expressions with roots, factor out perfect squares. (Perfect squares are the squares of integers.)

$$6\sqrt{18} = 6\sqrt{9 \cdot 2} = 6 \times \sqrt{9} \times \sqrt{2} = 6 \times 3 \times \sqrt{2} = 18\sqrt{2}$$

To divide, combine the expressions, then divide:

$$\frac{\sqrt{60}}{\sqrt{3}} = \sqrt{\frac{60}{3}} = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$$

That also means that if you need to take the root of a fraction, you can take the root of the top divided by the root of the bottom.

$$\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$$

ETS observes the convention that it's improper to leave a root in the bottom of a fraction. If a solution to a GMAT question has a radical sign in the bottom of a fraction, you'll need to rework the expression. You know that multiplying anything by 1 does not change the value, so you can multiply by a special form of 1. Form a fraction by placing that improper root in the top and the bottom of the new fraction and then multiply. For example:

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

A GMAT answer choice would look like the final form of the expression. This process is called **rationalizing the denominator**.

1. What is the value of $\left(2\sqrt{3} + 4\sqrt{3}\right)^2$?

- ☐ $6\sqrt{3}$
- ☐ 18
- ☐ 108
- ☐ 180
- ☐ 324

We first add the roots inside the parentheses. That gives us $6\sqrt{3}$. Now we square both parts of that expression. $36 \times 3 = 108$, which is answer choice (C).

$$2. \frac{(3\sqrt{3} + 8\sqrt{6}) + \sqrt{3}}{4\sqrt{3}} =$$

- ☐ $3\sqrt{3} + 2\sqrt{6}$
- ☐ $3\sqrt{3} + 2\sqrt{6} + 1$
- ☐ $1 + 2\sqrt{2}$
- ☐ $3 + 2\sqrt{6}$
- ☐ $4\sqrt{3} + 2\sqrt{2}$

Normally, we'd add up the expressions in parentheses first. However, since we can't add roots with different numbers under the radical sign, we'll make use of the fact that addition can be done in any order.

$$\frac{(3\sqrt{3} + 8\sqrt{6}) + \sqrt{3}}{4\sqrt{3}} = \frac{4\sqrt{3} + 8\sqrt{6}}{4\sqrt{3}}$$

Now, we'll split that into two fractions and use the division rule.

$$\frac{4\sqrt{3} + 8\sqrt{6}}{4\sqrt{3}} = \frac{4\sqrt{3}}{4\sqrt{3}} + \frac{8\sqrt{6}}{4\sqrt{3}} = 1 + 2\sqrt{\frac{6}{3}} = 1 + 2\sqrt{2}$$

That gives us answer choice (C).

There's one more thing to remember about roots. Look what happens when we take the square root of a number between 0 and 1.

$$\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$$

When you take the square root of a number between 0 and 1, you get a bigger number.

Quick Quiz: Roots

What happens to each of the following numbers when you find its square root?

| | <u>Gets bigger</u> | <u>Gets smaller</u> | <u>Stays the same</u> |
|---------------|-----------------------|-----------------------|-----------------------|
| 0 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $\frac{1}{4}$ | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 1 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 164 | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

Fractional Exponents

Suppose that we wanted to simplify an expression such as the following:

$$(x^2)^y = x$$

Multiplying the exponents gives $x^{2y} = x$, so $2y = 1$. Hence, $y = \frac{1}{2}$. What does $y^{\frac{1}{2}}$ mean? It means the square root of y . In general, fractional exponents tell you to take n th roots. In other words:

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

$$9^{\frac{1}{2}} = \sqrt{9} = 3$$

$$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$$

QUADRATICS

Quadratic equations involve exponents. You need to know how to multiply two terms to create a quadratic equation, and you need to now how to reverse the process by factoring a quadratic equation into its parts.

FOIL

You probably haven't done this since high school. FOIL stands for First, Outside, Inside, Last, and it is the method used to multiply two algebraic terms.


Here's an example:

$$(x - 5)(x + 8) =$$

$$\begin{array}{c} \swarrow \quad \searrow \\ \text{Firsts: } (x - 5)(x + 8) = x^2 \end{array}$$

$$\begin{array}{c} \swarrow \quad \searrow \\ \text{Outsides: } (x - 5)(x + 8) = x^2 + 8x \end{array}$$

$$\begin{array}{c} \swarrow \quad \searrow \\ \text{Insides: } (x - 5)(x + 8) = x^2 + 8x - 5x \end{array}$$



$$\text{Lasts: } (x - 5)(x + 8) = x^2 + 8x - 5x - 40$$

Combine the middle terms, and your final answer is $x^2 + 3x - 40$.

Finding Factors

If you already have the quadratic equation, reverse the FOIL process to find the factors. Follow these steps to factor $x^2 + 4x - 12$.

Set up the parentheses:

$$x^2 + 4x - 12 = (\quad)(\quad)$$

Find the first parts. To get x^2 , the F in FOIL must have been x multiplied by x :

$$x^2 + 4x - 12 = (x \quad)(x \quad)$$

Find the last parts. We need two numbers that have a product of 12. The possibilities are (1, 12), (2, 6), and (3, 4). Since the middle term is $4x$, the factors must have either a sum or a difference of 4. So, we need 6 and 2.

$$x^2 + 4x - 12 = (x \quad 2)(x \quad 6)$$

Determine the signs of the two numbers. Since we need a *positive* 4, the 6 needs to be positive and the 2 needs to be negative.

$$x^2 + 4x - 12 = (x - 2)(x + 6)$$

Finding Roots

Factoring a quadratic allows us to solve for x . Solving for x in a quadratic is called finding the roots or solutions. Let's go back to the previous example:

$$x^2 + 4x = 12$$

First, move the numbers and the variables to one side of the equation. Quadratics need to be set to zero before it's possible to find the roots by factoring.

$$x^2 + 4x - 12 = 0$$

Second, factor the quadratic.

$$x^2 + 4x - 12 = 0 = (x - 2)(x + 6)$$

Finally, we know the product of the factors must be 0, and it's only possible to get a product of 0 if one of the factors is 0. Therefore, we need to set each term equal to 0 and solve.

$$\text{If } (x - 2) = 0, x = 2.$$

$$\text{If } (x + 6) = 0, x = -6.$$

Thus, the roots of this equation are -6 and 2.

1. What is the value of x ?

$$(1) x^2 + 3x - 40 = 0$$

$$(2) x^2 - 10x + 25 = 0$$

The first equation has two roots, 5 and -8. Hence, BCE. The second equation factors to $(x - 5)(x - 5) = 0$. Since both factors are the same, its only root is 5. Therefore, the answer to the question is (B).

Common Quadratics

ETS has three quadratic polynomials of which they are especially fond. Memorize these so that you won't actually have to do the FOIL method when you see them.

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)(x - y) = x^2 - y^2$$

These patterns are really helpful in certain situations, such as this problem:

2. $(325)^2 - (324)^2 =$

- ☐ 1
- ☐ 324
- ☐ 325
- ☐ 649
- ☐ 650

ETS expects you will do the calculation and waste a lot of time. Notice, however, that this problem has the form $x^2 - y^2$, so we can rewrite it as $(325 + 324)(325 - 324) = (649)(1) = 649$, which is answer choice (D).

Factorials

The **factorial symbol** (!) is used when you need to multiply an integer by all the positive integers that are less than it. For example:

$$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$n! = (n) \times (n - 1) \times (n - 2) \times \dots \times 1$$

ETS sometimes throws factorials into questions to try to make them appear to be more work than they actually are. Don't be thrown off. Most factorial questions can be solved either by reducing or by breaking the question down into simple parts.

3. What is the value of $\frac{12!}{11!}$?

- ☐ 2!
- ☐ $\frac{4!}{2!}$
- ☐ 4!
- ☐ $\frac{6!}{3!}$
- ☐ 6!

Obviously, we wouldn't want to compute $12!$ and then divide it by $11!$. That would take too long. If we write out the problem, we get $\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$. Everything but the 12 in the numerator cancels out, leaving us with 12. Of course, now we have to compute the answer choices. You'll find answer choice (B) works out to 12.

Here are a few helpful things to remember about factorials when dealing with factorials.

$$0! = 1! = 1$$

$0!$ and $1!$ are the only odd factorials because all other factorials have 2 as a factor.

SUMMARY

Exponents

- When multiplying, add the exponents: $x^4 \cdot x^3 = x^7$
- When dividing, subtract the exponents: $\frac{x^8}{x^5} = x^3$
- When raising an exponent to a power, multiply the exponents: $(x^3)^2 = x^6$
- $x^0 = 1$
- Negative exponents are used to show reciprocals: $x^{-y} = \frac{1}{x^y}$
- Fractional exponents are another way of expressing roots. $x^{\frac{1}{2}} = \sqrt{x}$
- A fraction between 0 and 1 raised to a positive integer is smaller than the original fraction.

Roots

- The sum or difference of roots can be simplified only when they have the same number under the radical sign: $\sqrt{3} + 5\sqrt{3} = 6\sqrt{3}$ but $\sqrt{3} + \sqrt{5} \neq \sqrt{3} + \sqrt{5}$
- To multiply roots, use this general formula: $a\sqrt[r]{x} \times b\sqrt[r]{s} = ab\sqrt[r]{xs}$
- To divide roots, take the root formed by the fraction of the two numbers: $\frac{\sqrt[r]{x}}{\sqrt[r]{y}} = \sqrt[r]{\frac{x}{y}}$
- The square root of a number between 0 and 1 is larger than the original number.

Quadratics

- To find the factors of a quadratic, use the FOIL method (Firsts, Outsides, Insides, Lasts).
- The roots of an equation are the solutions. To find the roots of a quadratic, move all terms to the left-hand side (leaving 0 by itself on the right-hand side), then factor the quadratic polynomial. Set each of its factors equal to zero and then solve.

Factorials

- The definition of a factorial (!) is: $n! = (n) \times (n-1) \times (n-2) \times \dots \times 1$.
- When working with factorials, look for ways to rearrange the information.

DRILL

1. Which of the following is the smallest four-digit integer that is divisible by the first three prime numbers?

☐ 120
☐ 150
☐ 1020
☐ 1025
☐ 1060

2. Is integer x a multiple of 3?

- (1) The sum of the digits of x is 27.
 (2) x is a factor of 9990.

3. $\frac{x^2 + 2x - 8}{x^2 - 6x + 8} =$

☐ 1
☐ -1

☐ $\frac{1}{3}x$
☐ $\frac{x+4}{x-4}$
☐ $\frac{x+8}{x-8}$

4. After a recent stock split, a software tycoon finds that he owns 1.2×10^9 shares in his company. He decides to sell an average of 2.5×10^6 shares per month. If each share is worth an average of \$25, what are his net proceeds from the sale of stock for the year?

☐ 2.5×10^7
☐ 7.5×10^7
☐ 2.5×10^8
☐ 7.5×10^8
☐ 1.25×10^9

5. If $65x = 3^2 \times 5^2 \times 13$, which of the following is *not* a factor of x ?

☐ 3
☐ 6
☐ 9
☐ 13
☐ 15

6. What is the value of x^3 ?

- (1) $\sqrt[3]{x}$ is prime.
 (2) $\sqrt[3]{x}$ is even.

7. Which of the following equations has 7 and 2 as roots?

☐ $x^2 - 9x + 14 = 0$
☐ $x^2 + 14 = 0$
☐ $x^2 + 9x + 14 = 0$
☐ $x^2 - 7 = 0$
☐ $x^2 + 5x - 14 = 0$

8. If $a > 0$ and $b > 0$, which of the following is

equivalent to $\frac{a}{b} \div \sqrt{\frac{a}{b^2}}$?

☐ 1
☐ $\frac{\sqrt{a}}{\sqrt{b}}$
☐ \sqrt{a}
☐ $\frac{1}{\sqrt{a}}$
☐ $\frac{1}{\sqrt{b}}$

9. If $\sqrt[n]{(-64)} = -4$, then what is the value of n ?

☐ 2
☐ 3
☐ 4
☐ 5
☐ 6

10. $\frac{1}{\sqrt[3]{y}} =$

- ☐ $\sqrt[3]{y}$
- ☐ $\sqrt[3]{y^2}$
- ☐ $\frac{\sqrt[3]{y}}{y}$
- ☐ $\frac{\sqrt[3]{y^2}}{y^2}$
- ☐ $\frac{\sqrt[3]{y^2}}{y}$

11. If $2x^2 - 2x - 12 = 0$ and $y^2 - 5y + 6 = 0$ when $x = -y$, then what is the value of x ?

- ☐ -3
- ☐ -2
- ☐ 0
- ☐ 2
- ☐ 3

12. Which of the following gives the complete range

of values for which $\frac{x!}{y!}$ is an integer?

- ☐ $x > y$
- ☐ $x \geq y$
- ☐ $x < y$
- ☐ $x \leq y$
- ☐ $x = 0$

13. If x is an integer, what is the value of x ?

- (1) $x^{\frac{3}{2}} = 8$
- (2) $x(x - 2) = 8$

14. If $a > 0$, what is the value of $a^x a^y$?

- (1) $x = -y$
- (2) $a = 2$ and $x - y = 6$

15. If $a = 4$, what is the value of c ?

- (1) $(abc)^4 = (ab^2c^3)^2$
- (2) $b = 5$

ANSWERS AND EXPLANATIONS**Quick Quiz: Exponents**

| | <u>Gets bigger</u> | <u>Gets smaller</u> | <u>Stays the same</u> |
|-------------|----------------------------------|----------------------------------|----------------------------------|
| $(-26)^2$ | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $(-4)^5$ | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> |
| $(-0.14)^4$ | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| $(0)^{100}$ | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> |
| $(1)^0$ | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> |
| $(1)^{-2}$ | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> |
| $(0.52)^3$ | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> |

Quick Quiz: Roots

| | <u>Gets bigger</u> | <u>Gets smaller</u> | <u>Stays the same</u> |
|---------------|----------------------------------|----------------------------------|----------------------------------|
| 0 | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> |
| $\frac{1}{4}$ | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| 1 | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> |
| 164 | <input type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> |

Drill

1. **C** Use the divisibility rules and Process of Elimination. Get rid of the first two answer choices because they contain only three digits. (D) is not even, and we know that (E) is not divisible by three because the sum of the digits is not divisible by 3.
2. **A** The first statement is sufficient to answer the question because 27 is a multiple of 3, and that means that the number is divisible by 3. So, AD. The second statement, however, is not sufficient. For example, x could be 999 or x could be 10 since both are factors of 9990.
3. **D** Factor the top and the bottom of the expression.
That gives $\frac{(x+4)(x-2)}{(x-4)(x-2)}$, and then you can cancel the like terms. Note that you could also do this problem by Plugging In a number such as 2.
4. **D** First find the amount of stock sold during the year. Since $2.5 \times 12 = 30$, we know that $30 \times 10^6 = 3.0 \times 10^7$ shares of stock are sold. Now, multiply $3 \times 25 = 75$. Again the exponent will increase by one when we move the decimal point one place to the left.
5. **D** Since $65 = 13 \times 5$, we know that $x = 3^2 \times 5$.
6. **C** The first statement tells us that x^3 could be the cube of any prime number. For example, x^3 could be 8, 27, 125, etc. So, BCE. The second statement tells us that x^3 could be the cube of any even number. Eliminate (B). However, since the only even prime number is 2, we know that $x = 8$.
7. **A** Since the roots of the equation are 7 and 2, we know that factors of the equation are $x - 7$ and $x - 2$. Using FOIL on these factors will produce the correct equation.
8. **C** The quickest way to do this problem is to plug in. Make $a = 2$ and $b = 3$. That gives us

$$\frac{2}{3} \div \sqrt{\frac{2}{9}} = \frac{2}{3} \div \frac{\sqrt{2}}{3} = \frac{2}{3} \times \frac{3}{\sqrt{2}} = \frac{2}{\sqrt{2}},$$
which reduces to $\sqrt{2}$.
9. **B** The exponent must be odd because you can't take an even root of a negative number when using real numbers. Get rid of answer choices (A), (C), and (E). Plug in one of the remaining answer choices. Since $-4^3 = -64$, the answer is (B).
10. **E** Plug In. If $y = 8$, then $\frac{1}{\sqrt[3]{y}} = \frac{1}{2}$ and then evaluate the answers. This is an easy problem to get tripped up on if you do the algebra.
11. **B** Plugging In the Answers is the way to go on this problem. Answer (C) can't be right because 0 doesn't work for either equation. Plugging -2 into the first equation gives $[2 - 2^2 - 2 - 2 - 12 = 0]$ and plugging 2 into the second equation gives $[2^2 - (5 \times 2) + 6 = 0]$.
12. **B** Start by applying some Process of Elimination. Since a number divided by itself is always an integer, we can get rid of any of the answers that don't allow $x = y$. So, (A), (C), and (E) are gone. Now, Plug In a few small numbers to test the remaining answers. If $x = 4$ and $y = 2$, we get an integer, but if we reverse the numbers, we get a fraction that's less than 1. Get rid of (D).
13. **A** The first statement says that $\sqrt{x^3} = 8$, which means that $x = 4$. Remember, however, that the point of data sufficiency is simply to determine if you have enough information. It isn't really necessary to solve for x . You just have to know that there can only be one value that satisfies the statement. So, AD. The second statement does not give enough information. Distributing the x and rearranging shows that $x^2 - 2x - 8 = 0$. The factors of this polynomial are $x - 4$ and $x + 2$, so x can have two values.
14. **A** Statement (1) allows us to substitute $-y$ for x so that $a^x a^y = a^{-y} a^y = a^{-y+y} = a^0 = 1$. So, AD. The second statement gives the value of a but gives no value for $x + y$.
15. **E** The equation in Statement (1) reduces to $a^2 = c^2$. If $a = 4$, then c could equal either 4 or -4 . So, BCE. The second statement tells us nothing by itself. Combining the two statements still does not give sufficient information because the b 's cancel out in Statement (1).

ADMISSIONS INSIGHT

Letters of Recommendation

Almost all MBA program applications require two to three letters of recommendation from third parties capable of commenting on a student's qualifications for business school. Letters of recommendation serve an essential purpose in the application process by adding the perspective of others. They are meant to supplement the information, views, and experience set forth in the essays and personal interview.

PrincetonReview.com supplies the following tips for soliciting your letters of recommendation. First, present yourself truthfully throughout the application—so references support and reinforce the achievements mentioned. Second, choose the recommenders wisely. The best recommendations will be written by people who know you well in a professional capacity, sincerely like you as a person, and can discuss those same points you have already iterated in the essays and statement of purpose. Unlike other graduate programs, MBA admissions committees prefer professional recommendations to academic ones. If a school requires two letters of recommendation, try to get both from professional sources. If three letters are required, it is usually permissible for one to be from an academic source. At least one should be from your current immediate supervisor. For the others, make sure they have worked closely with you and are able to comment in detail about your qualifications. Finally, supply your recommenders with background information, such as a resume, statement of purpose, and talking points (if appropriate), to get them started. A list of schools to which you're applying and the recommendation deadlines is also helpful.

The Interview

Not all business schools attach equal value to the interview. For some, it's an essential screening tool. For others, it's used primarily to evaluate borderline cases. Still others strongly encourage, but do not require, the interview. Some schools make it simply informative. If an on-campus interview isn't a possibility, the admissions office may find an alum to conduct the interview close to home.

A great interview can tip the scale in the admit direction. So if an interview is offered, take it. Then, get set to present yourself as a compelling candidate. The interview may be your best opportunity to display the kinds of qualities—enthusiasm, sense of humor, maturity—that often fill in the blanks and sway a decision.

In general, business school interviews are not formulaic. The focus can range from specific questions about job responsibilities to broad discussions about life. Approach the interview as a conversation to be enjoyed, not as a question-and-answer ordeal. PrincetonReview.com gives these tips on interview preparation: Expect to discuss personal topics. Be ready to go into greater depth than you did in the essays (but don't assume the interviewer has read them). Put together two or three points that you want to impress on the interviewer. Go in with examples, or even a portfolio, to showcase your achievements. Practice speaking about past accomplishments without a lot of "I did this, I did that." Finally, be prepared to give a strong and convincing answer to the interviewer's inevitable question, "Why here?"

ASSIGNMENT 5

INFERENCES AND PARADOXES

In this lesson, we examine two new types of arguments questions: inference questions and resolve/explain questions. These types are a bit different from those you learned about in the first lesson because they don't require you to identify the conclusion, premises, and gap. To answer these questions correctly, you must stick closely to the topic and tone of the information provided in the passage.

INFERENCE QUESTIONS

You have already seen reading comprehension questions that ask what you can infer from the passage or what the author implies about a topic. The arguments section also contains inference questions. Inference question stems typically ask:

- Which of the following can be inferred from the information above?
- Which of the following conclusions is best supported by the passage?
- Which of the following conclusions could most properly be drawn from the information above?
- Which of the following must be true on the basis of the statements above?

The second question stem might look like a strengthen question at first glance. However, it asks you to find an answer based on information in the passage above the question. Whenever a question stem asks you to do this, you are dealing with an inference question. To determine whether you're dealing with a strengthen question or an inference question, pay careful attention to the wording. Strengthen questions require you to use the answer to support the passage, while inference questions ask you to use the passage to support the answer.

You will find that the passages for inference questions typically sound like a series of facts rather than an argument with a conclusion and premises. The answer you choose must fit the facts provided in the passage. However, the correct answer is often a minor point that can be deduced from the information provided. In this way, inference questions bear a similarity to specific reading comprehension questions. Inferences do not necessarily relate to the logical

Inferences should stick closely to the scope of the passage.

presentation of the argument, and because this is true, it is a waste of your time to break the argument down into its conclusion, premises, and gap.

The test writers expect you to be attracted to answers that draw broad conclusions or contain information that might be true, but these are precisely the answers you should avoid. When you work an inference question on the GMAT, approach the answer choices skeptically, and look for the choice that *must be true* based on the information provided.

Take a look at this question:

1. By studying the television-viewing habits of a number of average American households, a group of media analysts has discovered that people are less likely to switch channels during commercial breaks during rebroadcasts than they are during premieres and new episodes. The analysts have also found, however, that viewers pay substantially less attention to the broadcasts as a whole during rebroadcasts than they do during first-run episodes and premieres.

Which of the following is an inference that can properly be drawn from the passage above?

You need not spend time breaking the argument down into its parts. All you have to do is find an answer choice that you can conclusively support with information right there on the page. It helps to paraphrase and review the facts before you look at the answers. We know that:

1. People are less likely to change channels during reruns than during new shows.
2. People pay less attention to what's on TV during reruns than during new shows.

Let's look at some answer choices. It is usually hard to predict what inference the test writers will draw, so the best plan of attack involves going directly to the answer choices. As you read each choice, ask yourself, "Must this be true based on information in the passage?" Eliminate the choices you cannot prove using the information provided.

- ☐ The success of a commercial may not depend on whether the program during which it is broadcast is a premiere or a rerun.

This is a possible answer. People switch channels during commercials accompanying new broadcasts, and they don't pay attention to commercials during reruns. Therefore, you can't predict how well a commercial will be received based solely on whether it aired during a rerun or an original show.

- ☐ Commercials that are run during new episodes and premieres are seen in their entirety more often than not.

We don't know if this is true, so eliminate this choice. Nothing in the passage talks about how often commercials are seen in their entirety.

- ☐ Advertisers would be wise to run their commercials during rebroadcasts only.

We don't know that commercials should be shown only during reruns. While people are less likely to change channels during reruns, they also pay less attention. The facts don't support a choice with wording this strong, so eliminate it.

- ☐ There is an implicit trade-off in viewer response to first-run programs and rebroadcasts.

This choice might be tempting. However, we don't know that there is a trade-off in viewer *response*; we only know about channel switching and amount of attention. The passage doesn't provide sufficient support for this choice, so eliminate it.

- ☐ It is not whether the program is old or new, but whether it is interesting or not that influences a viewer's attention to the commercials shown during the program.

Get rid of this choice right away because it goes against information in the passage. The passage told us that whether a program is a rerun or an original does affect viewer attention.

Choice (A) is the correct answer. This seems fairly obvious, and it must be true. When you work inference questions, pick the safest answer, the choice that sticks most closely to the passage.

POE for Inference Questions

Making good use of POE is the key to working inference questions. Most of the time, you will not be able to come up with an answer in your own words, so you will have to find the right answer by eliminating wrong answers. An answer that paraphrases a fact from the passage is probably the credited response. The more obvious a choice seems, the more likely it is to be right. Avoid answers that:

- Go Beyond the Information Given (B.I.G). Eliminate choices that bring in new information, predict the future, require outside knowledge, or need additional assumptions.
- Are broader or more extreme than the passage. Be wary of choices with very strong wording.
- Could be true but cannot be proven using the facts in the passage.
- Go against information in the passage.

RESOLVE/EXPLAIN QUESTIONS

Another type of question on the GMAT asks you to resolve an apparent discrepancy or explain a perplexing situation. The discrepancies will not be of a comical Seinfeldian type such as “why is easy listening music so hard to listen to?” but will require an answer choice that explains how two seemingly incongruous statements can coexist.

Resolve/explain questions typically ask:

- Which of the following, if true, resolves the paradox outlined above?
- Which of the following, if true, does most to help explain the situation above?
- Which of the following, if true, best explains the apparent contradiction?

Resolve/explain questions are easy to recognize because the question stem contains the word “resolve” or “explain.”

Let’s look at a sample resolve/explain question.

1. One recent study of police records showed that the number of crimes committed in 1990 rose 10 percent over the previous year. Yet, another study showed that the number of people who reported being victims of a crime in 1990 rose 30 percent from the previous year.

Which one of the following resolves the apparent paradox?

As with inference questions, resolve/explain questions do not involve arguments with conclusions and premises. When you work these, begin by identifying the conflict between two facts. Most of the time you will see an indicator word that introduces the conflict. For this example, we know:

1. The number of crimes in 1990 rose 10 percent over the previous year.
Yet
2. The number of people who said they were victims of a crime rose 30 percent.

Once you have found the facts in conflict, state the discrepancy in the form of a question. We want to know how both the first fact and the second fact can be true, so we can ask, “Why did the number of crimes rise only 10 percent if the number of people claiming to be victims rose 30 percent?”

Phrasing the question is similar to looking for the gap in other arguments. The author cites two sets of statistics that seem to create a paradox, and the answer must explain how both statistics can be true. When you work a resolve/explain question, think about the common flaws in reasoning you learned earlier. The credited response must provide more information about the percentages, and it will likely address the sample populations involved. Consider each of the following answer choices, and decide which one best answers the question we phrased:

- ☐ People were more honest about the crimes they committed in 1990 than in the previous year.

The relative honesty of criminals does not explain why reported victims increased by a greater percentage than the actual number of crimes committed, so eliminate this choice.

- ☐ The proportion of corporate crimes increased from 1989 to 1990.

We have no idea if corporate crimes involve more victims per crime than other crimes do, based on the information provided. This choice is not relevant to the passage, so eliminate it.

- ☐ The number of criminals who were in turn victimized themselves rose from 1989 to 1990.

Were these criminals targets of a multiple-victim crime? Since that question is unanswered, this cannot be the credited response.

- ☐ The proportion of crimes that victimized several people at once increased from 1989 to 1990.

This clears up the paradox because it establishes an increase in multiple-victim crimes, which would in turn explain why crimes in general would increase by 10 percent while victims of crimes would increase by 30 percent.

- ☐ Because of worsening social conditions, people felt more victimized in 1990 than in 1989.

We're not interested in how people felt. Because there is not an explicit connection, this cannot be the credited response.

Choice (D) is the credited response. Both statistics could be valid if this is true, and it explains why the two facts are not in conflict.

POE for Resolve/Explain Questions

Stick closely to the facts in conflict when you consider the answer choices. The correct answer provides additional information that allows both facts to be true and clears up the paradox. Avoid answers that:

- Do nothing to clear up the conflict.
- Make the conflict worse.
- Address only one side of the conflict.

SUMMARY

Inference Questions

Step 1: Identify the question.

Typical question stems ask:

- Which of the following can be inferred from the information above?
- The information above best supports which of the following statements?
- Which conclusion can be logically drawn from the information above?
- If the statements above are true, which of the following must be true?

Step 2: Work the argument.

Read the passage. Pay attention to trigger words in the passage, especially those that indicate a change of direction.

Step 3: Predict what the answer should do.

Most of the time, you will not be able to come up with an answer in your own words. Simply keep in mind that you want the answer best supported by the facts.

Step 4: Use POE to find the answer.

Look for answers that **must be true**. An answer that paraphrases a fact from the passage is probably the credited response. The more obvious a choice seems, the more likely it is to be right. Avoid answers that:

- Go Beyond the Information Given (B.I.G). Eliminate choices that bring in new information, predict the future, require outside knowledge, or need additional assumptions.
- Are broader or more extreme than the passage.
- Could be true but cannot be proven using the facts in the passage.
- Go against information in the passage.

Resolve/Explain Questions

Step 1: Identify the question.

Look for key words like *resolve* and *explain*.

Step 2: Work the argument.

Read the passage. Identify the facts in conflict. Look for words like *but*, *yet*, and *however* to find the paradox.

Step 3: Predict what the answer should do.

Phrase the question that the correct choice will answer. Ask, "Why X but also Y?"

Step 4: Use POE to find the answer.

The correct answer will provide additional information that allows both facts to be true and clears up the paradox. Avoid answers that:

- Do nothing to clear up the conflict.
- Make the conflict worse.
- Address only one side of the conflict.

DRILL

Try a few inference and resolve/explain questions on your own.

1. Walking a mile at a slow pace, from 1.8 to 2.5 mph, burns about 100 calories. Walking a mile at a brisk pace, from 3 to 4 mph, also burns about 100 calories, though in a shorter amount of time. Jogging a mile, from 4.5 to 6 mph, burns about the same 100 calories. The difference in covering a mile at faster speeds is not in the number of calories burned, but in the greater degree of cardio-vascular benefit.

Which of the following conclusions is best supported by the statements above?

- ☐ Jogging builds more muscle tone than does walking at a slow pace.
- ☐ Speed is directly proportional to the number of calories burned.
- ☐ Walking at a brisk pace for a shorter amount of time burns more calories than does walking at a slow pace for a longer amount of time.
- ☐ There are benefits to walking at a slow pace other than burning calories.
- ☐ Jogging a mile offers greater cardiovascular benefit than does walking a mile briskly or at a slow pace.

2. Traditionally, the subway system has been subsidized by the state government because revenue from ticket sales is not sufficient to cover operating costs. The governor's plan proposes to cut financial support of the system.

Which of the following follows logically from the passage above?

- ☐ Although a bus fare costs the same as a subway fare, the buses do not travel to all the points that the subway does, so some people cannot travel by bus.
- ☐ If the subway system does not find another way of raising money, it will have to either raise ticket prices or cut back on service.
- ☐ If the price of a subway ticket is raised, some people will no longer be able to afford to ride the subway.
- ☐ If the subway system can find a way to cut costs, the state will maintain its level of support, causing surplus funding for the system.
- ☐ A group of investors is considering privatizing the subway system.

3. Poor anxiety management leads people to function less effectively in their jobs and relationships. Short-term psychotherapy, while it does not have the profound effect of psychoanalysis, can teach some concrete techniques for managing anxiety.

Which of the following can be correctly inferred from the statements above?

- ☐ People who can manage their anxiety reasonably well have no need for short-term psychotherapy.
- ☐ Psychoanalysis is more useful in promoting long-term change than is psychotherapy.
- ☐ If people can manage their anxiety well, they are able to be effective at work and in relationships.
- ☐ Short-term psychotherapy may help some people to manage their anxiety.
- ☐ Concrete techniques are better for anxiety management than is theoretical work.

4. Passenger boardings on trains almost doubled between 1985 and 1995, yet the actual number of trips made by passengers increased by only 38 percent over the same period. The reason for this discrepancy is that at least two boardings are counted when a passenger must take more than one train trip to get to the final destination of his or her trip.

Which of the following, if true, best helps explain why the increase in boardings exceeded the increase in the number of train passengers between 1985 and 1995?

- ☐ Between 1985 and 1995, the number of train stations increased dramatically.
- ☐ Between 1985 and 1995, the number of train lines decreased significantly.
- ☐ Between 1985 and 1995, the proportion of train trips requiring passengers to change trains en route to their final destinations increased significantly.
- ☐ The proportion of business travelers using trains increased significantly between 1985 and 1995 relative to the proportion of pleasure travelers.
- ☐ The average seating capacity of passenger trains increased significantly between 1985 and 1995.

5. In an effort to save energy and, thus, money, many people keep their electric freezers half-empty at all times, using them to store prefrozen foods bought at a store and nothing else. Yet freezers that are half-empty usually consume more energy than they would if they were kept fully stocked.

Which of the following, if true, contributes most to an explanation for the apparent discrepancy described above?

- ☐ A person who normally maintains a half-empty freezer would save a considerable amount of money by using a freezer that is half as large.
- ☐ An electric freezer can operate efficiently only if chilled air is free to circulate within the freezing compartment.
- ☐ A given volume of air in a freezer requires much more energy to be maintained at a temperature below freezing than does an identical volume of frozen food.
- ☐ The door of a full freezer is likely to be opened more often, and for a longer period of time, than is the door of a half-empty freezer.
- ☐ On average, it takes less energy to keep food frozen than it does to get it from room temperature to a frozen state.

6. All drivers who lease the cars they drive carry automobile collision insurance, but many who own the cars they drive do not.

Which of the following, if true, would best explain the cause of the situation described above?

- ☐ Automobile drivers are not required by law to carry collision insurance on the cars they drive, but automobile leasing companies require, as a condition of the lease, that the driver carry collision insurance.
- ☐ Automobile drivers are not required by law to carry collision insurance, and collision insurance is extremely expensive.
- ☐ Collision insurance is a sensible purchase only if the automobile it covers is under two years old.
- ☐ Most insurance companies prefer not to issue automobile collision insurance and are slow to pay when a policyholder makes a claim.
- ☐ Many drivers feel they would rather risk the cost of repairing their vehicles in the event of an accident than to pay the cost of collision insurance.

ANSWERS AND EXPLANATIONS**Drill**

1. **E** The difference between walking and jogging a given distance is the amount of cardiovascular benefit. (A) and (D) are never mentioned, and (B) and (C) contradict the premises.
2. **B** Current ticket sales don't cover costs. Without external funding, they must raise prices or cut costs. Use POE. (A) We don't know anything about buses. (C) seems reasonable, but it's never actually mentioned. (D) is never stated. In (E), Privatization is never mentioned.
3. **D** Anxious people don't function well in life, and psychotherapy can teach people how to manage their anxiety. (D) is nice and noncommittal. How can you argue with it? (A) is too extreme, and (B) long-term change and (E) theoretical work are never mentioned. (C) is too strong, since anxiety might not be a person's only problem.
4. **C** The argument states that it's possible for a passenger to account for two (or more) boardings while taking only one trip. Since boardings increased more than actual trips did, it must be true that more trips required switching trains. (A) is a common wrong answer; an increase in train stations does not necessarily ensure that the trains will always stop at them. (B) doesn't explain anything. The distinction in (D) between business traveler and pleasure travelers is irrelevant. (E) only explains why trips increased.
5. **C** If it's more expensive to cool a half-empty freezer than to cool a full freezer, air must be harder to cool than food is. In (A), the size of the freezer doesn't matter. (B) and (D) go against the premise that half-full freezers burn more energy. (E) doesn't distinguish between full freezers and half-full freezers.
6. **A** Lessees carry insurance because they have to, and owners don't carry insurance because they don't have to. (B), (D), and (E) don't explain why all lessees have the insurance, and (C) is irrelevant (where did two years old come from?).

GEOMETRY

If you have nightmares about your high school geometry class, you can relax a bit. The geometry tested on the GMAT is fairly basic—no proofs, no statements of theorems, and no trigonometry. The GMAT mainly tests fundamental concepts and shapes such as lines, angles, area, triangles, quadrilaterals, and circles. This doesn't mean GMAT geometry problems are always easy. As on the rest of the exam, ETS is capable of creating very tricky geometry problems. However, even the challenging problems test the same geometry fundamentals; they just combine them in different ways. In order to handle the harder questions, you first need to lay a foundation of basic geometry knowledge and skills.

CAN YOU TRUST THE DIAGRAM?

You might be wondering whether the diagrams you see on the computer screen are accurate. In problem-solving questions, the diagrams are drawn to scale, which means that you can trust what they look like on your screen. In data sufficiency questions, however, the diagrams are often misleading, inaccurate, and designed to confuse you. You may need to redraw the diagram in a different way using the information given in the question or the statements as your guide. You will nearly always need to transcribe geometry figures to your scratch paper in order to solve them. Be careful when you do this, and double-check your diagram. One mislabeled angle can lead to the wrong answer or a tremendous amount of wasted time. If a question describes a figure but doesn't provide a diagram, draw it yourself based on the information you're given.

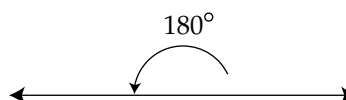
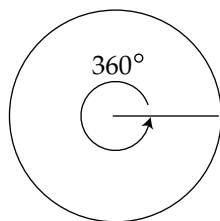
- Problem solving: You can trust what you see. Figures are drawn to scale unless a problem states otherwise.
- Data sufficiency: Be wary of what you see. Figures are not necessarily drawn to scale, and they may be drawn inaccurately in order to confuse you.
- Make sure you have correctly transcribed the diagram before you begin solving a problem. If no diagram is provided, draw one yourself.

LINES AND ANGLES

Two points determine a line.

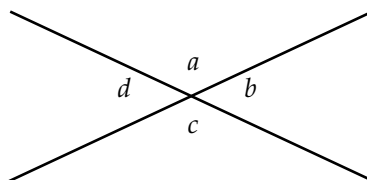


Angles are measured in degrees, and a complete circle contains 360° . A straight line contains 180° .



Intersecting Lines

When two straight lines intersect, the angles opposite each other are equal. These are called **vertical angles** or **opposite angles**.

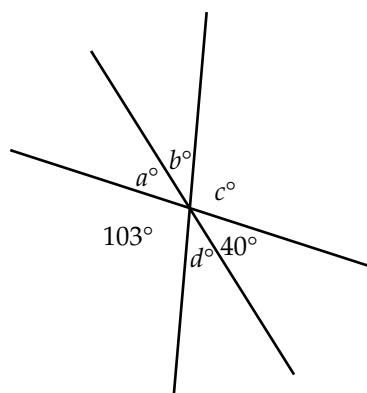


Vertical angles are equal.

$\angle a$ is equal to $\angle c$, and $\angle b$ is equal to $\angle d$. If $\angle d$ is 40° , what are the measures of the other three angles?

Angles d and b are vertical angles, so $\angle b$ must also be 40° . $\angle d$ and $\angle a$ form a straight line (as do d and c , a and b , and b and c). There are 180° in a straight line, so $\angle a$ must be equal to $180^\circ - 40^\circ$, which is 140° . $\angle c$ must also be 140° , since c and a are vertical angles.

Let's look at another question.



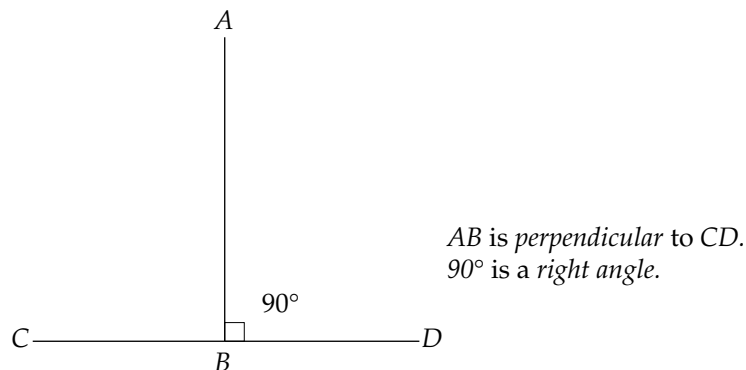
1. In the figure above, $a + b =$

- ☐ 30
- ☐ 40
- ☐ 55
- ☐ 77
- ☐ 103

We see that $\angle a$, $\angle b$, and the 103° angle form a straight line, which contains 180° . Therefore, $a + b + 103 = 180$. If you subtract 103 from both sides, you get $a + b = 77$. The answer is (D).

Perpendicular Lines

An angle measure of 90° is a **right angle**. Two lines that meet at a right angle are **perpendicular**, indicated by the \perp symbol.

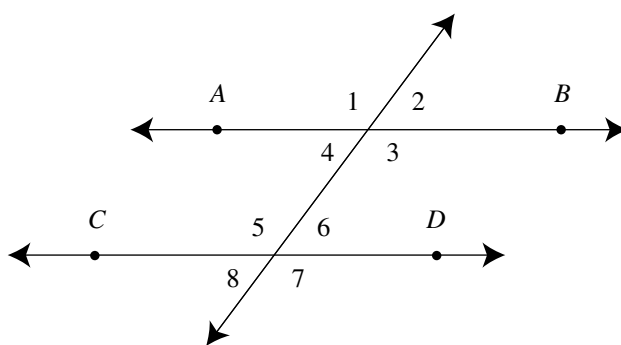


Perpendicular lines meet at a right angle.

Here, the right angle sign (which looks like a little box in the corner) indicates that the two lines are perpendicular. Line $CD \perp$ line AB .

Parallel Lines

If two lines in a plane never intersect, they are **parallel**, denoted by the symbol \parallel .

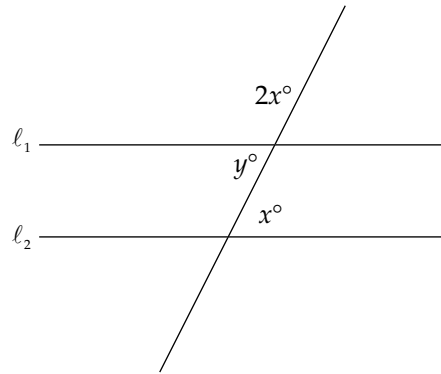


We've labeled eight different angles, but there are only two measures: The big angles all have the same measure, and the small angles all have the same measure. $\angle 1$, $\angle 3$, $\angle 5$, and $\angle 7$ are big angles, while $\angle 2$, $\angle 4$, $\angle 6$, and $\angle 8$ are small angles.

When a third line crosses two parallel lines, there are three important facts you need to know:

- Two kinds of angles are formed: big angles and small angles.
- All big angles are equal, and all small angles are equal.
- Any big angle plus any small angle equals 180° .

Let's try a problem.

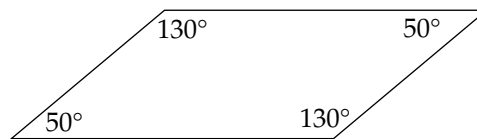


2. In the figure above, if $l_1 \parallel l_2$, what is the value of y ?

- ☐ 45
- ☐ 60
- ☐ 75
- ☐ 100
- ☐ 120

At first it may seem as though we can't solve this, because the problem doesn't give us any numbers. But look closer. The x° angle is a small angle, and the $2x^\circ$ angle is a big angle. Because any small angle plus any big angle equals 180° , we know that $x + 2x = 180$. Therefore, $3x = 180$, so $x = 60$. To find y , all we need to do is recognize that because x and y are both small angles, they have to be equal. Thus, $y = 60$ and the answer is (B).

You should also know that the same rules about big and small angles apply to parallelograms, which are four-sided figures made up of two pairs of parallel lines.



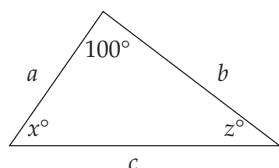
The opposite big angles are equal, the opposite small angles are equal, and the sum of any big angle and small angle is 180° .

TRIANGLES

A triangle is a three-sided figure with three corresponding angles. The points where the sides meet are called **vertices** (each corner is called a **vertex**). The **perimeter** of a triangle (or of any figure) is the sum of the lengths of its sides. The most important rule about triangles relates to the sum of their angles.

If you know two of the three angles of a triangle, you can always find the third. Simply subtract the sum of the two angles you know from 180° .

Now let's take a look at a data sufficiency problem to illustrate another feature of triangles.



The interior angles of any triangle add up to 180° .

1. In the figure above, is a greater than b ?

- (1) $x = 30$ and $z = 50$
- (2) $b = 6$

We need to determine if we have sufficient information to compare a and b . Because there are 180° in a triangle, we know that x and z must add up to 80. Fact (1) tells us the angle measures of x and z . Is this sufficient to answer our question? Yes, because there is a direct relationship between sides and angles in triangles. Since 50 is larger than 30, the side of length a must be longer than the side of length b . Cross off BCE. Fact (2) gives us the length of b . We know that c represents the longest side of the triangle because it's across from the biggest angle (100°), but we have no way of comparing a and b without further information. This fact is insufficient, so the answer is (A).

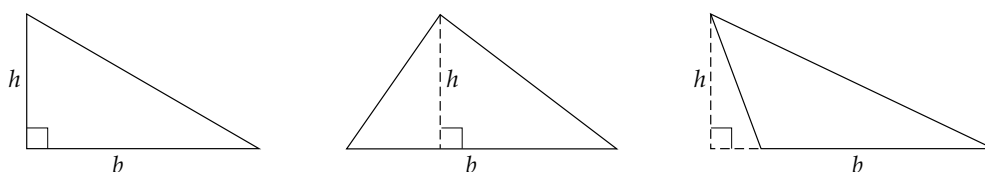
In any triangle, larger sides are opposite larger angles.
Smaller sides are opposite smaller angles.
Equal sides are opposite equal angles.

Area of a Triangle

One of the things you often have to do with triangles is calculate their areas. In order to do that, you need the triangle area formula.

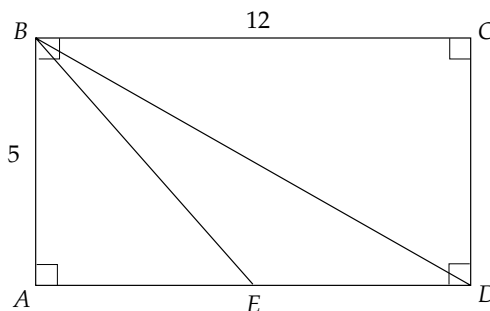
$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}bh$$

Any side of a triangle can serve as the base, although we usually use the one on the bottom for convenience. The key is to remember that the height must be perpendicular to the base.



All of the triangles above have the same base and the same height. Therefore, they all have the same area.

Look at the following problem.



2. In rectangle $ABCD$, E is the midpoint of AD . What is the area of triangle BED ?

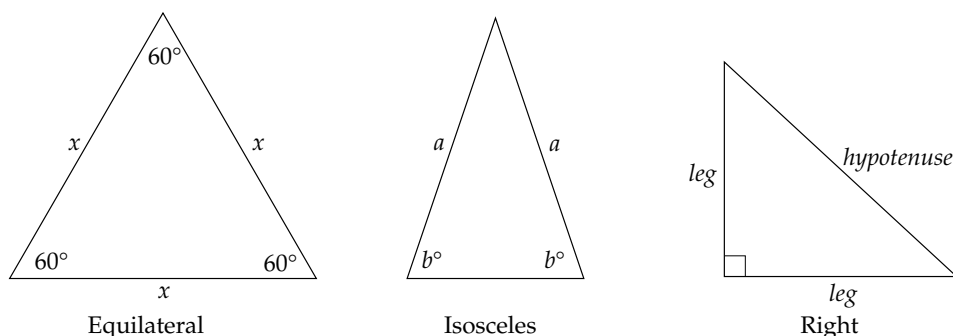
- ☐ 10
☐ 12
☐ 15
☐ 30
☐ 60

The formula for the area of a triangle is $A = \frac{1}{2}bh$, so we need to find the base and height of triangle BED . Since AD and BC have the same length, the length of AD is also 12. E is the midpoint of AD , so ED is 6. That means that the base of

triangle BED is 6. How do we find the height? The height must be 5 because BA is perpendicular to the base and touches the top of the triangle at point B . Thus, BA represents the height. Plugging those numbers into our area formula gives $A = \frac{1}{2}(6)(5)$, which equals 15. That's answer (C).

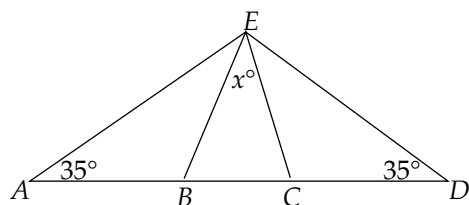
Special Triangles

There are a few different types of special triangles you might see on the GMAT.



Equilateral triangles have three equal sides and three equal angles. The measure of each angle is 60° . **Isosceles triangles** have two equal sides, and the two angles opposite those sides are also equal. **Right triangles** contain one right (90°) angle. The side opposite the right angle is called the **hypotenuse**, and it is the longest side in the triangle. The other two sides are called the **legs**.

You'll learn more about these triangles in class, particularly right triangles. In the meantime, let's look at another problem.



3. In the figure above, $AB = BE = EC = CD$. What is the value of x ?

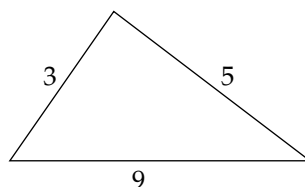
- ☐ 15
- ☐ 20
- ☐ 27
- ☐ 35
- ☐ 40

Because AB , BE , EC , and CD are all equal, there are several isosceles triangles in this diagram. Equal sides are across from equal angles, so you know that $\angle AEB$ and $\angle DEC$ must both be 35° , the same as $\angle BAE$ and $\angle CDE$. The angles of the big triangle AED must add up to 180° , so $35^\circ + 35^\circ + x + 35^\circ + 35^\circ = 180^\circ$. After subtracting, you get $x = 40^\circ$, which is answer (E).

Another way of approaching this problem is to use triangle BEC to find x . $\angle ABE$ and $\angle DCE$ must both be 110° because the angles of those outside triangles must add up to 180° . Then you know that $\angle EBC$ and $\angle ECB$ must both be 70° because there are 180° in a straight angle. Triangle EBC must also contain 180° , so if two of the angles are each 70° , the third one must be 40° .

Impossible Triangles

Why is the following triangle impossible?



The answer is that the length of any side of a triangle is limited by the lengths of the other two sides. This can be summarized by the *third-side rule*:

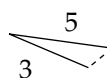
The length of any side of a triangle must be less than the sum of the other two sides and greater than the difference between the other two sides.

This rule is not tested frequently, but when it is, it's usually the key to solving the problem. Here's what that rule means in application. Take any two sides of a triangle. Add them together. Then subtract one from the other. The third side must lie between those two numbers.

Take the sides of 3 and 5 from the triangle above. What's the longest the third side could be? Try drawing the triangle with the two known sides as "open" as possible:



You can see that x , the third side, must be no more than 8 units long. In fact, it must be less than 8 units long. If it were exactly 8, you'd have a flat line, not a triangle. What's the shortest the third side could measure? Try drawing the triangle with the two known sides as "closed" as possible:



You can see that x , the third side, must be at least 2 units long. Actually, it must be greater than 2.

The easiest way to figure this out, of course, is to add and subtract: $5 + 3 = 8$ and $5 - 3 = 2$. Therefore, the length of the third side must be greater than 2 and less than 8.

Try the following question.

4. In isosceles triangle MNO , the length of MN is 5. What is the perimeter of the triangle?

- (1) The length of MO is 11.
- (2) NO is longer than MN .

With Fact (1) you know that two of the sides are 5 and 11. Because it's an isosceles triangle, the third side must be either 5 or 11. However, a 5, 5, 11 triangle wouldn't satisfy the third-side rule, because $5 + 5 < 11$. (The third side must be between $11 - 5 = 6$ and $11 + 5 = 16$, so 5 won't work.) So MNO must be a 5, 11, 11 isosceles triangle, and you could add up those sides to find the perimeter. Fact (1) is sufficient, so cross off (B), (C), and (E). Fact (2) is no help because it doesn't tell you exactly how long NO is, and it leaves MO out completely. Fact (2) is insufficient, so the answer is (A).

QUADRILATERALS

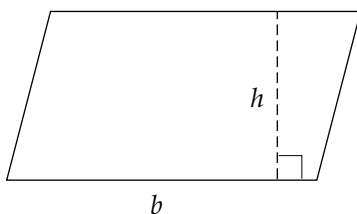
A quadrilateral is a four-sided figure. As with triangles, there is a rule about the sum of the angles.

This **rule of 360°** applies to any four-sided figure, no matter how it's drawn, but the GMAT focuses on a few particular quadrilaterals.

The interior angles of any quadrilateral add up to 360° .

Parallelograms

We briefly looked at a parallelogram when we discussed parallel lines.



Parallelograms have the following properties:

- Opposite sides are equal and parallel.
- Opposite angles are equal.
- Adjacent angles add up to 180° (big angle + small angle = 180°).
- Area = bh

Remember that, as with triangles, when you're calculating the area of a parallelogram, the height must be perpendicular to the base (as shown in the figure above).

Rectangles

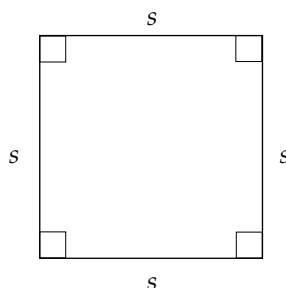


A rectangle is a parallelogram with four right angles. Therefore, it has all of the same properties that parallelograms do. Because the length and width are perpendicular, they're equivalent to the base and height. We simply express the area formula slightly differently. Also, instead of adding up all four sides to find the perimeter, we can just add the length and width and double the result. (This is true for parallelograms as well.)

For rectangles:

- Area = lw
- Perimeter = $2(l + w)$ or $2l + 2w$

Squares



A square is a rectangle with four equal sides. Therefore, it has all the properties of parallelograms and rectangles. Since the length and width are the same, we can just multiply one side by itself to find the area, and since all sides are equal, we can multiply one side by 4 to find the perimeter.

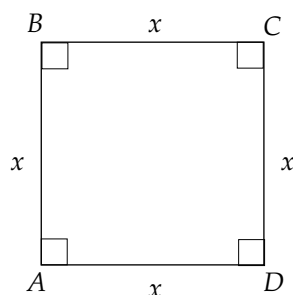
For squares:

- Area = s^2
- Perimeter = $4s$

1. A photographer has a rectangular photograph that he wants to frame, but he can only afford to frame three sides, leaving one 8-inch side open. If the area of the photograph is 112 square inches, how many inches of frame does he need for the other three sides?

☐ 14
☐ 28
☐ 36
☐ 44
☐ 84

We need to find the sum of three of the sides of this rectangular photograph. We know that one side of the rectangle is 8. We also know that the area of the rectangle is 112. The formula for the area of a rectangle is $A = lw$, so $112 = l(8)$. (We actually don't know if 8 is the length or width, but it doesn't matter.) $112 \div 8 = 14$, so the other dimension is 14. This is a 14 by 8 rectangle. We need to know the sum of three of the sides, leaving out an 8-inch side. So we have $14 + 8 + 14$, which is equal to 36. That gives us (C) as our answer.



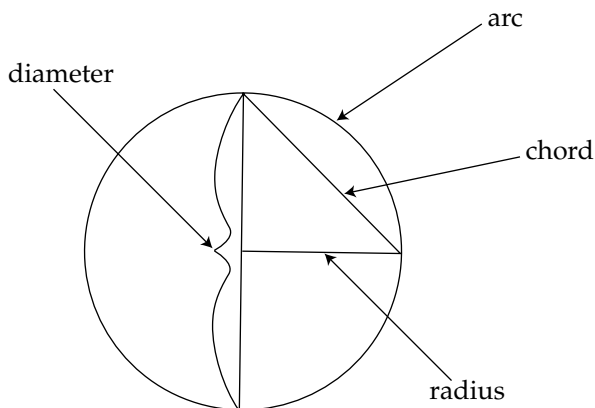
2. The perimeter of figure $ABCD$ is $16\sqrt{2}$. What is the area of the figure?

☐ 16
☐ 32
☐ 44
☐ 48
☐ 64

Four equal sides and four right angles tell us that $ABCD$ is a square. In order to calculate the area, we need to find the length of a side. We know that the perimeter is $16\sqrt{2}$. In a square, $P = 4s$, so dividing the perimeter by 4 will give us the length of a side: $16\sqrt{2} \div 4 = 4\sqrt{2}$. Now that we know the length of a side, we're ready to find the area. For a square, $A = s^2$, so the area of $ABCD$ is $(4\sqrt{2})^2 = 4^2 \times 2 = 32$. The answer is (B).

CIRCLES

There are a handful of things you need to know about circles. Let's start with some of the basic parts.



The most important part of a circle is the **radius**, the distance from the center of the circle out to the edge. All radii are equal because a circle is defined as a figure in a plane with all points equidistant from the center. A **chord** is a line segment from one point on the circle to another. A chord that passes through the center of the circle is called a **diameter**. The diameter is the longest chord in a circle.

A diameter is also equivalent to two radii placed end to end. Thus, you can write the equation $d = 2r$, where d is the diameter and r is the radius.

π = a little more than 3.

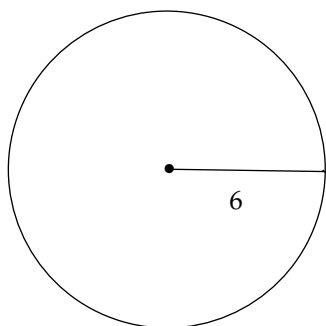
The **circumference** is the distance around a circle. Think of it as the circle's version of perimeter. An **arc** is a piece of a circle (so the length of an arc is a portion of the circumference). There is a relationship between the circumference of any circle and its diameter. In any circle, the circumference is slightly more than three times the diameter. To be precise the circumference is the diameter times 3.1415926535..., a number that we know more familiarly as **pi**, usually written with the Greek letter π . Most of the time, you can just use the symbol π and not worry about its numerical value. However, you shouldn't forget that π represents an actual number. For GMAT purposes, you only need to know that π is a little more than 3.

There are two formulas that you need to know for circles: one for circumference and another for area:

- Circumference = $\pi d = 2\pi r$
- Area = πr^2

In both formulas, r is the radius and d is the diameter. Because the diameter is twice the length of the radius, $C = \pi d$ and $C = 2\pi r$ are really the same formula. Use whichever one you like better. For data sufficiency questions it's important to note that because the radius, diameter, circumference, and area are all connected, knowing one of those values is sufficient to find all the rest.

Fill in the following missing values.



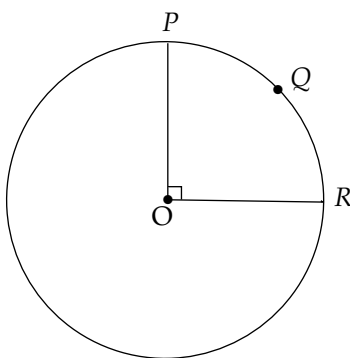
Diameter = ____

Circumference = ____

Area = ____

The radius is 6, therefore the diameter is **12**. The formula for circumference is $C = \pi d$ so the circumference is **12π** . The formula for area is $A = \pi r^2$ so the area is **36π** .

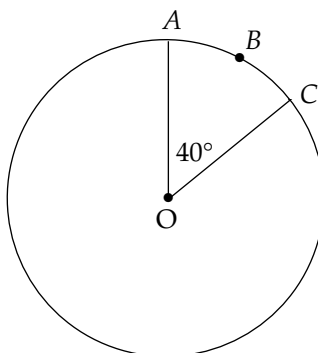
There's another important rule you should know about circles. Recall that a circle contains 360° (one complete revolution). Look at the following diagram.



An angle with a vertex at the center of a circle is called a **central angle**.

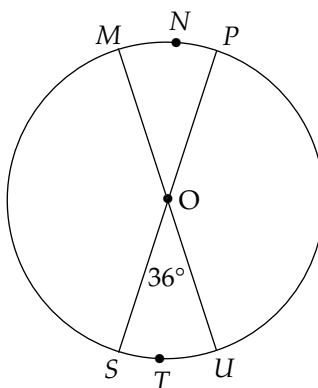
Consider the arc PQR . What fraction of the circumference of the circle do you think that arc represents? One quarter, right? And what fraction of the area of the circle do you think the “wedge” region $PQRO$ represents? It's the same thing, one quarter. All this is fairly easy to see, but the reason it's true is that the right angle of 90° is one-quarter of the total 360° of the circle: $\frac{90}{360} = \frac{1}{4}$. The length of an arc is a part of the circumference proportional to the measure of the central angle when compared to the entire circle. Similarly, the area of the wedge, or sector, created by an arc is a part of the area proportional to the measure of the central angle. Thus, if x is the measure of the central angle, the length of an arc is $\frac{x}{360}$ times the circumference of the entire circle, and the area of a sector is $\frac{x}{360}$ times the area of the entire circle.

Look at the following.



It's not as easy to determine by eye what the relationship is here, but let's apply the rule. We have a 40° central angle. $\frac{40}{360} = \frac{1}{9}$, so the arc ABC is one-ninth the circumference of the circle, and the area of sector $ABCO$ is one-ninth the area of the circle.

Let's look at a problem.



1. If the radius of the circle above is 5, then the sum of the lengths of the arcs MNP and STU are

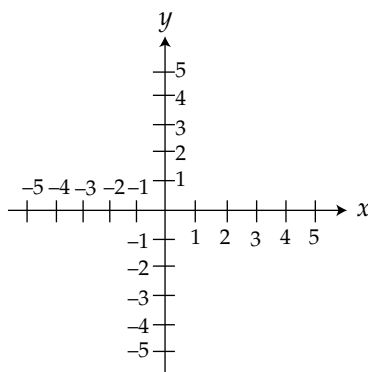
- ☐ 2π
☐ 4π
☐ 5π
☐ 12π
☐ 18π

The radius is 5, which means the diameter is 10. $C = \pi d$, so the circumference is 10π . This is enough to see that answer choices (D) and (E) are too large. $\angle SOU$ is 36° and so is $\angle MOP$ because they are vertical angles. $\frac{36}{360} = \frac{1}{10}$, so arcs MNP and STU are each one-tenth the circumference. Together, they are two tenths, or one fifth. The circumference is 10π , and $\frac{1}{5}(10\pi) = 2\pi$. The correct answer is (A).

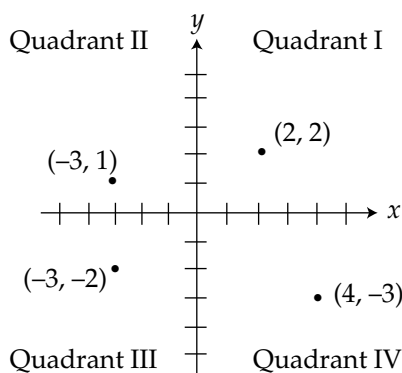
COORDINATE GEOMETRY

The GMAT will occasionally test your familiarity with the coordinate grid. You'll be learning more about coordinate geometry in class, but let's review the basics.

The coordinate grid consists of two perpendicular axes, labeled x and y .

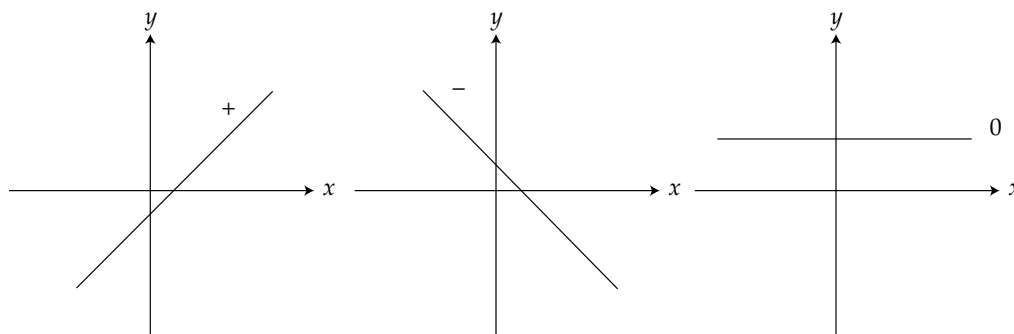


The horizontal line is the x -axis, and the vertical line is the y -axis. The point where the axes intersect is called the **origin**. The x -coordinate of any point tells us its horizontal position, while the y -coordinate tells us its vertical position. Positive x coordinates are located to the right of the y -axis, while negative x coordinates are to the left. Positive y coordinates are located above the x -axis, while negative y coordinates are below. The origin has the coordinates $(0, 0)$. Every point in the grid has an x -coordinate and a y -coordinate and can be expressed in the form (x, y) . The x -coordinate is always listed first, by convention.



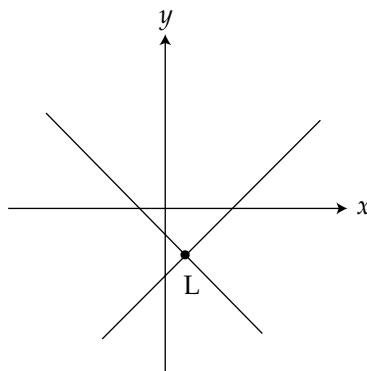
The x - and y -axes split the grid into four quadrants, starting with Quadrant I in the upper right and moving counterclockwise. All the points in Quadrant I have a positive x -coordinate and a positive y -coordinate $(+, +)$. All the points in Quadrant II have a negative x -coordinate and a positive y -coordinate $(-, +)$. All the points in Quadrant III have a negative x -coordinate and a negative y -coordinate $(-, -)$. All the points in Quadrant IV have a positive x -coordinate and a negative y -coordinate $(+, -)$.

You also need to know a little about handling lines on the coordinate grid. The most important concept is the **slope**. Slope is the measure of the steepness of a line, the vertical change over the corresponding horizontal change. You'll learn more about slope in class, so for now just note the following points.



Any line that rises from left to right has a positive slope. Any line that falls from left to right has a negative slope. A horizontal line has a slope of 0. And a vertical line has no slope at all (because a vertical line has no horizontal change, the formula for the slope would require you to divide by 0, which isn't allowed).

Try this question.



1. In the coordinate grid above, which of the following could be the coordinates of point L ?

☐ $(-3, 2)$
☐ $(-2, 3)$
☐ $(-3, -2)$
☐ $(2, 4)$
☐ $(2, -3)$

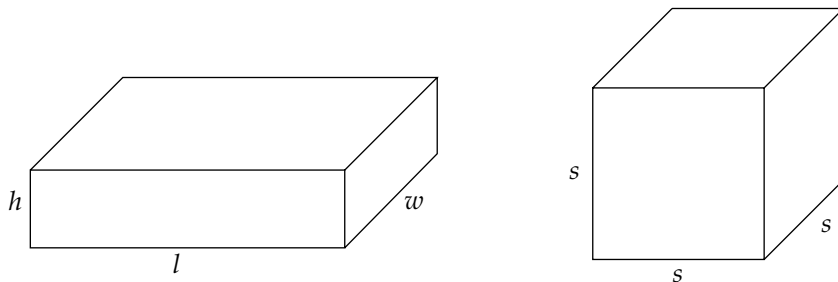
Point L is to the right of the y -axis, which means its x -coordinate is positive, and it is below the x -axis, which means its y -coordinate is negative. The only answer choice in the form $(+, -)$ is (E).

SOLID GEOMETRY

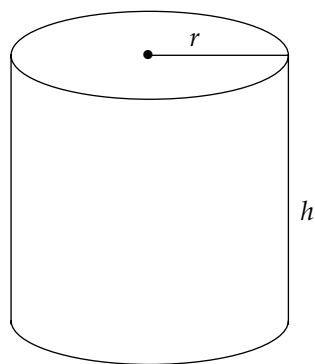
The GMAT may test you on some basic three-dimensional objects. There isn't anything automatically difficult about handling these problems. Solid geometry involves many of the geometry rules that have been covered in this chapter. We'll look at some of the fundamentals right now. In class, you'll learn more about how to handle these types of problems.

Volume

You should know how to find the volume of rectangular solids and cubes.



Given the length, width, and height of a rectangular solid, you simply multiply those dimensions together to calculate the volume. (Sometimes the dimension of width is called “depth.” It makes no difference.) Because all three dimensions of a cube are equal, to calculate the volume you merely have to multiply that dimension by itself three times—in other words, cube it.



One other solid you should be familiar with is a cylinder. To find the volume of a cylinder, first calculate the area of the circular base (πr^2) then multiply it by the height of the cylinder. If by some chance you need to find the volume of any other type of solid (cone, sphere, pyramid, etc.), the formula will be given to you in the problem.

For a rectangular solid: $V = lwh$

For a cube: $V = s^3$

For a cylinder: $V = \pi r^2 h$

Surface Area

Another topic that shows up once in a while in solid geometry is surface area, which is the combined area of all the surfaces, or faces, of a solid. Just find the area of each face and add them all together.

The opposite faces of a rectangular solid have the same area, so once you've found the area of each of the three different types of faces (front/back, left/right, top/bottom), you can just double them. The formula is $2(lw + lh + wh)$ or $2lw + 2lh + 2wh$.

Cubes have six equal faces, so you only have to find the area of one face and multiply it by six. The formula is $6s^2$.

1. A rectangular solid S has dimensions 4 by 9 by K . What is the value of K ?

(1) $4 < K < 9$

(2) A cube of side 6 has the same volume as rectangular solid S .

Fact (1) tells us that the length of K is between the lengths of the other two dimensions, but it could be anything between 4 and 9. That's insufficient, so cross off (D) and (A). Fact (2) allows us to calculate that the volume of the rectangular solid is 216 (the same as 6^3). Thus we could write the equation $216 = 4 \times 9 \times K$. That's enough to solve for K , so this fact is sufficient, and the answer is (B).

SUMMARY

Angles and Degrees

- There are 90° in a right angle.
- There are 180° in a straight angle.
- There are 180° in a triangle.
- There are 360° in a circle.
- There are 360° in a quadrilateral.
- When two lines intersect, opposite angles (vertical angles) are equal.

Perpendicular and Parallel Lines

- Perpendicular lines are lines that meet at a right angle.
- Parallel lines are lines in a plane that never intersect.

When a line crosses two parallel lines:

- Two kinds of angles are created: big angles and small angles.
- All big angles are the same. All small angles are the same.
- Any big angle plus any small angle is equal to 180° .

Triangles

- Larger sides are across from larger angles, smaller sides are across from smaller angles, and equal sides are across from equal angles.
- The third side of any triangle must be smaller than the sum of, and greater than the difference between, the other two sides.
- An isosceles triangle has two equal sides and two equal angles.
- An equilateral triangle has three equal sides and three equal (60°) angles.
- A right triangle has one 90° angle.
- Area of a triangle = $\frac{1}{2}bh$

Quadrilaterals

- Area of a parallelogram = bh
- Area of a rectangle = lw
- Area of a square = s^2
- Perimeter = sum of the sides of any figure

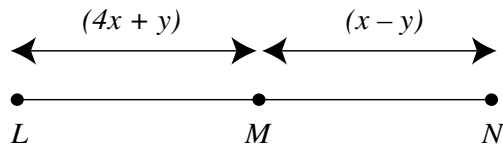
Coordinate Geometry

- The coordinate grid consists of a horizontal x -axis and a vertical y -axis.
- Coordinates are listed in the form (x, y) .
- The origin is the intersection of the axes and has the coordinates $(0, 0)$.
- A line that rises from left to right has a positive slope.
- A line that falls from left to right has a negative slope.
- A horizontal line has a slope of 0.

Solid Geometry

- Volume of a rectangular solid = lwh
- Volume of a cube = s^3
- Volume of a cylinder = πr^2h
- Surface area is the combined area of all the surfaces of a solid

DRILL



1. If segment LN is bisected by point M , which of the following must be true?

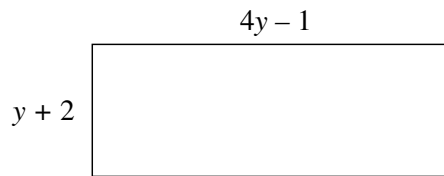
☐ $y = -\frac{3}{2}x$

☐ $y = \frac{2}{3}x$

☐ $y = \frac{2}{3}x$

☐ $y = x + 4$

☐ $y = x + 2$



2. What is the perimeter of the rectangle above?

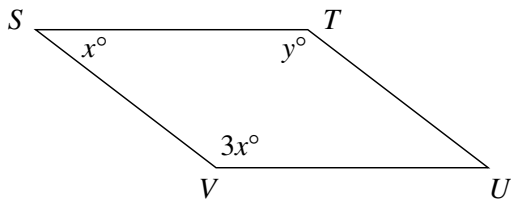
☐ $10y + 2$

☐ $10y + 1$

☐ $9y - 1$

☐ $5y - 1$

☐ $4y + 2$



3. If $STUV$ in the figure above is a parallelogram, $y =$

☐ 45

☐ 90

☐ 135

☐ 150

☐ 225

4. If the perimeter of square S is twice that of square T , then the area of square S is how many times the area of square T ?

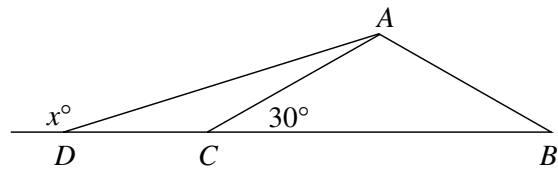
☐ $\frac{1}{4}$

☐ $\frac{1}{2}$

☐ $\sqrt{2}$

☐ 2

☐ 4



5. In the figure above, if $AC = CD$, then $x =$

☐ 105

☐ 120

☐ 135

☐ 150

☐ 165

6. The following are the dimensions of five rectangular fishbowls. All hold the same amount of water EXCEPT the one with dimensions

- ☐ $3 \times 7 \times 2$
- ☐ $2 \times 3 \times 10$
- ☐ $3 \times 5 \times 4$
- ☐ $2 \times 6 \times 5$
- ☐ $15 \times 2 \times 2$

7. The volume of a cube with edge 3 is how many times the volume of a cube with edge $\sqrt{3}$?

- ☐ $\frac{1}{3}$
- ☐ 1
- ☐ 3
- ☐ $3\sqrt{3}$
- ☐ 9

8. The degree measures of the three angles of a triangle are p , q , and r respectively. If p is the average (arithmetic mean) of q and r , then $p =$

- ☐ 30
- ☐ 45
- ☐ 60
- ☐ 90
- ☐ 120

9. Is the area of Circle A greater than the area of Circle B ?

- (1) The ratio of the circumference to the diameter is the same for Circle A as it is for Circle B .
- (2) The radius of Circle A is one-half the diameter of Circle B .

10. Jim designs a rectangular-banner with width w and height h . He then realizes that he must increase the width by 10 percent and the height by 20 percent. In terms of w and h , what is the area of the new banner?

- ☐ $30wh$
- ☐ $32wh$
- ☐ wh
- ☐ $1.3wh$
- ☐ $1.32wh$

11. Is quadrilateral $MNPQ$ a rectangle?

- (1) At least two of the four angles are right angles.
- (2) At least three of the four angles are right angles.

ANSWERS AND EXPLANATIONS

1. **A** If M is the midpoint, then $4x + y = x - y$. Once you simplify, you get $y = -\frac{3}{2}x$. To be sure, do some Plugging In: $x = 2$ and $y = -3$.
2. **A** The perimeter $= 2(y + 2) + 2(4y - 1)$. Simplify: $2y + 4 + 8y - 2 = 10y + 2$. You can also Plug In.
3. **C** Adjacent angles in a parallelogram total 180, so $x + 3x = 180$; $x = 45$, and $3x = 135$. Opposite angles in a parallelogram have the same measure, so y also equals 135.
4. **E** First, cross off (A) and (B) because S has to be larger than T . Plug In some numbers: If the perimeter of S is 32, its area is 64; if the perimeter of T is 16, its area is 16. The ratio is **4: 1**.
5. **E** The measure of $\angle DCA$ is 150° , and $\triangle DCA$ is isosceles. Therefore, $\angle ADC = \angle DAC = 15^\circ$, and $x = 165$.
6. **A** You can multiply 'em all out if you want: (A) = 42, and the other 4 equal 60. Here's a good hint: all but (A) have a factor of 5.
7. **D** The volume of a cube with edge 3 is 27, and a cube with edge $\sqrt{3}$ has a volume of $3\sqrt{3}$. When you divide 27 by $3\sqrt{3}$, you get $3\sqrt{3}$.
8. **C** Plug In the Answers: If $p = 60$, then $q + r = 120$ (because $p + q + r = 180$). If $q + r = 120$, then the average of q and r is 60.
9. **B** Fact (1) doesn't tell you anything except that the formula for circumference of a circle is the same for both circles. Fact (2) tells you that the circles have the same radius, so they would have the same area.
10. **E** Plug In $w = 10$ and $h = 10$. The new width is 11 and the new height is 12, so the new area is **132** (target answer).
11. **B** Fact (2) works; if three angles of a quadrilateral are right angles, the fourth angle must also be a right angle. Thus, $MNPQ$ is a rectangle. Fact (1) is no good, because the two right angles could be opposite one another and the figure could resemble a kite.

AWA

Located at the beginning of the exam, the Analytical Writing Assessment (AWA) is designed to test your ability to express yourself in a cogent, well-organized manner. On each test, you'll be given two essays—Analysis of an Issue and Analysis of an Argument. Each will take 30 minutes and be scored on a scale of 0 to 6. The AWA score is an average of the scores you receive from both essays, and it's reported separately from your overall score.

SCORING

Both the human graders and computer grading program (known as the e-grader) look for certain features. The essays are graded on organization and clarity, how well you address the questions, and your facility with the conventions of English. Don't worry about whether your ideas are original or creative; focus on writing a clear and organized essay.

The following information provides a general idea of the characteristics of the highest-scoring essays.

Analysis of an Issue

An essay that receives a 6:

- develops a position and explores ideas with perceptive reasons and/or convincing examples.
- is well organized.
- demonstrates outstanding control of language.
- shows facility with the conventions (grammar, usage, and mechanics) of standard written English.
- may have some minor flaws regardless of the high quality of the writing.

Analysis of an Argument

An essay that receives a 6:

- identifies important aspects of the argument and provides insightful analysis.
- displays logical organization and uses clear transitions to connect ideas.
- provides effective support for the critique of the main point.
- demonstrates outstanding command of language.
- shows facility with the conventions of standard written English.
- may have minor flaws regardless of the high quality of the writing.

As the scoring criteria suggest, you are not expected to develop a brilliant, sophisticated essay in 30 minutes. A well-organized, clearly-structured essay that uses straightforward language will receive a good score. Keep these points in mind:

- **Use simple, direct language.** Don't bother with unnecessarily big words, and don't use rhetorical style. Subtlety is not going to win many points. It could also be embarrassing if you use a word improperly.

- **Cater to a short attention span.** Because the readers look at so many essays, they spend about 2 to 3 minutes on each one. State your points directly in simple, short paragraphs. Dithering and long-windedness will only try your reader's patience. The last thing you want to do is make the reader's job any more unpleasant than it already is.
- **Be resolute.** Don't waver between two sides. Your job is to compel your reader to think the way you do. If you're not sure of yourself, how will your reader believe in what you have to say?

THE BASIC APPROACH

Use the same steps to approach both essays.

Step 1: Brainstorm

Take a few minutes to come up with as many ideas as you can that support your position. Jot your ideas down on your scratch paper. Choose the strongest 2–4 points to form the basis of your essay.

Step 2: Outline

Once you have your ideas, organize them into a coherent outline. Remember, good organization is one of the things the GMAT graders look for when they read your writing. Once you have your outline, the essay will flow much more easily. The brainstorming and outlining steps should take a total of about 5 minutes.

Step 3: Write

Use the standard format: introduction, supporting paragraphs, and conclusion. This will ensure that your writing stays focused. Write the “bookends” (introduction and conclusion) first, and fill the space in between with the body paragraphs. After all, you're using a word processor, so you can insert text. This also avoids the heartbreak of not finishing your essay because you ran out of time. Spend about 20 minutes writing the essay.

Step 4: Finish

Leave a few minutes to finish your essay. Read it over to check spelling and organization, and make any corrections.

THE ARGUMENT TASK

The argument task requires you to analyze an argument's logic. Give your opinion only on the structure of the argument, not on your opinion of the issue. This essay requires you to break apart an argument as you would on a critical reasoning question. Your job is to decide if the argument is persuasive and well reasoned; whether you agree or disagree with the author's conclusion is NOT relevant. Instead, you should:

- Identify the conclusion and premises of the argument.
- Identify the assumptions on which the author's conclusion relies.
- Decide whether the assumptions are valid and support the conclusion adequately.
- Indicate how you would make the argument more sound.

Brainstorming

Practice the brainstorming step with this sample prompt.

If a collision warning system were installed in all motorized vehicles, it would prevent the vast majority of low-impact driving accidents. When the system determines that a vehicle is on a collision course with another vehicle, a radio signal would be sent between both vehicles that would trigger an alarm. This alarm alerts the drivers to take evasive maneuvers in order to prevent the accident.

Brainstorm by breaking apart the argument, just as you would on a critical reasoning question.

Conclusion:

Premises:

What are some assumptions?

We'll talk more about how to construct the essay in class.

SAMPLE

Here's an example of a high-scoring essay written about the sample topic above. Note the structure of the essay and the way that key words emphasize this organization.

The author states that, "If a collision warning system were installed in all motorized vehicles, it would prevent the vast majority of low-impact driving accidents." This argument, however, is logically unconvincing. The following essay will explain the weaknesses of the argument and suggest ways to improve it.

The author's argument rests on the premise that the warning system would trigger an alarm which would alert the driver to "take evasive maneuvers." The author is making the assumption that drivers will actually take such maneuvers and that they will be successful. If this is not true, and the driver either does not or cannot respond to the alarm, accidents will not be prevented. Without further evidence that the alarm will actually cause drivers to act, it is impossible to conclude that accidents will be prevented.

The author also fails to define what he or she means by "low-impact driving accidents." If he means accidents in which both cars were moving at low speeds, the argument may have limited validity. However, if a low impact accident could be one in which a moving vehicle collides with a parked car, the warning system may not work. The fact that the system relies on a signal "between both vehicles" may be a problem if both vehicles need to be running in order for this signal to be sent from one to the other. If the system only

functions when both vehicles are running, it will not prevent any accidents in which one vehicle is parked.

The author also fails to take into account what would happen if drivers began to rely too heavily on the warning system. If drivers become lazy and are not as alert as they would have been without the warning system, many accidents may occur which never would have happened without the introduction of the early warning system. This would most definitely contradict the author's conclusion.

In conclusion, although the author's argument appears at first glance to be logically sound, it has many weaknesses. If the author could provide evidence that drivers would respond to the warning alarm, a better definition of "low-impact" accidents, and assurance that drivers would not become too lax in their driving habits, the argument would be strengthened. Without this evidence, however, the argument fails to convince the reader.

ADMISSIONS INSIGHT

Study Break

Visitors to our Business School Discussion Boards on PrincetonReview.com post messages about their experience with GMAT preparation and the business school admissions process. You can also see how your preconceptions compare to the reality of business school by taking our B-School Reality Quiz.

Business School Glossary

Business school students, graduates, and professors seem to speak a language all their own. Prepare to join the ranks with this MBA insider lingo:

B2B: "Business to Business": A company that sells not to retail consumers, but to other enterprises. With the renewed focus on more traditional industries, this now stands for "Back to Basics."

B2C: "Business to Consumer": A company that sells primarily to individual retail consumers. As with the above joke about B2B, business students occasionally say this really means "Back to Consulting."

Back of the Envelope: A quick analysis of numbers, as if scribbled on the back of an envelope.

Benchmarking: Comparing a company to others in the industry.

Burn Rate: Amount of cash a money-losing company consumes during a period of time.

Cycle Time: How fast you can turn something around.

Deliverable: Your end product.

Four Ps: Elements of a marketing strategy: Price, Promotion, Place, Product.

Fume Date: Date the company will run out of cash reserves.

Low Hanging Fruit: Tasks or goals that are most easy to achieve (consultant jargon).

Net Net: End result.

Pro Forma: Financial presentation of hypothetical events, such as projected earnings.

Quant Jock: A numerical athlete who is happiest crunching numbers.

Rule of Three: You should not talk more than three times in any given class, but you should participate at least once over the course of three classes.

Run the Numbers: Analyze quantitatively.

Slice and Dice: Running all kinds of quantitative analyses on a set of numbers.

Soft Skills: Conflict resolution, teamwork, negotiation, oral and written communication.

Take-aways: The key points of a lecture or meeting that participants should remember.

The Five Forces: Michael Porter's model for analyzing the strategic attractiveness of an industry.

Three Cs: The primary forces considered in marketing: Customer, Competition, Company.

Value-Based Decision Making: Values and ethics as part of the practice of business.

ASSIGNMENT 6

SENTENCE CORRECTION REVISITED

In this lesson, we'll revisit sentence correction questions. We'll examine some minor grammar rules and discuss how to work the most difficult questions.

BEYOND THE BIG SIX

Most sentence correction questions contain one or more of the Big 6 errors. However, there are a few other errors that can appear on the GMAT. You'll see these minor errors infrequently, but it's still a good idea to learn to recognize and correct them.

Clauses and Connectors

Occasionally, the GMAT tests your ability to use conjunctions and connect clauses correctly.

Many investors withdrew their money from the stock market
because it was erratic.

If you took *Because* away from the dependent clause, it would be an independent clause. *Because* serves as a connector of the two clauses. Whenever a sentence consists of two connected clauses, you need one and only one connecting word. If you have no connecting words, you get a run-on sentence:

Many investors withdrew their money from the stock market, it was
erratic.

If you have two connecting words, you get gibberish:

Since many investors withdrew their money from the stock market,
because it was erratic.

Gerunds

You already learned that when the *-ing* form of a verb takes on the role of a noun, it's called a gerund. Whenever you use a gerund, make sure that any nouns or pronouns are possessive, as in these sentences:

My learning Japanese made my trip to Tokyo much more enjoyable.

Adam's biting the apple caused his banishment from Paradise.

In general, it's best to avoid gerunds (remember the gerund *being*, which is almost always wrong). Whenever you see a gerund, there's usually a noun form that's better to use.

Okay: My *knowing* about calculus helped me fill out my tax forms.

Better: My *knowledge* of calculus helped me fill out my tax forms.

Where or When

People often misuse *where* in conversational sentences. The most common mistakes look something like these:

The essay section is one *where* I always have difficulty.

The harassment got to the point *where* she had to threaten to sue him.

In each of these cases, *where* is not used properly because you should use *where* only when referring to an actual location:

The linen closet is *where* I keep my sheets.

Here's how to fix the sentences above:

The essay section is one *in which* I always have difficulty.

The harassment got to the point *that* she had to threaten to sue him.

Similarly, people often use *when* inappropriately. *When* refers only to actual moments in time:

I'll apologize to that creep *when* Hell freezes over.

Passive vs. Active Voice

Notice the difference between the two sentences below.

A mistake was made.

I made a mistake.

Both sentences tell you that a mistake occurred. However, in the first sentence you don't know who made the mistake. The sentence is in the **passive voice** because the subject receives the action, and it's unclear who performed the action. The second sentence is in the **active voice**, meaning the subject performs the action. Some GMAT questions present you with a choice between the active and passive voices. Choose the active voice whenever possible.

Quick Quiz: Active vs. Passive

Take the following passive sentences and make them active. Ask yourself who or what performed the action, and make the actor the subject of the sentence.

1. Twelve new dresses were bought by Emily.
2. A good time was had by all.
3. The powerful performances were weakened by the clumsy staging of the play.

Subjunctive

In addition to tense, the GMAT occasionally tests your ability to use the subjunctive mood. The term “subjunctive” makes most people’s eyes glaze over. You might have learned the subjunctive in another language in high school. Relax. It’s not that complicated.

Subjunctive Part I

Use the subjunctive to express an idea that’s contrary to fact or to speculate about a hypothetical situation.

I *wouldn’t* do that if I *were* you.

If Air Force One *were* to crash with the president on board, the vice president *would* become president.

In the first sentence, I’m obviously not you, so the situation is contrary to fact. However, I’m speculating that if I were in your place, I would not do that. Normally, you wouldn’t use *were* with *I*, and the sentence might sound strange for that reason. However, subjunctive conjugation is different. No matter what the subject is, the “if” clause uses the verb *were*. The second example discusses the hypothetical situation of Air Force One crashing. In that case, the vice president would become president.

When you speculate about a situation that is hypothetical or contrary to fact, use the verb *were* in the “if” clause, no matter what the subject is. Use the verb *would* in the “then” clause. The format is:

- If *x* were, *y* would
- *Y* would if *x* were

Subjunctive Part II

Use the second form of the subjunctive with *that* clauses of order or recommendation.

The manager *requested that* the staff *stay* late to finish the project.

We *recommend that* you *arrive* on time for the GMAT.

The princess *demanded that* the frog *be brought* to her.

The regular conjugation in the first example would be *the staff stays late*, but subjunctive conjugation is different. The proper construction looks like this:

Subject Verb 1 *that* Object Verb 2 Rest of Sentence

You need the word *that* and Verb 2 must be in the infinitive form without the *to* in front of it.

Don't use the word *should* with the subjunctive.

Quick Quiz: Subjunctive

Read each of the following sentences and decide if the verb construction is correct. Make any changes needed to create a correct sentence using the subjunctive mood.

1. If the president was a woman, the country would see a real change in policy.
2. Jack suggested that Sylvia should buy a new couch.
3. The new law requires that all teachers undergo background checks.
4. I could visit ancient Rome if it were possible to travel back in time.

HARDER VERBAL PROBLEMS

As you know, if you answer questions on a CAT correctly, you will see increasingly difficult questions. This section looks at how difficulty is built into verbal questions. Although the specifics of difficult questions vary by the type of question (sentence correction, argument, or reading comprehension), there are some general patterns for hard questions of all three types.

Hard to Follow

Some questions are difficult because the text of the sentence, argument, or reading passage is hard to follow. A sentence correction sentence can be convoluted, with many clauses and phrases jammed in, seemingly at random. The logic of an argument can be opaque, with many strange assumptions and disconnected elements. A reading comprehension passage can be densely written, with long, twisting sentences and confusing terms and jargon (especially science passages).

What can you do to deal with hard-to-follow questions? Simplify. Don't get bogged down in all of the details. Focus on the parts that are most important. Also, take your time on these questions. Most test takers feel less pressed for time in the Verbal section than they do in the Math section. Allow yourself plenty of time to deal with the confusing questions.

Focus on the relevant parts of the text.

Several Good Answers

Other questions seem difficult because more than one answer looks reasonable. You know that there can't be two correct answers, but two (or three) of the choices seem pretty good to you. Process of Elimination becomes a critical skill on these questions.

The difference between the best answer and one of the distractor answers can be very subtle. Look for small differences between the answers. Often, just one word that doesn't quite match up is enough to make an answer wrong. Try putting yourself in the shoes of the question writer: You need to be able to justify why the wrong answers are incorrect. What things might you point out as incorrect in the answers you are considering?

Look for subtle differences between answer choices.

No Good Answers

Sometimes none of the answers seems acceptable. This often happens when you rely on how each answer "looks" or "feels." It can also occur when you're not connecting what the argument or reading passage means with what the answer choices say.

Start by eliminating those answers that you know are wrong for some concrete, definable reason. "It sounds bad" or "That doesn't seem right" is not enough. You must be able to say, "It's wrong because ____."

In the remaining choices, look for a connection that you might have missed on your first look at the answers. Often, an answer that you thought was "out of scope," or not connected to the topic, turns out to be correct, and you were misled because its wording was different from that of the passage. Remember that it's the meaning of the answer, not the particular words, that need to match the passage or argument.

In the rest of this lesson, you will learn how these hard verbal patterns apply to sentence correction questions.

Be sure to have a definite reason for eliminating an answer.

HARDER SENTENCE CORRECTION QUESTIONS

Some test takers make the sentence correction questions more difficult than they need to be. The worst way to handle tough questions is to “sound out” each of the answer choices in the sentence, looking for the one that seems right. Instead, master the rules you learn in class, and apply those to the questions. There are particular rules that make sentences good or bad.

Hard to Follow

Some sentences seem to go on forever, taking many twists and turns along the way. It seems very difficult to follow what they are saying, much less check for grammatical errors. This problem is compounded when all or most of the sentence is underlined and the answers are dissimilar to one another.

Simplify a sentence by looking for specific errors or checking particular rules. Then, you can focus on only the parts of the sentence that are relevant. For example, when checking for subject-verb agreement, isolate the subject and the verb, not the distracting prepositional phrase between them. When you are looking for misplaced modifiers, look at the introductory phrase and the following noun.

Do you really need to read the whole sentence in the following examples?

Eager to learn more about the GMAT, the course manual blah blah
blah blah the student.

Elmer blah blah blah *love* to hunt rabbits.

The previous sentences aren’t as tough as some of the questions you will see, but the principle of looking for particular errors or parts of the sentence is the same, whether the question is easy or hard.

See how these concepts apply to the following example.

1. Ella Fitzgerald’s unbridled delight in singing and her far-ranging versatility—encompassing styles as diverse as mournful ballads and show-stopping scat singing—lends credence to the numerous musicologists who claim her to be the greatest female vocalist of her generation.
 - ☐ lends credence to the numerous musicologists who claim her to be
 - ☐ has made believable the numerous claims of musicologists that she is
 - ☐ have lent credence to numerous musicologists that claim she is
 - ☐ have made believable the numerous musicologists’ claims of her to be
 - ☐ lend credence to the claims of numerous musicologists who hail her as

This sentence is long and twisted, so it can be hard to follow if you read the whole thing every time. It’s much better to pick some specific issue on which to focus. The differences among the answer choices usually provide the best clues as to what errors might be present. Notice that the first few words of the answer choices are varying forms of verbs. There is a 2/3 split between the “lend credence” answers and the “made believable” answers. However, that’s not an

Focus on one error at a time in the parts of the sentence relevant to the error.

easy decision to make. Most people would say that the “made believable” answers don’t sound as good as the others, but you don’t want to choose or eliminate answers based on the way they sound. Look for concrete rules first.

You should also notice that you have singular and plural forms of verbs, so focus on subject-verb agreement. What specifically is the subject of the sentence? It’s a compound subject: “delight and...versatility.” The “and” makes the subject plural, so you need a plural verb. Try testing the verbs with “they” and “it.” Eliminate answers (A) and (B) because they have singular verbs.

Another difference you might notice is between the “musicologists that claim” and the “musicologists who claim” answers. Musicologists are people, so use “who.” That eliminates answer (B), which you already eliminated, and answer (C).

You’re down to (D) and (E), so focus on differences between those answers. The “made believable” versus “lend credence” issue is still there, but look for something more concrete first. The difference between “musicologists’ claims” and “claims of...musicologists” also isn’t helpful. Either would work.

The ends of the two answers are significantly different. The “claims of” in (D) is unidiomatic. The correct idiom is “claims...that.” The “hail her as” in (E) is idiomatically correct. So, the best answer is (E).

The subject-verb agreement and “that” versus “who” issues are pretty straightforward. The trick is to notice them among all of the other things happening in this sentence. Once again, when the sentence gets confusing, try to simplify and focus on specific rules that you know. Don’t get distracted by all of the other changes.

Several Good Answers

When two answer choices (or more) seem correct, look for the subtle differences that make one answer better than the other. For sentence correction, the “almost, but not quite” answer often has some additional error in it that you didn’t notice, such as the wrong preposition for the idiom or a change in verb tense.

Look for the subtle differences among answers in the next example.

2. More common among friendly competitors than among bitter rivals, the undesirable consequences of tacit collusion often include oligopoly, which harms consumers by raising prices above efficient levels.
 - ☐ More common among friendly competitors than among bitter rivals, the undesirable consequences of tacit collusion often include
 - ☐ Leading to undesirable consequences, and more common among friendly rivals than bitter enemies, tacit collusion often include
 - ☐ Tacit collusion, which is more common among friendly competitors than among bitter rivals, leads to undesirable consequences, often including
 - ☐ More common among friendly competitors than bitter rivals, tacit collusion leads to undesirable consequences, such as
 - ☐ More common among friendly competitors than bitter rivals, the undesirable consequences of tacit collusion are often including

Little words often make a big difference!

As written, the sentence contains a classic misplaced modifier error. The introductory phrase “More common than...” is supposed to describe “tacit collusion,” not “the undesirable consequences.” You can, therefore, eliminate (A) and (E), which have the same problem. (D) clearly fixes the problem by placing “tacit collusion” immediately after the opening phrase. (B) is a little more convoluted, but it too fixes the misplaced modifier by moving “tacit collusion” to follow the descriptive phrases (“Leading to...” and “and more common than...”). Answer (C) also fixes the misplaced modifier issue, although in a different way. It rewrites the sentence so that it doesn’t have an introductory phrase.

Which of the remaining three answers is the best? Answer (D) seems to be the most straightforward fix. It corrects the misplaced modifier without changing everything else. But is it really better than the others? When you have several answers that seem reasonable, dig more deeply into the differences among them. After all, the test writer has chosen one to be better than the others, and he or she must have some reason to justify that decision.

Look closely at (B). It uses the verb “include,” which was correct in the original sentence. However, rewriting the sentence changed the subject from “consequences” to “collusion.” The new subject requires a singular verb, such as “includes.” The difference between the verb forms is very slight, only a single letter, and the problem is hard to notice unless you are looking very carefully. It is especially difficult when so much of the sentence is underlined and rewritten. Eliminate (B).

Another subtle difference among the answers is that some use “than among bitter rivals,” and some use “than bitter rivals.” Again, the difference is slight and hard to spot amidst all of the major restructurings of the sentence. Parallel construction requires that you use “than among bitter rivals.” Otherwise, the sentence could be interpreted as comparing the commonness of tacit collusion to the commonness of bitter rivals, rather than comparing the commonness of tacit collusion in two cases (among friendly competitors versus among bitter rivals). That eliminates (D), although it is an attractive answer in other respects. Answer (D) may sound better than (C), but you need to choose on the basis of grammar rules first. Stylistic concerns, such as awkwardness and simplicity, are less important than grammar issues. The correct answer is (C).

No Good Answers

Sometimes, none of the answers seem to make a good sentence. When this happens, you need to be careful with your elimination criteria. On the first pass, eliminate only the choices that clearly violate some rule that you know solidly. Don’t eliminate an answer just because it sounds bad.

Beware of making up your own grammar rules. People have a tendency to think that one word or phrase is better than another without a clear reason. For example, blanket rules such as “Never begin a sentence with ‘there’” usually have some exceptions; they are more guidelines than strict grammar rules.

Start by eliminating the answers that you are absolutely certain violate a rule you know. Don’t leap at the first difference in wording you see. Be prepared to hunt around for errors you recognize with confidence.

On the following example, be careful with your reasons for elimination.

3. Renowned economist Susan Albernaght once stated, as an illustration of opportunity cost, that a company that earned a return lower than the interest rate on a savings account ought to be liquidated and the proceeds distributed to the shareholders.

Don't eliminate an answer because it "sounds" bad.

- ☐ that a company that earned a return lower than the interest rate on a savings account ought to be
- ☐ that a company, earning a return lower than the interest on a savings account, should be
- ☐ that a company which earned returns lower than savings account interest rates ought to be
- ☐ a company that was earning a lower return than a savings account was
- ☐ a company earning a return lower than that of a savings account should be

After a quick glance at the answer choices, you may think that none of them is a clear-cut winner. That's okay; the "correct" answer doesn't have to be great. It merely needs to be better than the others. Start by looking for definite grammar errors.

The 2/3 split between "that a company" and "a company" is a good place to start. The correct idiom is "state...that." Eliminate (D) and (E). Notice that it is hard to identify the idiom words when they are separated by all of the other words in between. As you saw on earlier examples, you need to focus on the relevant parts.

You probably noticed that some answers have "ought to be" and some have "should be," and you may want to eliminate the "ought to be" choices. However, you shouldn't eliminate answers unless they clearly violate a grammar rule. Is "ought to be" breaking some rule, or do you merely not like the way it sounds? Look for a more concrete reason to eliminate some answers.

Another reason to eliminate (D) is the word "was." It implies that the liquidation actually occurred already, rather than being a hypothetical example or a recommendation for the future.

One difference among the remaining answers is "that earned" versus "earning" versus "which earned." You can eliminate (C) because of the "which." You use "which," as opposed to "that," when it is setting off a nonessential phrase. This phrase, however, is necessary to the sentence because it identifies the company that should be liquidated. If you throw out that phrase, it sounds like you're just picking some arbitrary company to shut down.

Between (A) and (B), you might be tempted again to eliminate "ought to be." However, there's no particular rule that it is breaking. In (B), however, the phrase set off by commas, "earning...savings account," creates a problem. Separating it from the rest of the sentence with commas is something you do with nonessential phrases. As you just saw, this phrase is essential to identify the company that should be liquidated. Also, you should generally be skeptical of *-ing* endings if there is a reasonable alternative. The best answer is (A), even though "ought to be" doesn't sound that great.

The questions for which (A) is the best answer often follow this pattern. None of the answers will sound good, but (A) doesn't make any clear grammatical errors. So, if all of the answers seem poor, reconsider (A). Make sure you didn't eliminate it for a bad reason.

ANSWERS AND EXPLANATIONS

Quick Quiz: Active vs. Passive

The sentences have been rewritten in the active voice.

1. Emily bought twelve new dresses.
2. All had a good time.
3. The clumsy staging of the play weakened the powerful performances.

Quick Quiz: Subjunctive

The sentences have been rewritten in the subjunctive mood.

1. If the president **were** a woman, the country **would** see a real change in policy.
2. Jack **suggested that** Sylvia **buy** a new couch.
3. The new law requires that all teachers undergo background checks. (No change needed.)
4. I **would** visit ancient Rome if it **were** possible to travel back in time.

MATH MISCELLANEOUS

GROUPS, FUNCTIONS, SEQUENCES, AND SUMMATION

There are a few topics that appear only occasionally in GMAT math questions. None of these topics is terribly difficult once you know how to attack the problem.

Groups

A GMAT problem may involve different groups of people or things. These questions can seem tricky because they deal with overlapping groups. Fortunately, there is an easy way to deal with these questions.

Group Equation

Look at an example:

1. A country club has 230 members, 60 of whom play tennis. If 130 members play golf, and 40 members play both golf and tennis, how many members play neither golf nor tennis?

- ☐ 10
☐ 40
☐ 60
☐ 80
☐ 100

You can easily identify this type of question because the question refers to **two groups** (in this case, golfers and tennis players), **neither**, and **both**. You may remember Venn diagrams (the overlapping circles) from high school, and you could try to solve the problem using those. However, there is an easier way. Just use this equation:

$$\text{Total} = \text{Group 1} + \text{Group 2} + \text{Neither} - \text{Both}$$

There are 230 members **total**, 60 **tennis players**, 130 **golfers**, 40 who play **both**, and the question asks how many play **neither**. Using the formula above, we get: $230 = 60 + 130 + \text{Neither} - 40$. Rearrange to get $\text{Neither} = 230 - 60 - 130 + 40 = 80$, or (D).

Group Grid

Look at the other type of group question.

2. A manufacturing company employs 125 people. Each employee is either a blue-collar employee or a white-collar employee. 67 of the employees are female, and 75 of the employees are blue-collar. If there is a total of 35 male white-collar employees, then the number of female white-collar employees is

- ☐ 15
☐ 23
☐ 52
☐ 50
☐ 58

This question is different from the last one because each member is classified according to two sets of either/or characteristics. In other words, there are actually four groups: blue collar, white collar, male, and female. To solve this question, draw a grid and fill in the information from the question.

| | male | female | total |
|--------------|------|--------|-------|
| white-collar | 35 | | |
| blue-collar | | | 75 |
| total | | 67 | 125 |

To complete the grid, make each row and column add up to the total.

| | male | female | total |
|--------------|------|--------|-------|
| white-collar | 35 | 15 | 50 |
| blue-collar | | | 75 |
| total | | 67 | 125 |

Circle the box you're solving for to help figure out the quickest way to the answer—you don't have to fill in every box. We are looking for female white-collar employees, and we know that there are 50 total white-collar employees (because $125 \text{ workers} - 75 \text{ blue-collar workers} = 50$). If 35 of those white-collar employees are male, then $50 - 35 = 15$ females, (A).

Functions

You may encounter questions with symbols or formulas that you've never seen before. Don't panic! Follow the directions and plug the numbers into the equation.

A function question might look like this:

- If, for all x and y , $x \# y = xy^2$, which of the following is equal to $3 \# 4$?

- ☐ 4 # 3
- ☐ 1 # 6
- ☐ 2 # 6
- ☐ 12 # 1
- ☐ 12 # 2

Think of a function as a set of directions: plug the values into the function, and follow the directions to get the solution. Substitute 3 and 4 for x and y , and you get $3 \# 4 = 3 \times 4^2 = 3 \times 16 = 48$. You need an answer that also works out to 48, so you'll need to perform the function on the answer choices. Since $12 \# 2 = 12 \times 2^2 = 12 \times 4 = 48$, (E) is the answer.

Sets and Sequences

A **set** is a general term for a group of things. There can be some rule that defines a set, for instance, the set of all prime numbers would include {2, 3, 5, 7, 11, 13....} You can also simply define a set by listing the numbers, for example, the set of numbers {8, 9, 22, 64}. That's a pretty meaningless set of numbers, but it is a set if the GMAT says it is. Each thing in a set is called an **element**.

A **sequence** is a set of numbers that are related to each other by some pattern or equation. If you know one value and the equation or pattern, you can figure out other terms in the sequence. Working sequence questions is very similar to working function questions.

1. If a sequence of 8 consecutive even integers with increasing values has 36 as its eighth term, then the least number in the sequence is

☐ 50
☐ 48
☐ 24
☐ 22
☐ 20

We have a list of eight numbers: {___ ___ ___ ___ ___ ___ ___ 36}. Starting with the value we know (36), we can apply the pattern (consecutive even integers with increasing values) and fill in the other values. If 36 is the last term, then 34 is the term before that, 32 is the term before that, and so on, all the way down to 22. The sequence is {22, 24, 26, 28, 30, 32, 34, 36}. Since the question asks for the least number, the answer is 22, choice (D).

Some sequence questions may look more complicated, but they work the same way. For example:

2. If in sequence a , $a_n = 3a_{n-1}$ and $a_2 = 5$, what is the value of a_4 ?

☐ $\frac{5}{3}$
☐ 5
☐ 15
☐ 30
☐ 45

The pattern is $a_n = 3a_{n-1}$, which simply means that each term is three times as much as the term before it. The term a_n means the n th term in the sequence. That

is, a_1 means the first term, and a_2 means the second term, and so on. We're told the second term is 5, and we can use this information to find the next term by plugging the information into the formula. Thus, $a_3 = 3a_2 = (3 \times 5) = 15$. The question asks for the fourth term, so we need to repeat the operation one more time. Since $a_4 = 3a_3 = (3 \times 15) = 45$, the answer is (E).

Summation

Once in a while, a problem will ask you to find the sum of a long list of numbers. To find the sum of a symmetrical list of numbers, such as "all integers from 51 to 100, inclusive," or "the list of consecutive odd numbers between 24 and 40," use the formula:

$$\text{sum} = \frac{\text{number of numbers in the set}}{2} (f + l)$$

where f and l are the first and last terms in the sequence.

For example, the sum of all integers from 1 to 50 is $\frac{50}{2}(1 + 50)$, or 1,275.

If you need to determine how many numbers are in a range, subtract the lower number from the higher number and add 1. For example, the range of numbers from 27 to 64 is $(64 - 27) + 1 = 38$, so the sum of all integers from 27 to 64 is $\frac{38}{2}(27 + 64)$, or 1,729.

HARDER PERMUTATIONS AND COMBINATIONS

You learned the basics of combinations and permutations earlier in the course. The two formulas you learned will get you through the easy to intermediate problems involving these concepts. However, the more difficult problems require you to understand how combinations and permutations fit together so you can adapt the formulas to the specifics of the problem.

Take a look at the following problem.

1. Elmo wants to make a compact disc containing his three favorite songs. He has identified six songs—A, B, C, D, E, and F—as candidates, and now he must choose the best three and decide in what order to record them on the CD. How many different versions of this 3-song CD are possible?

You can solve this problem using methods you learned earlier in the course. You could approach it by viewing it as three decisions and then multiplying the number of options for each decision together. There are 6 candidates for the first song, which leaves 5 possibilities for the second song and 4 possibilities for the third song. That creates $6 \times 5 \times 4 = 120$ possible versions of the CD.

Alternatively, you could use the permutation formula to get the same result. There are 6 candidates, so $n = 6$. Elmo is choosing 3 songs, so $r = 3$.

When you plug these numbers into the formula, you get

$$P = \frac{n!}{(n-r)!} = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} = 120.$$

A third possibility is to break this problem into two sub-problems: a combination and a permutation. It's not the most efficient way to answer the questions but it will provide some insight into the way combinations and permutations fit together. This understanding can be critical for answering more difficult questions that do not fit nicely into the patterns you've seen up to now.

You can analyze Elmo's decision in two stages:

1. He must decide which 3 songs he wants to include. This is a combination because order is not involved.
2. Once he has chosen the 3 songs, he must decide in what order to record them. This is a permutation because order is involved.

In the first phase, you find the number of 3-song groups (combinations) that are possible. Then, you multiply by the number of ways to arrange a given 3-song group. That will give you the total number of permutations.

| | | | | |
|---------------------------------|---|--|---|----------------------------|
| # of combinations of 3 songs | × | # of ways to order a group of 3 songs | = | Total # of permutations |
|---------------------------------|---|--|---|----------------------------|

The first step is to determine how many 3-song combinations are possible.

You could use the group formula for this:

$$C = \frac{n!}{r!(n-r)!} = \frac{6!}{3!3!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 3 \times 2 \times 1} = 20. \text{ The 20 possible groups include}$$

ABC, ABD, ABE, and so forth.

Next, suppose that Elmo has selected his 3 most favorite songs. He needs to determine the order in which to record these songs on the CD. This a permutation in which you choose 3 out of 3 items and arrange them. There are $3 \times 2 \times 1 = 6$ choices. You could also use the permutation formula, with $n = 3$ and $r = 3$. You will get the same result. (Remember that $0! = 1$.)

So, there are 20 possible groups (combinations), each of which could be arranged in 6 different ways, for a total of $20 \times 6 = 120$ permutations.

The key insight about combinations and permutations is that a permutation is just a combination that has been arranged in some order. Conversely, a combination is just a permutation with the ordering factored out. That's why there is an extra $r!$ in the denominator of the combination formula.

$$P = \frac{n!}{(n-r)!} \quad C = \frac{n!}{r!(n-r)!} = \frac{P}{r!}$$

CIRCULAR PERMUTATIONS

Questions involving circular permutations are very rare. In fact, it's fine to skip this section. It's only worth spending time on this topic if you've mastered everything else.

Compare these problems:

- Four honorees are to be seated along one side of a rectangular head table for a banquet in their honor. If there are exactly four places along the side where the honorees are to be seated, in how many different orders may they be arranged?
 - ☐ 4
 - ☐ 6
 - ☐ 12
 - ☐ 24
 - ☐ 64
- Four honorees are to be seated around a circular head table at a banquet in their honor. If there are exactly four places around the table, in how many different orders may they be arranged?
 - ☐ 4
 - ☐ 6
 - ☐ 12
 - ☐ 24
 - ☐ 64

At first glance it may seem like both are asking the same question, but they're not. Problem #1 is linear—that is, the head table has two ends. If we call our four honorees A, B, C, and D, then these two arrangements should be counted separately:

A B C D *is different from* D A B C

Problem #2, however, is circular. Since the arrangement into which you're placing A, B, C, and D hasn't got ends, these two arrangements should **not** be counted separately:

| | | |
|---|---|--------|
| A | | B |
| D | B | A C |
| C | | D |

is the same as

As you can see, in both arrangements A is sitting with D to the left and B to the right, B is sitting with A to the left and C to the right, and so on.

We already know how to work #1:

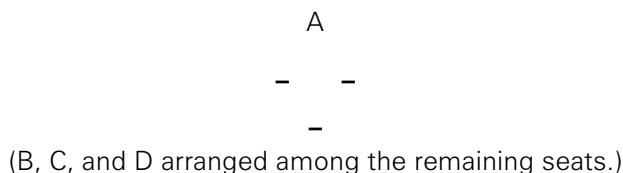
possibilities, seat 1 × remaining possibilities, seat 2 × remaining possibilities, seat 3 × last possibility, seat 4
 $= 4(3)(2)(1) = 4! = 24$

Or, in terms of the formula we learned earlier:

$$P = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{24}{1} = 24$$

(Don't forget that $0! = 1$.)

When you're working a circular problem, the method isn't a great deal different. The one thing we have to do is find a way to "fix" our circle so we don't count the same permutation more than once. The way we do that is to count all the permutations with one of the elements—say, A—sitting in one particular seat, like this:



This way, we won't count any permutation twice. We simply do the calculation:

$$\begin{aligned}
 &\text{single possibility, seat 1} \times \text{remaining possibility, seat 2} \times \text{remaining} \\
 &\quad \text{possibilities, seat 3} \times \text{last possibility,} \\
 &\text{seat 4} = 1(3)(2)(1) = 3! = 6
 \end{aligned}$$

If you want to think in terms of the permutations formula, all we really have to do is *divide by* the number of separate times the formula for linear permutations counts each circular permutation. Basically, this means dividing by the number of "seats around the table"—the number of different locations in which the first object can be placed.

In this case, for each permutation, there are 4 different places where A could be seated, and therefore 4 "different-looking" versions of each circular permutation. So we take our linear answer (4!) and divide by 4 (the number of seats at the table) to get the answer: 6.

For those of you who are keeping score at home, those 6 possibilities are:

- | | |
|---------|---------|
| 1. ABCD | 2. DCBA |
| 3. ACDB | 4. BDCA |
| 5. ADBC | 6. CBDA |

(The honoree on the leftmost end is seated next to the honoree on the rightmost end.)

In most contexts, you'll be able to apply the formula below to get the right answer in circular cases.

$$\text{Number of circular permutations} = \frac{p(n, r)}{l} = \frac{n!}{(n-r)!l}$$

n = Number of objects in the source group

r = Number of objects selected

l = Number of possible locations for first object placed

As always, understanding the idea behind the formula is the only sure way to apply it correctly on more difficult problems.

SUMMARY

Groups

- When there are 2 different groups, and people (or objects) can belong to one or the other, both, or neither, use the Group Equation: $\text{Total} = \text{Group 1} + \text{Group 2} + \text{Neither} - \text{Both}$
- When there are 2 characteristics, each with “either/or” options, use the Group Grid. Add down columns and across rows to get totals.

Functions

- Function problems test how well you follow directions. Plug the numbers provided into the function to solve the problem.

Sets and Sequences

- A **set** is a general term for a group of things. Each thing in a set is called an **element**.
- A **sequence** is a list of numbers that are given by an equation or related to each other by some pattern.

Drill

- Of 110 customers at a restaurant, 63 order hamburgers and 71 order French fries. If 22 customers order neither hamburgers nor French fries, how many customers order both hamburgers and French fries?
 - ☐ 8
 - ☐ 30
 - ☐ 31
 - ☐ 46
 - ☐ 88
- On Thursday night, 120 college students are drinking beer at a local bar. Each student drinks either light beer or dark beer. One-third of the students are female, and one-fifth of the students drink dark beer. If 10 of the students are male and drink dark beer, how many female students drink light beer?
 - ☐ 14
 - ☐ 26
 - ☐ 40
 - ☐ 70
 - ☐ 96
- Of the 200 customers who make purchases in the men's department of a clothing store on a certain day, 120 purchase dress shirts, 100 purchase neckties, and 40 purchase both dress shirts and neckties. How many of the 200 customers purchase neither dress shirts nor neckties?
 - ☐ 60
 - ☐ 50
 - ☐ 40
 - ☐ 30
 - ☐ 20
- 160 performers show up for an open audition. Of these 160, 95 are singers and the rest are actors. Half the performers belong to Equity and the rest do not. If 23 of the singers belong to Equity, how many of the actors do not belong to Equity?
 - ☐ 8
 - ☐ 23
 - ☐ 57
 - ☐ 80
 - ☐ 137
- How many workers does Factory X employ?
 - 120 workers at Factory X install batteries, and 200 workers knit socks.
 - Every employee of Factory X either installs batteries or knits socks, or both.
- How many first graders have loose teeth?
 - There are 60 children in the first and second grades combined, and 30 of the children have loose teeth.
 - The 20 students in second grade make up $\frac{1}{3}$ of the total students in the first and second grades combined, and 9 of the second graders do not have loose teeth.
- S is a sequence $s_1, s_2, s_3, \dots, s_n$ in which every term after the first is one less than three times the previous term. If $s_5 - s_3 = 28$, which of the following is the first term in the sequence?
 - ☐ $\frac{2}{3}$
 - ☐ $\frac{8}{9}$
 - ☐ 1
 - ☐ $\frac{5}{3}$
 - ☐ 2

$S = (4, x, y, \dots)$
- In the sequence above, each term is the product of the previous term and a fixed positive integer. What is the fourth term in the sequence?
 - $x = 12$
 - $y = 36$
- The sum of all integers from 1 to 50, inclusive, is 1,275. What is the sum of all integers from 51 to 100, inclusive?
 - ☐ 1,275
 - ☐ 2,550
 - ☐ 3,775
 - ☐ 3,825
 - ☐ 5,100
- What is the sum of the list of consecutive odd numbers between 24 and 40?
 - ☐ 255
 - ☐ 256
 - ☐ 448
 - ☐ 512
 - ☐ 544

ANSWERS AND EXPLANATIONS

1. **D** Use the group equation. Total = Group 1 + Group 2 + Neither – Both. Plug In and you get $110 = 63 + 71 + 22 - \text{Both}$. Make sure you keep track of your signs, and Both = 46.

2. **B** Use the group grid.

| | Light | Dark | Total |
|--------|-------|------|-------|
| Female | 26 | 14 | 40 |
| Male | 70 | 10 | 80 |
| Total | 96 | 24 | 120 |

The 40 comes from one-third of the total students (120). The 24 comes from the one-fifth of the people who drink dark beer. Just fill in, adding across and adding down to get the totals. $120 - 24 = 96$ is the number of light beer drinkers. $120 - 40 = 80$ is the number of males. If 10 of those males drink light beer, then $80 - 10 = 70$ males drink light beer. Finally, the number of female and male light beer drinkers add up to 96, so $96 - 70 = 26$ females who drink light beer.

3. **E** $120 (\text{shirts}) + 100 (\text{neckties}) - 40 + \text{Neither} = 200$.

4. **A**

| | Singers | Actors | Total |
|------------|---------|--------|-------|
| Equity | 23 | 57 | 80 |
| Not Equity | 72 | 8 | 80 |
| Total | 95 | 65 | 160 |

5. **E** There isn't enough information available to finish the formula. Fact (2) tells you that there's no *Neither*, but you still don't know how many workers do *Both* tasks: $120 (\text{batteries}) + 200 (\text{socks}) - \text{Both} = \text{Total}$.

6. **C** From Fact (1), you need to know that 30 have loose teeth and 30 don't. From Fact (2), you need to know that 9 second graders don't have loose teeth. Given the information in both facts, you can fill in the chart like this:

| | 1st Grade | 2nd Grade | Total |
|-----------|-----------|-----------|-------|
| Loose | 19 | 11 | 30 |
| Not Loose | 21 | 9 | 30 |
| Total | 40 | 20 | 60 |

7. **B** This looks pretty nasty, and it is tedious, but doable if you Plug In the Answers. Each term is one less than three times the previous term. So, basically, the formula we are plugging into is $3x - 1$, where x is the previous term. Start with C. If $s_1 = 1$, then the next term is $3s_1 - 1 = 2$, which is s_2 . Use this new term to find the next term. $s_3 = 3s_2 - 1 = 3(2) - 1 = 5$. Then $s_4 = 3s_3 - 1 = 3(5) - 1 = 14$. And $s_5 = 3s_4 - 1 = 3(14) - 1 = 41$. Finally, we are trying to match $s_5 - s_3$, which in this case would be $41 - 5 = 36$. We're looking for 28, so (C) is out. Then, notice a pattern in the numbers: they are getting bigger and bigger, so we can also knock out (D) and (E). Trying again with (B) will give you the correct solution: $s_1 = \frac{8}{9}$, then the next term is $3(\frac{8}{9}) - 1 = \frac{5}{3}$, which is s_2 . $s_3 = 3s_2 - 1 = 3(\frac{5}{3}) - 1 = 4$. Then $s_4 = 3s_3 - 1 = 3(4) - 1 = 11$. And $s_5 = 3s_4 - 1 = 3(11) - 1 = 32$. Finally, $s_5 - s_3 = 32 - 4 = 28$. Woohoo!

8. **D** The key bit of missing info is the *fixed positive integer*. Fact (1): If $x = 12$, then the integer must be 3 (and the fourth term is 108). Narrow down to AD. Fact (2) tells you the same thing, although in a more roundabout way. If $y = 36$, then $4 \times (\text{fixed integer}) \times (\text{the same fixed integer}) = 36$. Again, the integer must be 3.

9. **C** Just apply the summation formula. There are 50 numbers between 51 and 100, inclusive, so take $(25)(151)$, and your answer is 3,775.

10. **B** Just apply the summation formula. There are 8 odd numbers between 24 and 40. Remember, the smallest is 25 and the largest is 39. So the formula is $(4)(64)$, which is 256.

ANALYSIS OF AN ISSUE ESSAY

One mistake that test takers commonly make is to write the same type of essay for both questions. The two kinds of essay topics are different and require different approaches.

Previously, we discussed the finer points of writing an Analysis of an Argument. Now, we shall look at the Analysis of an Issue.

BASIC APPROACH

Use the same approach for the issue essay as for the argument essay.

Step 1: Brainstorm

Step 2: Outline

Step 3: Write

Step 4: Finish

ISSUE TASK

The Issue question asks you for your opinion on some topic. Your job is to take a side and support that position. Do not take a “middle-of-the-road,” “both are partly right” approach. The graders aren’t so much concerned with “right or wrong” here, but they are concerned with how well you take a side and defend that side.

Your job here is straightforward. The essay will present a topic for discussion, and you’ll be asked to express your opinion about it. Your goal is to present an essay that:

- Expresses your viewpoint clearly and resolutely
- Includes examples or other reasoning that supports your views
- Compels the reader to feel the same way you do

Brainstorming

Let’s use the following example of an issue to be analyzed:

The main goal of a business must be to maximize profit for its owners or shareholders.

First, do you agree or disagree with this statement?

Second, list some reasons (and examples that support those reasons) why the statement is or is not true.

We’ll talk more about how to construct the essay in class.

SAMPLE

Here's an example of a high-scoring essay written about the sample topic from earlier in this section. Note the structure of the essay and the way that key words emphasize this organization.

Although some would argue that a company has goals other than profit, the primary objective for a business must be to maximize value for its owners. Some of the reasons for the primacy of the profit goal are problems associated with divided ownership and the subordinate nature of other objectives.

One reason that profit should be the primary objective is the division of ownership. The shareholders of a large corporation may number in the tens or hundreds of thousands. The more ownership is divided, the more likely that some of the owners' goals may conflict. For example, one shareholder may support environmental conservation. Another shareholder may prefer that land be developed to support a growing economy. If the management of a corporation favors one side over the other, it is harming one shareholder for the benefit of another. The only goal that all owners are certain to share is the desire for a profit. If the corporation maximizes profit, then the individual owners are free to use that money to support whichever other objectives they desire.

Second, other objectives are often merely a means to increase profit. The company desires to reach those goals because their fulfillment will lead to greater profit. For example, a company may state that it wishes to become more socially responsible. A closer look at its motivations, however, may reveal that the company desires this social responsibility because it believes that prospective customers will respond favorably by rewarding the company with increased demand for its products. Thus, the company's stated goal is subordinate to the implied goal of greater profit. Similarly, a company may work to treat its employees well. The underlying reason for this goal is the belief that those employees will respond with increased loyalty and productivity, leading ultimately to higher profits.

In conclusion, profit maximization should be, and usually is, the top priority for any company. Any other goals will lead to a conflict of interest among the owners or are merely a component of a broader plan to increase income.

ADMISSIONS INSIGHT

Financial Aid

Many students assume they won't receive financial aid and therefore don't apply. But more than 70 percent of students today receive some form of financial aid. A little effort now can go a long way toward a financially sound future.

Apply for aid when you apply for admission to school. Consult each school's literature since filing procedures and deadlines can vary tremendously. Financial aid forms will be used to determine whether or not you demonstrate "need"—and if so, they also determine the amount of aid that you are eligible to receive for the first year of school. The institution's financial aid officers (FAOs) will try to put together a package of grants, scholarships, and/or student loans to meet that need. They'll send this information in a document known as an award letter that comes with the acceptance letter for that school. Unfortunately, not all schools are able to meet your need in full.

When it comes to financial aid, PrincetonReview.com houses helpful tips on saving money, tax forms, aid, loans, scholarships, and personal finance, in addition to a host of financial tools. Get started with the **Tuition Cost Calculator**, which estimates business school tuition for a particular school, including the yearly tuition increases, for the duration of enrollment. The **Scholarship Search** tool provides access to millions of dollars of scholarship money and a way to match your qualifications to timely scholarships. Once you're accepted into business school, our **Aid Comparison Calculator** compares the different aid award packages schools have offered you to determine which is the best deal.

If the award package isn't enough to cover business school expenses, there may be some room for negotiation, particularly if your family's financial situation has changed since you submitted the paperwork. Be sure to discuss alternative funding sources with your FAO. Also look into lending options, such as unsubsidized student loans or home equity loans, which are not dependent on "need" as defined by the aid formulas.

Typically, aid is disbursed at the start of each semester. The financial aid office will have exact dates. You will need to complete the U.S. Department of Education's Free Application for Federal Student Aid (FAFSA) and any other required forms for each and every academic year. Many scholarships and grants may be granted only to cover freshman year, so be aware of this and plan accordingly.

Lesson on Loans

Few families or individuals can afford to pay for business school without loans. The key is choosing the loan that works best for your particular situation, both now and down the road.

There are many different kinds of education loans, but they fall into two main categories: need-based loans, which are designed to help meet part of a family's remaining need (as determined by the federal government or your school); and non-need-based loans, which are designed to help pay part of the expected family contribution when the family doesn't have the cash on hand. The loans that will be offered as part of the school's financial aid package are primarily need-based loans.

The best need-based loans (the federal Perkins and subsidized Stafford loans) are such good deals that we feel families should always accept them if they are

offered. No interest is charged while the student is in school, and repayment does not begin on Perkins or subsidized Stafford loans until the student graduates, leaves school, or drops below half-time status. Even if you have money in the bank, we would still suggest taking the loans. Let the money earn interest in the bank. When the loans are due, pay them off immediately, in full if you like, without penalty. Most loans, with the exception of the Perkins loans, have some kind of an origination fee (a one-time-only cost that averages about 3 percent of the value of the loan) and perhaps an insurance fee as well. These fees are deducted from the proceeds of the loan itself; you will never have to pay them out of your pocket.

Non-need-based loans, or alternative loans, are those developed by lenders to help families cope with the expense of an education. These alternative loans often mimic some of the best features of the federal programs; however, there is usually no cap on the interest rate charged.

Another approach is to use a home equity loan. Because they have the advantages of low interest rates and usually mean an interest deduction on your taxes, many families find that these loans are an excellent financial strategy for meeting education costs. Moreover, the funds need not be spent only on direct school costs, but also can be used for big-ticket items such as computers or travel.

Research and compare as many loans as possible before committing. Register on PrincetonReview.com to allow our partner lenders to contact you with their best offers. It's the easiest way to learn about different loan options available for business school.

ASSIGNMENT 7

HARDER VERBAL QUESTIONS

In the last lesson, you learned how to approach challenging sentence correction questions. This lesson discusses how to work difficult critical reasoning and reading comprehension questions.

HARDER ARGUMENTS QUESTIONS

Even the toughest arguments questions can be successfully attacked if you take time to carefully read the argument and analyze its logical structure. Even if you have trouble identifying assumptions, having a precise sense of the conclusion will help you eliminate the wrong answers.

Hard to Follow

Some arguments are difficult to read because they concern esoteric topics, contain tough terminology, or have a disjointed logical structure. With the strange topics and terminology, you need to slow down and piece together what the author is saying. Sometimes drawing diagrams or writing notes can help. Keep track of the key elements in the premises and conclusion so that you can find the logical gaps that the assumptions will bridge.

When the logic of the argument is disjointed and hard to follow, focus on the conclusion: what the author is trying to prove, even if he or she did a lousy job of explaining it. That will help you break down the pieces of the argument, or at least provide better ammunition for eliminating answers.

Work slowly and carefully through the following example.

Read slowly and carefully to find the conclusion, premises, and assumptions of difficult arguments.

1. Companies that dominate an industry usually do so by developing a competitive advantage, often control of a unique resource or a superior technology, that allows them to manufacture products at a lower cost than their competitors can. Nondominant companies that seek to increase their share of the market generally must endure drastically lower profit margins as they win customers away from the dominant companies by matching their prices. Companies that increase their market share in this way and do not change their disadvantage in production costs relative to those of the dominant companies will, therefore, eventually lose their recently won market share as prices return to normal levels.

Which of the following is an assumption upon which the conclusion of the argument depends?

- ☐ Few companies lacking competitive advantages in costs of production that have increased their market share will sustain price margins lower than those of firms with production cost advantages.
- ☐ Dominant companies generally cannot maintain their competitive advantage over long periods of time unless they acquire additional unique resources or develop improved technology.
- ☐ Nondominant companies can improve their competitive positions by developing unique resources or technological innovations similar to those of dominant companies.
- ☐ A dominant company with a competitive advantage generally will not lower its prices to undercut those of a firm that lacks competitive advantages in production costs.
- ☐ Acquiring unique resources or developing superior technology is a difficult undertaking that requires substantial investment on the part of a company seeking to gain a competitive advantage in production costs.

The conclusion is that companies without a competitive advantage will not be able to retain an increase in market share. It helps if you simplify the argument to a degree. Some companies (dominant) have lower costs (competitive advantages) than others (non-dominant). If the higher-cost companies lower their prices to attract customers away from the lower-cost companies, they won't earn as much (lower profit margins). Therefore, they will lose the market share as they raise their prices back to original levels. The missing piece is "Why do they raise their prices back to original levels?" The author assumes that they aren't willing to continue with lower profit margins.

Answer (A) says that nondominant companies won't sustain lower margins. That matches the assumption you identified, so keep it. (B) talks about whether companies maintain their advantages over time. That's irrelevant to whether the high-cost companies can maintain a higher market share, so eliminate it. (C) sounds nice, but the scope of the conclusion is limited to companies that don't have competitive advantages. If they can develop one, the argument doesn't apply to them anymore; eliminate (C). Answer (D) might make it tougher for a high-cost company to maintain market share. That strengthens the conclusion, but is not necessary to it. Even if the low-cost companies maintain their prices, the high-cost company might lose the market share. Eliminate (D). Answer (E) is like (C); companies that can develop competitive advantages are irrelevant to the conclusion. Eliminate (E). The best answer is (A).

Several Good Answers

In many cases, an argument's question seems difficult because there is more than one answer that seems to work. The test writers have created several distractor answers in the hopes that you will choose one of those instead of the correct answer. In some instances, these trap answers will even look better than the correct answer unless you look very closely.

The first step in dealing with this type of difficult question is to carefully consider all five answer choices. Too many times, a test taker will latch on to the first answer that looks reasonable and not give enough consideration to the ones that follow it. Be sure to check all five answers equally closely.

Once you realize that there are several answers that seem attractive, you need to switch your frame of reference. Instead of looking for things that would make an answer good, you need to scrutinize the remaining answers and look for reasons to eliminate one or more of them. For weaken, strengthen, and assumption questions, carefully review the argument's conclusion so that you can make accurate judgments about which answers are relevant and which are not. Some distractors will require additional assumptions to do what the questions ask (e.g., weaken or strengthen).

In the following problem, identify the distractor answers and the reasons that you can eliminate them.

Look for subtle problems when choosing among several attractive answers.

2. A computer wholesaler increased its cash flow last year by accelerating the collection of its accounts receivables, money owed to the company by retailers that had purchased products but not yet paid for them, which had accumulated because the company had neglected to follow up with the retailers and seek payment. Last year, the wholesaler hired additional staff for the purpose of improving the collection of these receivables and, even with the increased expenses from hiring the new employees, the company's net cash flow increased by 20 percent from the prior year because it successfully collected much of its accumulated receivables. The company proposes to increase its cash flows by a similar amount in the current year by continuing its more vigorous collection efforts.

Which of the following, if true, most clearly identifies a weakness in the computer wholesaler's proposal to improve net cash flow in the current year?

- ☐ The wholesaler projects sales growth for the current year that is only a marginal increase over sales growth for last year.
- ☐ Last year, the wholesaler retained the services of an external collection agency, in addition to its internal staff, to collect payments for the accounts most egregiously in arrears.
- ☐ The retailers that showed the highest propensity in the past to pay late are the ones most likely to do so in the future.
- ☐ Net cash flow and net profits are different accounting measures, and an increase in one may not correspond to an increase in the other.
- ☐ The earlier collection efforts greatly reduced the amount owed by the retailers at the beginning of the current year.

The conclusion of the argument is the company's plan to increase net cash flows again by repeating its collection efforts. Why do they think that will work? Because it worked so well last year. The company is assuming that conditions this year are similar to those of last year. This is a typical argument by analogy. The weakness probably has something to do with how conditions are different this year. Doing a good job of initially analyzing the argument will help you avoid many of the distractor answers.

On the first pass through the answers, (A) seems reasonable; if sales growth isn't high, it may be hard to boost cash flow. (B) also seems okay; maybe the outside collection agency accounted for the cash flow increase, not the internal staff. You can eliminate (C) because which customers pay late is irrelevant; you only care about whether the company can collect enough to boost cash flow. (D) is true, but out of scope; the conclusion is about net cash flow. Whether or not net profit also increases is irrelevant. (E) seems okay; it shows one way that the current year is different from the preceding year.

For your second pass through the answers, you have to get tough. Look for flaws within each answer choice. Only one of the three remaining answers is correct. The other two must have some problems that the test writers can use to justify why they are wrong. You just need to find those flaws.

Answer (B) can be eliminated because the use of an outside agency is something the company could do again this year if it were responsible for the boost in cash flow. You need something that is different between the two situations. (A) is a little tougher, but it can be eliminated, too. Although you would like for sales growth to be high, it is not necessary for boosting cash flow. Also, the answer suggests that this year will be similar to last year, at least in terms of sales growth. (E) shows a relevant difference between the current year and last year: For the last year, there was an accumulation of receivables that would boost cash flow when collected. This year, there is not as much of a backlog to draw on, so it will be difficult to collect receivables that aren't there. The best answer is (E).

No Good Answers

With some questions, none of the answers seem to be any good. There must be a correct answer, so one answer you thought was irrelevant must actually be connected to the argument. What typically happens is that you identified a different assumption from the one tested by the question. That doesn't mean you were wrong; it just means that there might be several assumptions.

Test each answer for relevance to the argument.

If you find that none of the answers seems reasonable on your first look at the answers, you need to reassess them more slowly, checking for things that are relevant to the argument and its conclusion. Ask yourself whether accepting or denying the statement in the answer makes a difference to the validity of the conclusion.

Apply these concepts to the following example.

3. A state government is concerned about recent increases in the number and severity of injuries caused by automobile accidents. Because most serious injuries arising from automobile accidents occur on highways within city limits, where traffic is heavy and automobiles travel at high speeds, the government proposes new laws significantly lowering the speed limit on highways within a 10-mile radius of a city and imposing harsh fines on any driver guilty of exceeding them. The state government predicts that these measures would significantly reduce the number of severe injuries due to automobile accidents.

Which of the following, if true, would cast the most doubt on the accuracy of the state government's prediction?

- ☐ The new penalties will be especially harsh to drivers with low incomes who commute into the city from less expensive suburbs.
- ☐ The fines for violating the speed limits increase for repeat offenders.
- ☐ Drivers will seek to avoid the new penalties by taking alternate routes, increasing the volume of traffic on narrow and poorly lit roads not designed for heavy use.
- ☐ Speed limits in the state are already lower than those in neighboring states.
- ☐ Injuries arising from automobile accidents could be reduced more effectively by laws mandating better safety features on cars sold within the state.

The conclusion of the argument is that lower speed limits and higher fines on highways within cities will reduce injuries due to auto accidents. The premise is that most serious injury-causing accidents occur on highways within a city. Some of the assumptions in the argument are that speed causes traffic accidents and that the fines will make people drive more slowly. To weaken the argument, you want to attack these assumptions.

At first glance, most of these answers seem irrelevant or otherwise incorrect. The income issues in (A) were never mentioned in the argument. If answer (B) were true, it would probably strengthen the argument, or at least not weaken it. The alternate routes in (C) were never mentioned in the argument. The neighboring states in (D) seem irrelevant. (E) might be tempting at first glance, but it doesn't prove that the new laws won't work, just that there might be other ways to achieve the desired result.

At this point, you need to closely reexamine each answer. The test for relevancy is not “Did they mention that?” but “Would that matter?” You should care only whether the law will or will not reduce the number of injuries. In (A), the unfairness of the laws doesn’t affect whether or not it will reduce injuries. Eliminate it. Answer (B) still supports the argument; it does not weaken it. In (C), if lots of drivers start taking these inappropriate routes, they might have more accidents (and therefore injuries). A bit of a stretch, but it’s a possible answer. In (D), we don’t know enough about the neighboring states, their accident rates, and so forth to tell whether this affects the conclusion. Eliminate it. Answer (E) again seems out of scope because it doesn’t say the law won’t work as intended. The best answer is (C).

HARDER READING COMPREHENSION

As with sentence corrections and arguments, tough reading comprehension questions don’t require a radically different approach, just a more focused and disciplined attack using the methods you already know.

Hard to Follow

Certain reading comprehension passages are harder to read than others. These include the science passages, which are filled with difficult terminology, and any with densely written text and convoluted sentences.

When reading one of these passages, it’s important to keep track of the major viewpoints or elements of the passage. Questions about theory *x* will try to trap you with answers pertaining to theory *y*. It’s also important to watch multiple names for the same thing or idea. In general, take your time on the tougher passages, but don’t get bogged down by the details.

Identify the major elements, but don’t get lost in the details.

Read the following passage and identify the major elements.

- Transgenic agriculture is the use of genetically modified crops to enhance the production of food and other farm products. Genes from another species are added to a crop to introduce new traits, such as resistance to herbicides,
- 5 that are not present in the original. One widespread application involves the addition of genes from *Bacillus thuringiensis*, a soil bacterium that produces insecticidal toxins, to corn, producing a hybrid that is resistant to insect pests.
- 10 Proponents of this agricultural biotechnology argue that it has enormous potential benefits. The corn hybrids produce delta endotoxins, which are generally thought to be harmless to humans and other nonpest species, but toxic to certain insect pests. Production of the insecticide
- 15 within the plant reduces crop losses from the pests and also reduces the need for application of conventional insecticides and other agrochemicals, which can harm humans and cause other environmental damage. Tradi-

tional insecticide treatments involve spraying of the delta
 20 endotoxins on the vulnerable crops. However, the chemicals break down rapidly when exposed to ultraviolet light, a problem avoided when the toxins are produced internally to the plant.

The growth of genetically modified crops has sparked
 25 opposition on a number of counts. The toxins produced in the plant remain in the soil when the crop is plowed under, accumulating over time with unknown effects at high levels of concentration. The insecticide has the potential to spread upwards through the food chain, starting with
 30 predators that consume contaminated insects and, in turn, become prey for other species. Transgenes can spread to nearby wild plants through crop-weed hybridization, a process that is extremely difficult to control, transmitting superior traits to unintended recipients. Fears of these
 35 "superweeds" motivate a number of the opponents of genetic modification. Finally, transgenic agriculture encourages monocultures, genetically homogeneous fields, which are ironically more vulnerable to pests and disease because they lack the genetic diversity that provides the
 40 natural defense of evolution.

The passage is about transgenic agriculture, which is synonymous with genetically modified crops and agricultural biotechnology. One application of this technology is to produce corn that makes its own insecticide, also called delta endotoxins. There are some other terms, but those are the major ones.

The passage presents two sides of the issue, those advocating the benefits of the technology and those highlighting its drawbacks. These positions correspond to the second and third paragraphs, respectively. That's the basic summary of the passage and should be enough to get you started on the questions.

Several Good Answers

Some questions will include one or more trap answers that are designed to catch your eye. Be careful that you're not lured to an answer just because some of the words in it are familiar from the passage. When you have several answers that seem reasonable, start looking for reasons to eliminate some of them.

Look for small things
 that can make an
 answer wrong.

1. According to the passage, delta endotoxins

- ☐ are added to a crop, such as corn, to introduce new traits found in another species
- ☐ are one of several types of herbicide
- ☐ are generally perceived to be harmful to humans
- ☐ decompose quickly under certain forms of radiation
- ☐ can spread to nearby plants through crop-weed hybridization

A number of the answers are attractive at first glance, so you need to look more closely at them. Look for reasons to eliminate answers, not keep them. Answer (A) is true, but about transgenes, not the toxins. Make sure you keep the major elements clearly separated. Eliminate (A). Answer (B) is trying to trap you. It would be a great answer if it said “insecticide” instead of “herbicide.” Eliminate it. Answer (C) sounds tempting when you think about what toxins are, but the passage specifically said that the delta endotoxins are generally thought to be harmless to humans. Eliminate (C). Answer (E) is similar to (A): true, but for transgenes, not endotoxins. Eliminate it. Answer (D) is a classic case of using different words to say the same thing. Play the matching game: “Decompose quickly” means “break down rapidly” and “certain forms of radiation” means “ultraviolet light.” The best answer is (D).

No Good Answers

If no answers seem to match the passage, that often means that the correct answer is disguised. You’ll need to carefully check each of the answers to see which one can be translated into something resembling what the passage says.

2. With which of the following statements would the author of the passage be most likely to agree?

Look for paraphrases of the passage.

- ☐ The dangers of transgenic agriculture outweigh any possible benefits.
- ☐ Genetic modification can create crops that are particularly vulnerable to the very dangers it was trying to reduce.
- ☐ Traditional insecticide treatment carries less risk to other species than does transgenically produced insecticide.
- ☐ Proponents of transgenic agriculture cite the creation of “superweeds” as evidence of the technology’s potential benefits.
- ☐ Plants that are contaminated with delta endotoxins should be plowed under the soil.

Although the passage cites many potential dangers of transgenic agriculture, it never gives a clear indication that they outweigh the benefits. The author is taking a more neutral, descriptive position. You can eliminate (A). You can eliminate (C) because both the traditional and transgenic insecticide treatments involve delta endotoxins; they should have similar risks to other species. You can eliminate (D) because superweeds are among the dangers, not the benefits. You can eliminate (E); although the passage mentions plowing the plants under the soil, it doesn’t say that’s the response when they are contaminated. Answer (B) paraphrases the passage. Transgenic modification is trying to reduce insect pest damage, among other things. Yet, the end of the passage points out that these crops may actually become more vulnerable to pest damage. The best answer is (B).

HARDER MATH

In this lesson, you will learn all about difficult math questions: what makes them different from easier questions, what patterns they follow, and, most importantly, how to attack them successfully.

How are “hard” math problems different from easy math problems? Although there are many factors, the following four are the most important.

Complexity

Harder problems involve more steps from start to finish, and they often require you to integrate several concepts. Each individual step isn’t too bad, but cumulatively they can be nightmarish.

Difficult Phrasing

Some questions seem to be written in another language. Translating the words and determining what information they provide can be a major obstacle.

Tricks and Traps

The infamous “trick questions” test your understanding of the exceptions to the usual math rules and patterns. Often, some seemingly minor detail makes a big difference between the right answer and a trap answer.

Difficult Topics

Some areas of math are tougher because you are less familiar with them or because they are just plain difficult. Once you’ve mastered the basics, you need to dig into these advanced topics.

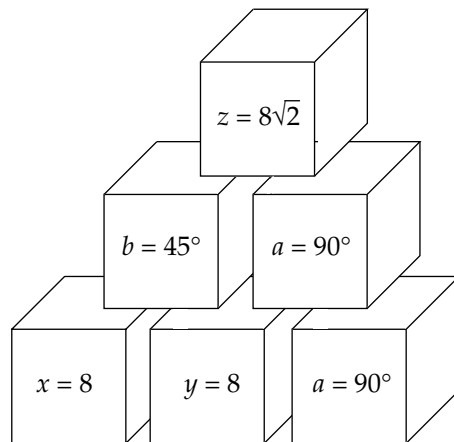
Let’s look at these factors in more detail.

COMPLEXITY

Many math questions are difficult because they are complex. They layer several concepts within a single problem. Each individual layer, or building block, may be relatively simple, but working through so many of them can be confusing.

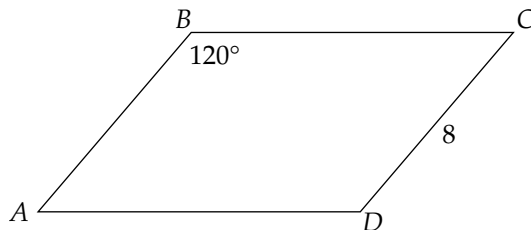
- Use building blocks. Tackle the problem in small, manageable steps.
- Know your goal. Work toward the piece(s) of information you need to answer the question.

When tackling a complex problem, you should think of it as a series of building blocks, or sub-problems. Use the information provided to find one additional piece. Then use that piece to find another, and so on until you can answer the question.



As you work on each building block, remember your ultimate goal: to answer the question. Determine what piece of information would help you solve the problem. Then direct your building block steps toward it. One danger of complex problems is the potential for confusion from all of those steps. Keep your eyes on the prize!

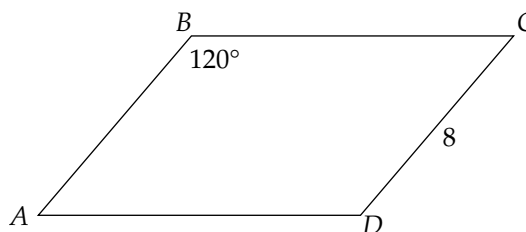
Geometry problems are often constructed with many layers, as shown in the following example.



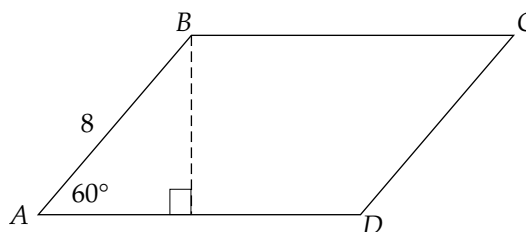
- In the diagram above, $ABCD$ is a parallelogram with an area of 96 square inches. If CD has a length of 8 inches and $\angle B$ measures 120° , what is the length of BC ?
 - ☐ 10
 - ☐ 12
 - ☐ $8\sqrt{3}$
 - ☐ $10\sqrt{3}$
 - ☐ 18

First, fill in what you know from the information provided. This is the first building block. In parallelograms, opposite sides are equal in length. Since CD is 8 inches long, AB is too. Also, BC and AD will have equal lengths, although you don't know that number yet.

In parallelograms, opposite angles are equal and a large angle and a small angle must add up to 180° . You're told that $\angle B$ is 120° , so it follows that $\angle A$ is 60° , $\angle C$ is 60° , and $\angle D$ is 120° .



Know your goal. Note that you are given the area of the parallelogram (96), so you could use the area formula (Area = base \times height) to find BC . If you use BC for the base, you can fill in the formula: $96 = BC \times \text{height}$. So, you need to find the height of the parallelogram. When you sketch in the height, your diagram should look like this:



Because the height is perpendicular to the base, you have created a 30:60:90 triangle on the left side. The ratio of sides in the triangle must be $1:\sqrt{3}:2$. AB is 8, and it corresponds to the 2 side in the ratio. So the multiplier for the ratio is 4, giving you $4:4\sqrt{3}:8$. The height is the $4\sqrt{3}$ side.

Finally, you can solve the area formula for BC . With an area of 96 and a height of $4\sqrt{3}$, you get $96 = BC \times 4\sqrt{3}$. Divide both sides by $4\sqrt{3}$ to get $BC = \frac{96}{4\sqrt{3}} = \frac{24}{\sqrt{3}}$. Although that is the value of the answer, it's not one of the answer choices. You need to multiply the top and bottom of the fraction by $\sqrt{3}$ (called rationalizing the denominator). This gives you $BC = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}$. That gives you the right answer, (C).

Solving this problem required three major building blocks: filling in the angles and sides of the parallelogram, finding the sides of a 30 : 60 : 90 right triangle, and using the formula for the area of a parallelogram.

The critical insight was that you needed the height of the parallelogram to solve the area formula for BC . The “Know your goal” guideline leads you to this idea.

Take a look at another example. This one involves several layers of number theory concepts.

2. The average of a set of five distinct prime numbers is 10. If the median of the set is 7, which of the following is the greatest possible value for a number in the set?

- ☐ 13
☐ 17
☐ 19
☐ 23
☐ 29

For the first building block, take a look at what you know. There are five numbers, and the one in the middle is 7. If the 5 numbers have an average of 10, the total must be $5 \times 10 = 50$. You could write out the problems as:

$$__ + __ + 7 + __ + __ = 50$$

Next, figure out what you need to do. “Know your goal.” To make the last number as big as possible, you need to make the other three as small as possible, subject to the requirement that they be prime numbers and distinct (i.e., no duplicates).

What else can you figure out about the missing numbers? The remaining four numbers must add up to 43. You know that most prime numbers are odd, but the sum of four odd numbers will be even. So, one of the missing numbers must be the only even prime number, 2.

$$2 + __ + 7 + __ + __ = 50$$

The second number must be either 3 or 5. Remember that you want to make all of the numbers except the last one as small as possible. So, try 3 to see whether it works. If you have 2, 3, and 7, the last two numbers must add to $50 - 7 - 3 - 2 = 38$.

$$2 + 3 + 7 + __ + __ = 50$$

Start checking other prime numbers to see what adds up to 38. If the fourth number is 11, the last number would be $38 - 11 = 27$, but that’s not prime. If the fourth number is 13, the last number would be $38 - 13 = 25$; again, not prime. If the fourth number is 17, the last number would be $38 - 17 = 21$; still not prime. If the fourth number is 19, the last number would be $38 - 19 = 19$, but then they wouldn’t be distinct. Anything higher and the fourth number would be bigger than the last number, and that won’t work. So, you can conclude that 3 doesn’t work for the second number. That means 5 must be the second number.

$$2 + 5 + 7 + __ + __ = 50$$

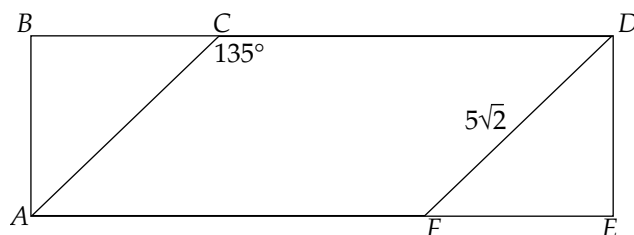
Try out more primes to see what will work for the remaining two blanks. You know that they must add up to $50 - 7 - 5 - 2 = 36$. If the fourth number is 11, the last number would be $36 - 11 = 25$, but that's not prime. If the fourth number is 13, the last number would be $36 - 13 = 23$, so that is a possibility. If the fourth number is 17, the last number would be $36 - 17 = 19$, so that is also a possibility. If the fourth number is any larger, it would be bigger than the fifth number.

So the two possibilities are 13 and 23 or 17 and 19. The greatest possible value is therefore 23. The correct answer is (D).

This example, like the first, required you to apply several concepts and work through many steps. You needed to understand averages, medians, prime numbers, and the definition of distinct. Although you could definitely determine a couple of the numbers, you had to simply try out various combinations for the others and eliminate your way to the correct answer.

Quick Quiz: Complex Problems

To solve the following problems, use the building block steps below each one.

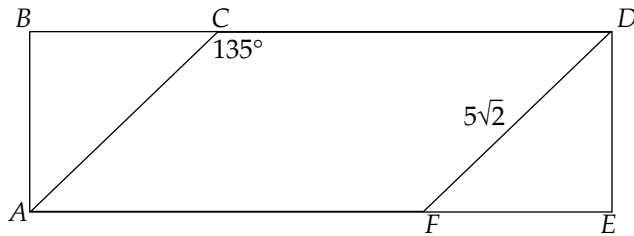


1. In the figure above, DF has a length of $5\sqrt{2}$ and $\angle ACD$ measures 135° . If the area of parallelogram $ACDF$ is 75, what is the area of rectangle $ABDE$?

- ☐ $50\sqrt{2}$
- ☐ 100
- ☐ $75\sqrt{2}$
- ☐ $100\sqrt{2}$
- ☐ 150

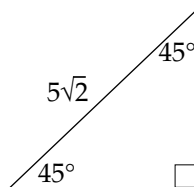
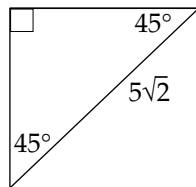
Building Block #1

Given that $ABDE$ is a rectangle and $ACDF$ is a parallelogram, find the measures of all of the angles in the diagram below.



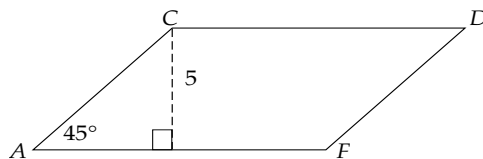
Building Block #2

Find the lengths of the sides of the triangles below.



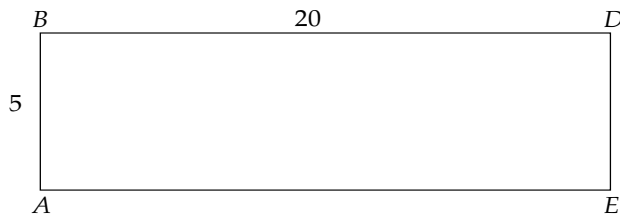
Building Block #3

Given that the area of parallelogram $ACDF$ below is 75, find the lengths of AF and CD .



Building Block #4

Find the area of rectangle $ABDE$ below.



Now, put all of the building block steps together to answer the original question.

2. In the sequence defined by $S_n = S_{n-1} + x$, $S_1 = 3$ and S_5 is equal to the sum of the distinct prime factors of 462. What is the average of the first five terms in the sequence?

- ☐ 5
☐ 13
☐ 23
☐ 26
☐ 65

Building Block #1

Find the prime factors of 462.

Building Block #2

Add 2, 3, 7, and 11.

Building Block #3

Solve the equation $3 + x + x + x + x = 23$ for the value of x .

Building Block #4

Write out the numbers 3, $3 + 5$, $3 + 5 + 5$, $3 + 5 + 5 + 5$, and 23. Then find the average of those numbers.

Now, put all of the building block steps together to answer the original question.

DIFFICULT PHRASING

Another way that the test writers can make a question difficult is to use convoluted wording. Once you understand what is going on in the problem, it's not too hard. The challenge is to decipher the information presented.

- Translate the terms. Determine which numbers you can use.
- Take small steps. Work on one piece at a time.

When the question uses math jargon, you need to translate the terms into numbers that you can work with. Keep asking yourself, "What does that mean?" and try to narrow it down to particular numbers whenever possible. Often, a convoluted description can be replaced by a single number. For example, the "least prime number" must be 2.

Even when you can't narrow things down to one particular number, you can determine some characteristic of the number(s). For example, if $x = 2$ and y is a positive integer, you can determine that xy must be a positive, even integer. On some problems, that type of information is the key to finding the answer.

As you saw with the building block approach, you often need to tackle tough problems in smaller chunks. Don't try to solve the whole thing in one massive calculation. Break it down into manageable pieces.

1. Is positive integer n a multiple of 6?

- (1) 3 is a factor of n .
- (2) $3n + 3$ is divisible by 6.

The question stem tells you that n is a positive integer (e.g., 1, 2, 3...). The question is whether it is a multiple of 6 (e.g., 6, 12, 18...). Notice that the question stem and the fact statements mention factors, multiples, and divisibility. Those terms are closely related. A number is divisible by each of its factors and a number is a multiple of its factors. For example, 5 is a factor of 10, so 10 is divisible by 5 and it is a multiple of 5.

Statement (1) says that 3 is a factor of n . In other words, n is divisible by 3. So n could be something such as 6 or 12, which are divisible by 6, or something such as 3 or 9, which are not. You can't definitively answer the question, so eliminate (A) and (D).

Statement (2) says that $3n + 3$ is a multiple of 6. From this, you can tell that $3n$ is not a multiple of 6. Take any multiple of 6 (e.g., 6, 12, 18, 24...) and subtract 3. The result will not be divisible by 6. To get another multiple of 6, you would have to subtract 6 or a multiple of 6. So, 6 is not a factor of $3n$. If 6 is not a factor of $3n$, then it isn't a factor of n either. When you divide $3n$ by 3 to get n , you are removing factors, not adding them.

Since you can tell that n is not a multiple of 6, you have answered the question with Statement (2). The correct answer is (B).

Quick Quiz: Translating Difficult Phrasing

For each of the following questions, translate the terms and determine as much information as you can about the numbers involved.

- Even integer n is divisible by 3. What can you say about n ?
- Even integer q is the product of two distinct prime numbers. What can you say about the factors of q ?
- If x is the least possible product of two negative integers, what can you say about x ?
- If m and n are each a number between -1 and 0 , exclusive, what can you say about mn ?
- If $0 < a < 1 < b$, what can you say about a^b ?

TRICKS AND TRAPS

Some math problems are tough because there is a trick involved. Either the problem requires a quirky method for solving it, or it contains traps for unwary test takers. In both cases, you must adapt your thought process to the tricks inherent in the problem.

- Impossible problems. Look for a different angle of attack.
- Traps. Beware of common math mistakes.

Some problems that seem impossibly difficult turn out to be much simpler if you understand the trick involved. These problems are extremely frustrating, because you think you know a general method for solving it, but that method turns out to be very complicated or time-consuming when you apply it to the problem.

In other problems, the trick is some esoteric concept or an exception to a rule that creates a trap. If you don't notice the trap, you will happily pick the wrong answer the test writer provided for you. The test writers prey on the common mistakes that people make when solving math problems.

A simple example of a tricky question is "If $x^2 = 100$, what is the value of x ?" Many people will choose $x = 10$ without thinking twice about it. However, x could be 10 or -10 . Those people know about positive and negative roots for squares, but they didn't think to apply that knowledge when they saw the problem. You must be on the alert for tricky concepts and trap answers at all times on the math section.

1. Jack and Christina are standing 210 feet apart on a level surface. Their dog, Lindy, is standing next to Christina. At the same time, they all begin moving toward each other. Jack walks in a straight line toward Christina at a constant speed of 3 feet per second and Christina walks in a straight line toward Jack at a constant speed of 4 feet per second. Lindy runs at a constant speed of 10 feet per second from Christina to Jack, back to Christina, back to Jack, and so forth. What is the total distance, in feet, that Lindy has traveled when the three meet at one place?

- ☐ 105
- ☐ 210
- ☐ 300
- ☐ 420
- ☐ It cannot be determined from the information provided.

This question falls into the "impossible problem" category. An obvious approach is to measure how far Lindy runs from Christina to Jack, then add the distance back to Christina, plus the distance back to Jack, and so forth. However, this is difficult because you're measuring distance between moving objects. Also, breaking the distance down into individual segments makes it very time-consuming to calculate.

A simpler, but less intuitive, approach is to use the rate formula for Lindy's entire travel. You know how fast she runs (10 ft./sec). If you knew the total time she was running, you could easily calculate the distance. So, the trick is to find out how long she runs.

Lindy runs the whole time until everybody arrives at the same point. You can figure out how long that will take by looking only at the other two. Jack and Christina are walking towards each other at 3 ft./sec. and 4 ft./sec., respectively. So their combined rate is $3 + 4 = 7$ ft./sec. In other words, they get 7 feet closer together every second.

Next, use the rate formula: $\text{rate} \times \text{time} = \text{distance}$. Plug in the numbers you have to get $7 \times \text{time} = 210$. So, the time is $210 \div 7 = 30$ seconds. That means that everybody meets after 30 seconds.

Now, use the rate formula for Lindy's travel: $10 \times 30 = \text{distance}$. The distance is $10 \times 30 = 300$ feet. The correct answer is (C).

2. If $x^{a+3} = y^{b+2}$, where x and y are distinct prime numbers, what is the value of ab ?

- ☐ -6
☐ 0
☐ 5
☐ 6
☐ It cannot be determined from the information provided.

It's very tempting to pick (E) for this problem. The problem never tells you the value of any of the numbers, so how could it be possible to solve for ab ? However, you should realize that an answer like (E) is too easy to be correct for a hard problem. The path of least resistance will never be the answer for a tough question. If it were correct, most people would guess that answer, and the problem wouldn't qualify as hard anymore.

Once you decide that (E) is a trap and that there must be a way to figure out the value of ab , you can try to identify it. The key is that x and y are distinct prime numbers, such as 3 and 5. Taking 3 to some power is the same as multiplying a bunch of 3's together. Is it ever going to equal a bunch of 5's multiplied together? The two numbers will be equal in only one place: if the exponent of both is 0. You know that any number to the zero power is 1. So, making $x^0 = y^0 = 1$ is the only way to make the equation true. That means $a + 3 = 0$ and $b + 2 = 0$. Solving those equations gives you $a = -3$ and $b = -2$. That makes $ab = (-3)(-2) = 6$. The correct answer is (D).

The hardest part of dealing with tricks and traps is realizing when there is likely to be one present. Once you know that, you can look carefully and try to determine what it is. However, if you don't realize that there is a trick, you can't do anything about it.

Quick Quiz: Tricks and Traps

Answer the following questions. If you don't know an answer, mark it so that you can review it later.

1. If $0 < x < 1$, which of the following is true?

☐ $x > x^2$
☐ $x < x^2$
☐ $x = x^2$
☐ I don't know.

2. If $x < 0$, which of the following is true?

☐ x^2 is positive.
☐ x^2 is negative.
☐ x^2 could be either positive or negative.
☐ I don't know.

3. If $x < 0$, which of the following is true?

☐ x^3 is positive.
☐ x^3 is negative.
☐ x^3 could be either positive or negative.
☐ I don't know.

4. If $-1 < x < 0$, which of the following is true?

☐ $x^2 < -1 < x < 0$
☐ $-1 < x < x^2 < 0$
☐ $-1 < x < 0 < x^2$
☐ I don't know.

5. If $0 < x < 1$, which of the following is true?

☐ $x < \sqrt{x} < 1$
☐ $x < 1 < \sqrt{x}$
☐ $\sqrt{x} < x < 1$
☐ I don't know.

6. Which of the following is equivalent to x^{-2} ?

☐ $-x^2$
☐ $\frac{1}{x^2}$
☐ \sqrt{x}
☐ I don't know.

7. Which of the following is equivalent to $x^{\frac{3}{2}}$?

☐ $\sqrt{x^3}$
☐ $\frac{3}{2}x$
☐ $\frac{x^3}{x^2}$
☐ I don't know.

8. If $x = \sqrt{25}$, which of the following is true?

☐ $x = 5$
☐ $x = -5$
☐ $x = \pm 5$
☐ I don't know.

9. If $x^2 = 25$, which of the following is true?

☐ $x = 5$
☐ $x = -5$
☐ $x = \pm 5$
☐ I don't know.

10. Line 1 is defined by $y = 3x + 7$, and line 2 is perpendicular to line 1. What is the slope of line 2?

☐ -3
☐ $\frac{1}{3}$
☐ $-\frac{1}{3}$

☐ I don't know.

11. Line 1 is defined by $y = 3x + 7$, and line 2 is parallel to line 1. What is the slope of line 2?

☐ -3
☐ 3
☐ $\frac{1}{3}$
☐ I don't know.

12. Which of the following expressions is equivalent

to $\frac{12!}{15!}$?

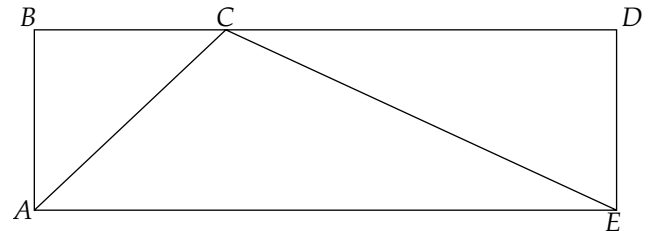
☐ $\frac{4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$
☐ $\frac{1}{15 \times 14 \times 13}$
☐ $\frac{4}{5}$
☐ I don't know.

13. In 40 seconds, a mountain biker travels 230 feet. What is her speed in miles per hour? (Note: 1 mile = 5280 feet)

☐ $\frac{230 \times 60 \times 60}{40 \times 5280}$
☐ $\frac{230 \times 5280}{40 \times 60 \times 60}$
☐ $\frac{40 \times 60 \times 60}{230 \times 5280}$
☐ I don't know.

14. If a , b , and c are distinct prime numbers, how many factors does abc have?

☐ 3
☐ 5
☐ 8
☐ I don't know.



15. In the figure above, $ABDE$ is a rectangle with an area of 18. What is the area of triangle ACE ?

☐ 9
☐ 10
☐ It cannot be determined from the information provided.
☐ I don't know.

SUMMARY

Complex Problems

- Use building blocks.
- Know your goal.

Difficult Phrasing

- Translate the terms.
- Take small steps.

Tricks and Traps

- Look for shortcuts on “impossible” problems.
- Watch for trap answers.

Other Methods

Don't forget about other strategies you already know:

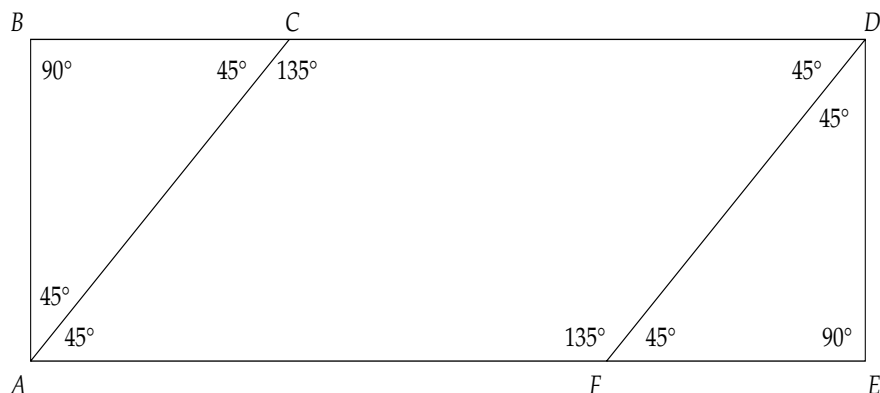
- POE
- Plug In
- Plug In The Answers
- Ballparking

ANSWERS AND EXPLANATIONS

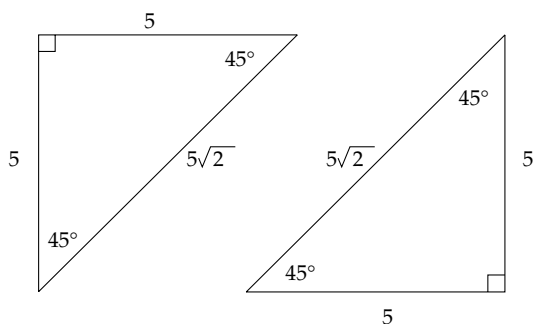
Quick Quiz: Complex Problems

1. B

Building Block #1:



Building Block #2:



Building Block #3: $AF = CD = 15$

Building Block #4: Area of rectangle $ABDE$ is 100

You are told that $\angle ACD$ is 135° , so all of the small angles in parallelogram $ACDF$ must be 45° and the large angles 135° . That means both of the triangles are $45 : 45 : 90$ triangles. AC and DF are both $5\sqrt{2}$ in length, because opposite sides of a parallelogram are equal. Using the ratio $(x : x : x\sqrt{2})$ or the Pythagorean theorem, you can find that $AB = BC = DE = EF = 5$. Draw in the height for the parallelogram; it must also be 5 because it matches AB . For parallelograms, $A = bh$. Plug in the known values for the height and the area: $75 = b(5)$, and you can solve for AF (the base) = 15. You know that $AF = 15$ and $EF = 5$, so $AE = 15 + 5 = 20$. Finally use the area formula for the rectangle $A = lw = 5 \times 20 = 100$. The correct answer is (B).

2. B

Building Block #1: 2, 3, 7, and 11

Building Block #2: $2 + 3 + 7 + 11 = 23$

Building Block #3: $x = 5$

Building Block #4: The average of 3, 8, 13, 18, and 23 is 13.

Start by finding the prime factors of 462. Using a factor tree can be helpful. The prime factors of 462 are 2, 3, 7, and 11. So $S_5 = 2 + 3 + 7 + 11 = 23$. Using the formula for the sequence ($S_n = S_{n-1} + x$), you know that each term is x more than the previous term. If you start at $S_1 = 3$ and take 4 “steps” to $S_5 = 23$, you can write that as $3 + x + x + x + x = 23$. Solving, you get $x = 5$. Plug that into the formula and find the first 5 terms of the sequence: $S_1 = 3$, $S_2 = 8$, $S_3 = 13$, $S_4 = 18$, and $S_5 = 23$. To find the average, add the numbers and divide by 5: $\frac{5+8+13+18+23}{5} = \frac{65}{5} = 13$. The correct answer is (B).

Quick Quiz: Translating Difficult Phrasing

1. **n is a multiple of 6.** If n is even, that means it is a multiple of 2. You also know that it is a multiple of 3. So, it must be a multiple of 6, if it is divisible by 2 and 3.
2. **q has 4 factors: q , 1, 2, and the other prime number.** Every integer is divisible by 1. q is even, so it must be divisible by 2, which tells you that 2 is one of the prime numbers. Every integer is divisible by itself. So, q has a total of 4 factors: 1, 2, the other prime number, and q .
3. **x is 1.** The product of two negative numbers must be positive. To make x as small as possible, you want it to be close to 0 (since it can't be negative). The smallest x can be is $(-1)(-1) = 1$.
4. **$0 < mn < 1$** Because m and n are both negative, the product mn must be positive. You also know that m and n are both fractions between -1 and 0 . When you multiply fractions, they get closer to 0. So mn must be closer to 0 than either m or n . You don't know for certain what it is, but it must be less than 1. Put both of those facts together, and you get $0 < mn < 1$.
5. **$0 < a^b < 1$** You know that a is a fraction between 0 and 1. When you multiply it by itself (b is a power greater than 1), it just gets closer to 0. That means $0 < a^b < 1$.

Quick Quiz: Tricks and Traps

1. **A** When you multiply a fraction between 0 and 1 by another such fraction, the result is smaller than either of the original numbers. Try Plugging In an example: $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, which is less than $\frac{1}{2}$. The correct answer is (A).
2. **A** Squaring x is multiplying a negative by a negative. That gives you a positive result. The answer is (A).
3. **B** Cubing x is multiplying a negative by a negative by a negative. Two of the negatives cancel and give you a positive, but the third makes it negative again. The answer is (B).
4. **C** If x is negative, then x^2 is positive. Answer (B) is a trap. It correctly reflects the fact that squaring a fraction gets closer to 0, but it leaves out the squaring-a-negative part. The answer is (C).
5. **A** Just squaring a fraction between 0 and 1 makes it smaller, taking the square root makes it bigger. However, it still remains less than 1. Try an example: $\sqrt{\frac{1}{4}} = \frac{1}{2}$, which is greater than $\frac{1}{4}$, but still less than 1. The correct answer is (A).
6. **B** A negative exponent is the reciprocal of the positive exponent. For example, $\frac{x^3}{x^5} = \frac{1}{x^2}$ if you cancel terms on the top and bottom of the fraction. However, $\frac{x^3}{x^5} = x^{3-5} = x^{-2}$ when you use the “dividing the base means subtract the exponents” rule. The two expressions are equal, so $x^{-2} = \frac{1}{x^2}$. The correct answer is (B).
7. **A** With a fractional exponent, the numerator represents a power and the denominator represents a root. So, $x^{\frac{3}{2}}$ means a power of 3 and a 2nd (or square) root. The correct answer is (A).
8. **A** The radical sign implies only the positive root. So, $\sqrt{25} = 5$ but not -5 . The correct answer is (A).
9. **C** When you take the square root yourself (i.e., there is no radical sign in the problem), you must allow for both positive and negative roots. If $x^2 = 25$, then $x = \pm \sqrt{25} = \pm 5$. The correct answer is (C).
10. **C** The slope of line 1 is 3 (taken from the $y = mx + b$ line equation). The slopes of perpendicular lines are negative reciprocals. The reciprocal of 3 is $\frac{1}{3}$, so the negative reciprocal is $-\frac{1}{3}$. The correct answer is (C).
11. **B** The slope of line 1 is 3 (taken from the $y = mx + b$ line equation). Parallel lines have the same slope. That means the slope of line 2 is also 3. The correct answer is (B).
12. **B** If you expand out the factorials, you get $\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times \dots \times 2 \times 1}{15 \times 14 \times 13 \times 12 \times 11 \times \dots \times 2 \times 1}$. All of the terms on the top will cancel, leaving just $\frac{1}{15 \times 14 \times 13}$. The correct answer is (B).

13. **A** When converting units, it can be helpful to write them out as fractions: $\frac{230 \text{ feet}}{40 \text{ seconds}}$. Then multiply by other fractions that convert the units. The units on the top should cancel with the units on the bottom, leaving you with the desired form $\frac{230}{40 \text{ seconds}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}}$. Notice that feet, seconds, and minutes all cancel, leaving you with miles/hours. The correct answer is (A).
14. **C** This one is tricky. The numbers a , b , and c are all factors of abc , but so are 1, ab , ac , bc , and abc . It may help to Plug In some numbers. Let $a = 2$, $b = 3$, and $c = 5$. That means $abc = 2 \times 3 \times 5 = 30$. The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30. That's 8 factors. The correct answer is (C).
15. **A** The formula for the area of a rectangle can be written as $A = bh$ and the formula for a triangle is $A = \frac{1}{2}bh$. If a triangle and a rectangle have the same base and height (as these do), the area of the triangle should be one-half the area of the rectangle: $\frac{1}{2} \times 18 = 9$. Answer (C) is the trap; don't take the path of least resistance. The correct answer is (A).

PARTING WORDS

Here's some information that will help you complete your GMAT preparation.

MENTAL PREPARATION

Even though you've spent the last several weeks preparing for the exam, you're probably feeling slightly anxious about the GMAT. In addition to developing an approach to each section of the exam, you need to be mentally prepared for the challenges presented by the GMAT. Most test takers feel some anxiety, and the most prepared are those who have learned to manage that anxiety. Having a plan to manage stress is essential to achieving your goal score.

Be Positive

If you encounter a situation expecting to be successful, you are much more likely to be successful than if you expect to fail. Consider the following two statements:

- 1) I'm never going to get this. If I bomb the GMAT, I will be a failure.
- 2) I am well prepared and deserve to do my best. I know what to expect and I am ready to succeed.

It shouldn't take too much to figure out which student is going to do better. Whether it's looking in a mirror and saying affirming statements or writing a positive thought on your scratch paper on test day, it's very important to go into the actual exam expecting to be successful. If you expect to fail, why would you be at the test in the first place? This can be a difficult exercise at first, but you must get yourself in a frame of mind to succeed. When you dwell on negative thoughts, your mind isn't free to work on the GMAT. Trust that you are well prepared. If you've attended all the classes and done the work, you are better prepared than most of the population. Have confidence that you are going to be great!

Also, don't place excessive importance on the exam; keep the GMAT in perspective. Remember that the GMAT does not measure your intelligence or your ability to succeed in business school. A GMAT score only reflects how well you performed on the test on a given day. This is not the day that determines the rest of your life. If the test does not go as well as planned, you can always take it again next month. In fact, repeat exams make up about twenty percent of all GMAT's administered. Repeating the exam is unlikely to affect your chances for admission because most admissions committees look at your highest score.

Know the Test

When you take a computer adaptive test like the GMAT, you have a lot of work to complete in a limited amount of time. Also, the computer program adapts to your performance and presents you with challenging questions all the way through the test. Thus, by its very design, a CAT can be a stressful experience.

However, you've learned how a CAT works, and you know what to expect. Keep in mind:

- Seeing hard questions on this test is a *good sign*, not a bad sign. Remember that you have to “earn” the hard questions on this test by getting questions right. When you see something difficult, remind yourself that this is an achievement, not a pitfall.
- The exam adapts to your performance, so you will encounter questions that challenge you. That's part of the game. If a question looks really strange or difficult, take a breath and remain calm. Try to figure out what it's testing, and apply the appropriate technique. If you're absolutely stumped, guess and move on. Maintain the pacing and approach you've learned from your practice tests.
- One in four questions is experimental. Twenty questions on this test do not factor into your final score. If you see something you have never seen before, it probably does not count. If you see a killer question, there is a 25 percent chance it doesn't count either. Do not let it shake your confidence.

Visualize Success

As much as your Princeton Review practice tests serve as “dress rehearsals,” practice tests are not quite the same as the “real” GMAT. This is where visualization techniques come into play. If possible, visit the test center before the day of your actual exam. Get a feel for the layout of the center. At most test centers, you'll be able to see the testing room through a window in the lobby. This will help you simulate the actual GMAT in your mind's eye in your last few days of preparation.

Imagine yourself in the examination room on test day. Get a good picture in your mind. In the picture you created in your mind, do you see yourself in the test room or do you see the situation through your eyes as if you were actually in the test room? Getting comfortable seeing the situation through your own eyes helps manage potential test anxiety. You are not scared; you are excited. This is the day that you have been training for over the last several weeks. You have prepared. You can't wait to get started. After the administrative rigamarole, you sit down in front of the computer. You feel a rush of adrenaline as you begin the first essay. You take a few deep breaths and execute the strategy that you have been planning for the few weeks leading up to today. Some of the questions are hard, but you expected that.

There are two important keys to visualization: See yourself succeeding and imagine yourself overcoming every type of obstacle. You are unstoppable. You have prepared to be successful and you deserve to be successful.

Control the Physiological Responses to Anxiety

It's normal to feel a little nervous on the day of a big event. Your breathing gets shallow, and you may even feel a little sick to your stomach. Something that you can do that will ameliorate these symptoms is *deep breathing*. Close your eyes and imagine that your torso is an empty cylinder. Take a deep breath, filling

the cylinder from the bottom (as if you were filling your torso with air through your navel). Slowly release all the air from the top of the cylinder to the bottom. You will feel yourself start to relax within the first few breaths. Your breathing should be deep and regular. This exercise will generally take about half a minute. It's time well spent because folks who are highly stressed are not going to give their best performance. Once you have given your brain that little extra oxygen and gotten yourself focused back on the task at hand rather than on your stress, get back to the test and start kicking butt.

COUNT DOWN TO THE GMAT

All your hard work will be put to the test on the day of the actual GMAT. The final week can be very stressful, and events on the day of the test have the potential to rattle unprepared testers. To help guarantee that you will emerge successfully from the GMAT, here is some advice for the time leading up to your exam.

The Week Leading Up to the GMAT

Continue to practice regularly until you take your exam. Finish the last remaining online test a few days before your exam. It's best not to do it the day before, as you'll want time to review the last test and work on any last-minute problem areas. If any specific topics are still giving you trouble, do some targeted work in those areas. Review your class notes, and practice questions dealing with those topics. Continue to work problems in the *Official Guide* and online drills.

Visit the Test Center

If possible, take a field trip to your test center. If you're unfamiliar with the area, figure out how to get to the center ahead of time. It's much better to get lost on your dry run than on the day of your exam. Walk inside to check out the waiting area and peek into the testing room. Seeing the center ahead of time can have a calming effect, especially if you're picturing the testing room as a chamber of horrors.

Take Care of Yourself

In the days leading up to the test, try to get regular exercise and adequate sleep. Exercise, even a short walk, helps you manage your stress. You may have a little trouble sleeping the night before the test, so you want to be well rested in the days leading up to the test. To be at your best, your body must be conditioned to be awake and ready to work at the time that your GMAT is given. If you scheduled your exam first thing in the morning, become accustomed to waking up at the proper time for the entire week leading up to the test. It is also important not to go to bed at a ridiculously early hour the night before the exam. Fourteen hours of sleep the night before the test is not necessary, and any deviation from the sleep schedule you have established in the final week is a dangerous idea.

Continue with Your Visualization Techniques

The final week leading up to the GMAT can inspire all kinds of negative thinking. Being excited about the test is normal, even helpful. Letting the importance of the GMAT inspire feelings of dread is not. Use your stress management techniques to keep yourself in proper focus through the end of the exam: Now is not the time to start thinking that you are going to fail.

The Night Before the Test

Don't study the evening before your test! Your performance depends on your work over the last several weeks, not in the last few hours before the exam. Go out to dinner or watch a movie. Do not do any more GMAT work because you are as ready as you are going to be.

Make a test-day kit that includes everything you might need. Be sure to include:

- Directions to the test center. Ideally, you will have already been there to check out the accommodations.
- A mid-test snack and beverage
- Some warm-up problems. Bring your *Official Guide* at the very least.
- Telephone numbers of taxi services in case you have car trouble
- A photo ID
- Something to pass the time if you have to wait for the test to begin
- List of schools you want to receive your scores. Your test fee covers up to five score reports. If you want to send scores to more than five schools or if you don't choose score recipients on the day of your test, you will be charged \$25 per school to send your scores later.

Fuel for Body and Mind

On the day of your test, eat something before your exam. It's a long test, and your brain needs fuel. Also, bring a snack with you to the exam. You'll have to leave it in your locker during the test, but you can have it during one of your breaks. Stay away from excessive amounts of caffeine unless you are comfortable with the idea of either losing test time to bathroom trips or wearing a diaper.

What to Wear

Be prepared for a wide range of temperatures. Dress in layers, and keep in mind that you are dressing for comfort.

Warm Up

Just as an athlete warms up before a race, you may want to do some warm-up problems on the day of your exam. Do a few *Official Guide* questions during breakfast or while you're waiting in the test center. These help you get into test-

taking mode so that you can hit the ground running when the test begins. Work problems you've done before; if you do new problems and get them wrong, it could shake your confidence.

At the Test Center

Arrive at the test center 30 minutes before your test appointment. If you are late, you may not be permitted to take your test. When you arrive, a staff member will check you in and verify your identification. You'll be given a confidentiality statement to write out and sign. It states that you are taking the GMAT solely for the purpose of applying to a graduate program and that you agree not to disclose any material from the exam. You will be provided a locker into which you must place all of your belongings. You will not be allowed to bring anything with you into the testing room. Though the official policy states that you cannot remove anything from your locker during the breaks, we've found most test centers allow you to visit your locker during the break.

Time for Your Test

The proctor will give you six sheets of scratch paper and pencils. These will be the only items you are allowed to bring with you into the testing room. You will be taken into the testing room, which consists of computers separated by dividers. If you have any questions for the proctor, ask them before your test begins.

You must begin your test as soon as you are seated at your workstation. Remember, the first section of the test is the tutorial. You have unlimited time here; make sure to take the time you need to prepare yourself to take the test and set up your scratch paper.

Your computer will keep track of the time remaining during the two five-minute breaks. You may not take longer than five minutes. You may eat or drink only on the break.

You Made It!

At the end of the exam, you'll face the moment of truth. A window will appear on your computer screen, and the text will ask you whether you want to accept or cancel your score. You must decide whether to cancel or accept your scores before you see them. Even though you may feel uncertain, you probably did fine and should accept your score. Cancel your score only if you're certain something truly awful happened during the exam. For example, if you pushed the start button and threw up all over the keyboard, maybe it wasn't your day. If you cancel your scores, you won't find out what you scored and you won't receive a refund of the test fee. ETS will send a cancellation notice to you and to any schools you designated to receive score reports.

Upon exiting the testing room, the test administrator will give you your unofficial score report. You will receive your official score report, via mail, when the AWAs have been scored. The official score report usually arrives about two weeks after you take the exam, and schools you designated to receive score reports get them around the same time.

If Anything Goes Wrong

It's very rare to encounter technical problems during the exam. If technical problems occur that prevent you from completing your exam, you will be offered the chance to schedule another appointment at no charge.

If you want to complain about test center conditions or any testing irregularities, begin by filing a complaint before you leave the test center. If possible, get the test center staff to corroborate your complaint. You'll also need to file a complaint with ETS as soon as possible after leaving the test center. Explain the situation in a letter, and address it to "Attention: GMAT Complaints." Submit it by fax (609-883-4349), email (gmat@ets.org), or mail (GMAT—Attention GMAT Complaints, Educational Testing Service, P.O. Box 6103, Princeton, NJ 08541-6103).

If you have any other questions about test regulations, go to www.mba.com/mba/TaketheGMAT to see the official policies.

A Final Word

Congratulations on completing the course. If you have committed all the information we've covered during the course to memory, you now know everything you need to know to excel on the GMAT. Does this mean that you are destined for a perfect score on the test? Of course not. It means that the only thing holding you back now is the speed and accuracy with which you can work test questions. If you keep practicing, you should expect to see some more improvement in your score. As always, if you have any questions or concerns, your local Princeton Review office can help you.

ADMISSIONS INSIGHT

FINANCIAL AID TIMELINE

Get the most money available to you by taking the financial aid process step by step and staying on track.

Here are some things to keep in mind throughout the process:

- Register on PrincetonReview.com to allow our partners to contact you with loan offers and complete our online **Scholarship Search**.
- If you still need more money, look into alternative loans.
- Sign up for our **Financial Aid and Business School Newsletters**, which come with important financial aid tips in every issue.

September to November

- Focus on GMAT test preparation and applications—these are the most important things to worry about now.
- Ask about financial aid during campus visits and meet with financial aid officers.
- Look into merit-based grants and fellowships. Take note of financial aid deadlines at the schools to which you are applying.
- Get ready to finish your tax returns so that the FAFSA can be completed and filed as soon as possible after January 1.

December

- Submit all applications on time.
- Learn about the Federal loan process with articles on PrincetonReview.com's **Apply for Loans** area.
- Begin preparation for the FAFSA form by calculating your expected family contribution. The EFC is an important aspect of the FAFSA form, so you will want to be sure you get this done first. (Check out our online **EFC Calculator**.)

January to Mid-February

- File the FAFSA as soon as possible after January 1 as you can (your family's tax returns must be completed in order to do so). The sooner you submit the FAFSA, the more likely you are to get a larger financial aid package. Before filling out the worksheet, check back with us for our FAFSA Worksheet. We'll have it online in late December so you can start thinking about your answers for the 2003–04 FAFSA.

Mid-February to March

- Learn about the alternative loan process; you need more money in addition to your aid package.
- If you haven't completed the FAFSA, do so immediately.
- If you're not working already, look for summer jobs and/or internships in your chosen field.

April to June

- Compare aid awards carefully to determine the best aid package. Use the **Aid Comparison Calculator** on PrincetonReview.com to compute the best deal.
- Appeal the aid package at your favorite school if the amount of money does not meet your need.
- If you still need more loan money, look into alternative loans.
- Explore other financial needs for school including bank accounts, credit cards, and various forms of insurance. Read more on PrincetonReview.com's **Smart Money** section.

July to August

- If you still need more loan money, continue to explore from our selected lenders.
- Get ready to start school!

IN-CLASS LESSONS

LESSON 1

WELCOME

We want to make this course as helpful as possible for you. Do not hesitate to tell your teacher (the scintillating person at the front of the classroom) if there is anything he or she can do to make this course better for you.

Your teacher has some information (name, contact information, etc.) he or she wants to pass along. You can write it down here:

UNDERSTANDING YOUR SCORE

Your overall score, which can range from 200 to 800, is a scaled score calculated by combining your Math and Verbal scores and comparing that result to a chart. You can use the chart below as an approximate guide. These scores are comparable from test to test. In other words, a score of 580 implies the same level of performance no matter which test you take.

The two axes below represent your Math and Verbal scores; either axis can be used for either score. Make a goal score by taking into consideration your starting score, what score you need to get into your schools, and how much work time you have to put into this course. Your instructor will help you set a realistic goal.

| | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 10 | 210 | 260 | 300 | 330 | 360 | 400 | 430 | 470 | 500 |
| 15 | 260 | 300 | 330 | 360 | 400 | 430 | 470 | 500 | 550 |
| 20 | 300 | 330 | 360 | 400 | 430 | 470 | 500 | 550 | 580 |
| 25 | 330 | 360 | 400 | 430 | 470 | 500 | 550 | 580 | 610 |
| 30 | 360 | 400 | 430 | 470 | 500 | 550 | 580 | 610 | 650 |
| 35 | 400 | 430 | 470 | 500 | 550 | 580 | 610 | 650 | 690 |
| 40 | 430 | 470 | 500 | 550 | 580 | 610 | 650 | 690 | 720 |
| 45 | 470 | 500 | 550 | 580 | 610 | 650 | 690 | 720 | 760 |
| 50 | 500 | 550 | 580 | 610 | 650 | 690 | 720 | 760 | 800 |

The Graduate Management Admission Council (GMAC) is emphatic in its recommendation of how a GMAT score should be interpreted: A score isn't to be seen as an individual number or used as a strict cutoff for admission. The test writers acknowledge a standard error of measurement of 28 points, which means the score you get on any given exam may represent a "true" score of 28 points higher or lower.

Your *Official Guide* lists the percentile ranks that correspond to scaled scores. The percentiles indicate the percentage of examinees who score below a particular level. Looking at the percentiles can help you put your score in perspective. For example, a student who enters our course with a score of 530 ranks in the 45th percentile. If that student were to improve her score by 80 points to a 610, she would rank in the 74th percentile. Her score moved from the lower half of all GMAT scores to nearly the top quarter, an impressive achievement.

There are two big messages here:

- **Understand your score** and its changes as you go through the course. Your ultimate score depends on your starting score and how much you put into the course.
- **Commit yourself** to preparing for GMAT once, getting your best possible score, and moving on with the rest of your life.

INTRODUCTION TO PACING

The GMAT CAT calculates your score based on three considerations:

- Number of questions you answer correctly
- Difficulty of the questions you answer
- Number of questions you complete

Developing a pacing plan is just as important as knowing how to answer the questions. The basic guidelines are:

1. Start slowly and carefully. The early questions carry the most weight, so be as accurate as possible. Eliminate careless mistakes.
2. Gradually pick up speed so that you can finish the section.
3. Don't waste time on killer questions. Guess and move on.
4. Don't let time run out without answering all the questions. Leave at least enough time to mark a guess for every question.

Below are guidelines for how long to spend on each section of the test. Learn them well. Notice that the focus is on how long you should take in any one section, not how long you should spend on any one question.

MATH

Question numbers

| Score | 1–10 | 11–20 | 21–30 | 31–37 |
|-----------------|---------|---------|---------|---------|
| Under 35 | 30 min. | 25 min. | 15 min. | 5 min. |
| 35–42 | 30 min. | 20 min. | 15 min. | 10 min. |
| Above 42 | 25 min. | 20 min. | 20 min. | 10 min. |

VERBAL

Question numbers

| Score | 1–10 | 11–20 | 21–30 | 31–41 |
|-----------------|---------|---------|---------|---------|
| Under 28 | 30 min. | 25 min. | 10 min. | 10 min. |
| 28–34 | 27 min. | 20 min. | 18 min. | 10 min. |
| Above 34 | 25 min. | 20 min. | 15 min. | 15 min. |

Guessing

You will see some questions that you cannot solve, even if you are planning to get a perfect score in a section. Guessing is essential in order to maintain the score and level of difficulty that you earn in any section. Every test taker will benefit from learning how to guess. It is a skill that requires practice!

If you see a killer question or if you are running short on time, improve your chance of getting a question right before guessing. If you can spot any trap answer choices, eliminate them before you guess. If you increase your odds of getting a question right, you will also increase your odds of raising your score. Your instructor will be teaching you many strategies for guessing throughout this course.

Don't let killer questions
kill your confidence!

Applying to Business School

Applying to business school is a long process. Preparing for and taking the GMAT is just the first hurdle. Information on researching, applying to, and paying for school can be found on PrincetonReview.com. Visit the site with business-school questions or to browse the archive of articles. Another way to keep on top of the process is to subscribe to our free Business School Newsletter. Each month, we send business school students an email with timely tips, advice, and up-to-the-minute reminders. Information on business schools' competitive environment, post-grad job market, and steps for success are included, as well as admissions and financial aid news.

PrincetonReview.com's Online Bookstore carries a number of titles relevant to prospective business school students. Our comprehensive line of books spans from business vocabulary and internships to careers, interviewing, networking, resumes, cover letters, and job hunting.

The site's Internship Search tool provides access to thousands of internships at top companies around the world, whether you're searching before, after, or during business school.

SET YOUR GOALS

Please fill out the information below and turn the questionnaire in to your teacher. Knowing your goals and concerns will allow your teacher to work with you more effectively.

Name

Goals and Expectations

What were your scores on the first practice test?

Math_____ Verbal_____ Overall_____

What score range do you hope to reach?

What parts of the GMAT give you the most trouble?

How many hours outside of class per week do you plan to spend preparing for the GMAT?

What things could interfere with your GMAT preparation time?

How will you handle possible interferences?

VERBAL INTRODUCTION

In the Verbal section, you will have 75 minutes to answer 41 questions. You will receive a separate Verbal score that ranges from 0 to 60 with an average of 27. This Verbal score is a factor in your overall score on the 200 to 800 scale.

A SYSTEMATIC APPROACH

The Verbal questions strike fear into many hearts because the credited answers seem very subjective and arbitrary. Fortunately, this isn't the real story. The questions and answer choices, both the correct and incorrect ones, follow definite patterns.

The key to a good Verbal score is learning to take apart the questions and answers, find the patterns, and use them to choose the best answer. Everything in the Verbal section conforms to certain rules, and once you learn those rules you will be able to work the problems.

THE IMPORTANCE OF POE

The multiple-choice format of the Verbal section actually works to your benefit. Instead of writing down your own free-form responses to the questions, all you have to do is tell the difference between good and bad answers. Once you learn the ways ETS creates bad answer choices, this process becomes more and more straightforward.

The Process of Elimination (POE) is especially important in the Verbal section because you will hardly ever find an answer that says what you want to say. Don't worry about "liking" the credited response. Just concentrate on finding the things that make an answer incorrect.

The "right" answer is right only because it's better than the others.

SENTENCE CORRECTION 1

The sentence correction questions supposedly test your knowledge of grammar. Hundreds of different grammatical rules could be tested, but the GMAT concentrates on only a few. We will look at the most commonly tested mistakes.

THE FORMAT

Each sentence correction question presents a sentence with some or all of the sentence underlined. The first answer choice repeats the underlined portion exactly as it is in the original sentence. Each of the other answers provides a variation on the underlined portion.

Your task is to choose the answer that creates the best sentence according to the rules of grammar.

1. The Texas longhorn, a breed of cattle descended from herds brought by the Spanish conquistadors, is a symbol of the Wild West era of American history.
 - ☐ is a symbol of the Wild West era
 - ☐ are symbols of the Wild West
 - ☐ are symbolizing the Wild West era
 - ☐ is symbolic in the Wild West
 - ☐ are a symbol of the Wild West era

The 2/3 Split

Notice the pattern of the answer choices. Two of the answers begin with *is* and the other three begin with *are*. This pattern is a “2/3 split,” and it gives you a big clue about the grammar rule the question tests. Once you choose between *is* and *are*, you can eliminate either two or three of the answer choices. Then, you can look for another error and repeat the Process of Elimination.

BASIC APPROACH

The key to success with sentence correction is to identify one error at a time and eliminate choices that contain the error. Most questions contain more than one error, so you’ll usually have to repeat the process two or three times.

Let’s work a question one error at a time. Look at the italicized words in the answer choices, and identify the first error.

The 2/3 split between *is* and *are* is a clue for which error?

2. Each team of business students are responsible for creating a marketable business plan.
- ☐ *are* responsible for creating a marketable business plan
 - ☐ *are* responsible to create a marketable business plan
 - ☐ *is* responsible for creating a business plan that is able to be marketed
 - ☐ *is* responsible for creating a marketable business plan
 - ☐ *is* responsible to create a marketable business plan

Once you identify the error, eliminate the choices that contain it. Compare the remaining choices and identify a second error.

2. Each team of business students are responsible for creating a marketable business plan.
- ☐ is responsible *for creating* a business plan that is able to be marketed
 - ☐ is responsible *for creating* a marketable business plan
 - ☐ is responsible *to create* a marketable business plan

What’s the second error?

What's the difference between the remaining choices?

Eliminate choices with the second error. Since two answer choices remain, we need to repeat the identify-and-eliminate process one more time.

2. Each team of business students are responsible for creating a marketable business plan.

- ☐ is responsible for creating a *business plan that is able to be marketed*
- ☐ is responsible for creating a *marketable business plan*

Step 1: Identify an error.

Identify a grammar problem or rule that the sentence is testing. Look for:

- A 2/3 split in the answer choices
- The common types of errors
- Differences among the answer choices

Step 2: Use POE.

Eliminate all answer choices that contain the error.

Step 3: Identify an error in the remaining choices.

Look for another grammatical error in the remaining answer choices. Once again, compare the answer choices and look for differences.

Step 4: Use POE.

Eliminate the remaining choices that contain the secondary error.

Step 5: Keep going until only one answer choice remains.

Try another question using the basic approach.

Errors can occur at the beginning, in the middle, or at the end of the answer choices.

3. Concerned by the increasing trade deficit, the government passed legislation which prohibits the consumer to purchase computers that contain micro-chips manufactured overseas.

- ☐ which prohibits the consumer to purchase computers that contain
- ☐ prohibiting the consumer from computer purchases which contain
- ☐ that prohibit the consumer to purchase computers containing
- ☐ that prohibits the consumer from purchasing computers that contain
- ☐ prohibiting the consumer to purchasing computers which are containing

Rely on POE

The multiple-choice format of sentence correction questions works to your advantage. Avoid the temptation to rewrite the sentence in your head before looking at the answer choices. Instead, concentrate on using POE to eliminate answers that contain grammatical errors.

When presented with two answer choices that are both grammatically correct, stick with the answer choice that conserves the original meaning of the sentence. An *unnecessary* change in meaning denotes an incorrect answer and should be eliminated.

THE BIG SIX

How will you know which errors to look for in sentence correction questions? As we mentioned earlier, ETS concentrates on a handful of grammar errors. We'll show you how to spot each type of error and use the grammar rule to eliminate incorrect answer choices. Learn these rules backward and forward because most sentence correction questions involve one or more of them.

The Big Six

The most common types of errors are:

- Verb Tense
- Pronouns
- Idioms
- Misplaced modifiers
- Paralellism
- Subject-Verb Agreement

SUBJECT-VERB AGREEMENT

While the rule for subject-verb agreement is simple, spotting errors is not always so easy.

The Rule: A singular subject requires a singular verb. Plural subjects require plural verbs.

Let's try a few questions.

Use the 2/3 split to identify the error.

1. Federally-imposed restrictions on how much they pay small savers has created difficulties for savings banks as they are competing with such unregulated investment vehicles as money market certificates.

- ☐ has created difficulties for savings banks as they are competing with such
- ☐ has made it difficult for savings banks competing with such
- ☐ have created difficulties for savings banks as they are competing with
- ☐ have made it difficult for savings banks to compete with such
- ☐ have made it difficult for savings banks as they are competing with such

Isolate the subject and the verb, and check for agreement.

2. Neither my mother nor my father, both of whom are English professors, agree with my contention that James Joyce's novel *Ulysses* is overrated.

- ☐ both of whom are English professors, agree with my contention that James Joyce's novel *Ulysses* is overrated
- ☐ both of whom are English professors, agrees with my contention that James Joyce's novel *Ulysses* is overrated
- ☐ both of whom are English professors, agree with my contention that *Ulysses*, a novel by James Joyce, is overrated
- ☐ each of whom is an English professor, agree with my contention that James Joyce's novel *Ulysses* is overrated
- ☐ English professors the both of them, agrees with my contention that James Joyce's novel *Ulysses* is overrated

3. Approximately seventy-five percent of the global freshwater supply is stored in glaciers, which cover roughly ten percent of land area.
- ☐ is stored in glaciers, which cover roughly ten percent of land area
 - ☐ is stored in glaciers, which covers roughly ten percent of land area
 - ☐ is stored in glaciers, that covers roughly ten percent of land area
 - ☐ are stored in glaciers, which cover roughly ten percent of land area
 - ☐ are stored in glaciers, which covers roughly ten percent of land area
4. Attempts to maintain the current level of funding for museums, though impressive, has not resulted in the continuation of financial backing for the coming year.
- ☐ Attempts to maintain the current level of funding for museums, though impressive
 - ☐ The attempt to maintain the current level of funding for museums, though impressive,
 - ☐ Maintaining the current level of funding, though an impressive attempt,
 - ☐ The impressive attempts to maintain the current level of museum funding
 - ☐ Attempts to maintain the level of funding for museums currently, though impressive,
5. A number of military personnel who served in Operation Desert Storm has encountered a series of unexplainable symptoms that were attributed to Gulf War syndrome.
- ☐ has encountered a series of unexplainable symptoms that were attributed to
 - ☐ has unexplainably encountered a series of symptoms that have been attributed as
 - ☐ have encountered a series of unexplained symptoms that have been attributed as
 - ☐ have encountered a series of unexplainable symptoms that have been attributed to
 - ☐ have encountered, unexplainably, a series of symptoms that were attributed as

To spot subject-verb agreement errors, look for:

To fix subject-verb agreement errors:

VERB TENSE

In most instances, verb tense should be kept consistent. Verb tense errors are usually easy to correct once you decide which tense to use.

The Rule: Sentences should use only one tense (past, present, or future) unless the meaning of the sentence requires a shift.

What changes in the answer choices?

1. The Department of Education has concluded that if children learn to read and did math from an early age, they will require less remedial work in later school years.

- ☐ did math from an early age, they will require less remedial work in later school years
- ☐ did math from an early age, they require in later school years less remedial work
- ☐ do math from an early age, they require less remedial work in later school years
- ☐ do math from an early age, they will be required in later school years to do less remedial work
- ☐ did math from an early age, less remedial work will be required in later school years

What helps you determine which tense to choose?

2. Although Smith once championed the charity, he changed his mind after a journalist linked its origin to a corrupt individual.

- ☐ once championed the charity
- ☐ had once championed the charity
- ☐ has once championed the charity
- ☐ was a champion of the charity once
- ☐ championed the charity at one time

3. So far this year, twenty elected legislators had opted not to run for reelection in the aftermath of the Bathwater scandal.

- ☐ had opted not to run for reelection
- ☐ had opted to not run for reelection
- ☐ have opted to not run for reelection
- ☐ have opted not to run for reelection
- ☐ have opted to not run again for reelection

To spot verb tense errors, look for:

To fix verb tense errors:

PRONOUNS

Because pronouns are small words, you must read carefully to spot pronoun errors.

The Rule: Singular pronouns replace singular nouns, and plural pronouns replace plural nouns. Also, a pronoun must unambiguously refer to one noun.

If a sentence contains multiple errors, it doesn't matter which one you attack first.

Isolate the pronouns and the nouns they replace.

1. Each of the dogs now in the animal shelter had been neglected by their former owner before they were abandoned.
 - ☐ had been neglected by their former owner before they were abandoned
 - ☐ was neglected by its former owner before it was abandoned
 - ☐ was neglected by their former owner before they were abandoned
 - ☐ had been neglected by its former owner before it was abandoned
 - ☐ was abandoned, but before that they had been neglected by their former owner
2. Although aspirin irritates the stomach, it can be avoided if the aspirin tablet is given a coating that will not dissolve until the tablet reaches the intestine.
 - ☐ Although aspirin irritates the stomach, it
 - ☐ The irritation of the stomach caused by aspirin
 - ☐ The fact that aspirin causes irritation of the stomach
 - ☐ Aspirin causes stomach irritation, although it
 - ☐ Aspirin irritates the stomach, which

3. Although the bite of diamond-backed adders are rarely fatal, they cause chronic flesh wounds, posing the greatest danger to the infant and elderly, who are particularly vulnerable to its poison.

- ☐ diamond-backed adders are rarely fatal, they cause chronic flesh wounds, posing the greatest danger to the infant and elderly, who are particularly vulnerable to its
- ☐ diamond-backed adders are rarely fatal, they cause chronic flesh wounds, and pose the greatest danger to the infant and elderly, who are particularly vulnerable to their
- ☐ the diamond-backed adder is rarely fatal, it causes chronic flesh wounds, posing the greatest danger to the infant and elderly, who are particularly vulnerable to their
- ☐ the diamond-backed adder is rarely fatal, it causes chronic flesh wounds, and poses the greatest danger to infants and the elderly, who are particularly vulnerable to its
- ☐ the diamond-backed adder is rarely fatal, they cause chronic flesh wounds, and they have posed the greatest danger to infants and the elderly, who are particularly vulnerable to their

4. Unsure to whom the teacher was talking, the students stared mutely at the algebra problem on the board.

- ☐ Unsure to whom the teacher was talking
- ☐ Unsure of whoever the teacher was talking to
- ☐ Being unsure of which of them the teacher was talking to
- ☐ As they were unsure to whom the teacher was talking
- ☐ Unsure of who was being talked to by the teacher

Once in a while, the GMAT tests *who* vs. *whom*.

To spot pronoun errors, look for:

To fix pronoun errors:

RED PENCIL FEVER

Don't eliminate something just because you don't like the way it sounds. Often, the correct answer won't sound very good, and you don't want to eliminate it hastily. Start by eliminating those answers for which you can identify a definite flaw.

1. Depending on which scholar you consult, either Daniel Defoe's *Robinson Crusoe*, Henry Fielding's *Joseph Andrews*, or Samuel Richardson's *Pamela* is believed to have been the first English novel ever written.
 - ☐ is believed to have been the first English novel ever written.
 - ☐ is believed as being the first English novel ever written.
 - ☐ are the English novels believed to be the first written.
 - ☐ are the English novels which were believed as the first written.
 - ☐ are the first English novels ever believed to be written.

Because you're looking for errors, it's easy to find something wrong with every sentence. Don't forget that the sentence can be correct as is. Answer choice (A) always leaves the sentence unchanged, and it will be correct on approximately 20 percent of the sentence correction questions.

MATH INTRODUCTION

The Math section of the GMAT follows the two 30-minute AWA essays and a five-minute break. You will have 75 minutes to answer 37 questions. You will receive a separate Math score that ranges from 0 to 60.

Smart strategies can help you guess wisely on difficult questions or when time is short.

Trap Answers

Pick any number and write it down here: _____

Determine the trap answers for these questions:

1. The original price of an article was reduced by 25 percent. During a special sale the new price was decreased by 10 percent. By approximately what percent would the price now have to be increased in order to restore the price of the article to its original amount?
 - ☐ 32.5%
 - ☐ 35%
 - ☐ 48%
 - ☐ 65%
 - ☐ 67.5%
2. A wooden crate has inside dimensions 3 meters by 4 meters by 12 meters. What is the length, in meters, of the longest, straight, inflexible rod of negligible diameter that can be placed completely within the crate?
 - ☐ 12
 - ☐ 12.6
 - ☐ 13
 - ☐ 19
 - ☐ 24
3. The average (arithmetic mean) of x , y , and z is 50. What is the sum of $(4x + y)$, $(3y + z)$, and $(3z)$?
 - ☐ 150
 - ☐ 200
 - ☐ 600
 - ☐ 800
 - ☐ It cannot be determined from the information given.

Ballparking

Although the Math section involves some calculation, testing your computational skills is not its primary purpose. Instead, the questions are designed more to test whether you understand the concepts well enough to set up the problems. Often, you don't need to find precise answers. After all, the GMAT is a multiple-choice test. You can use approximate numbers to make your calculation quicker and easier. Then choose the answer that's "in the right ballpark."

Ballpark to simplify the calculations.

4. Four containers of flour are on a table: The first contains $\frac{1}{3}$ of a pound, the second contains $\frac{1}{6}$ of a pound, the third contains $\frac{1}{9}$ of a pound, and the fourth contains $\frac{1}{18}$ of a pound. If each container can hold one pound of flour, how many additional pounds of flour are required to fill all four containers?

☐ $\frac{2}{9}$

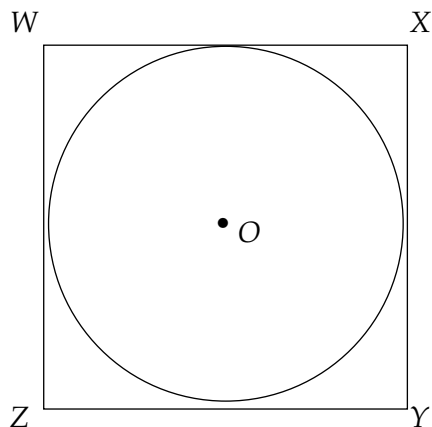
☐ $\frac{2}{3}$

☐ $\frac{11}{9}$

☐ $\frac{25}{9}$

☐ $\frac{25}{9}$

Even when you do not feel confident enough to ballpark away all of the wrong answers, Ballparking can help you avoid obviously wrong answers.



5. In the diagram above, circle O is inscribed within square $WXYZ$. If the square has area 400, what is the area of the circle?
- ☐ 20π
- ☐ 50π
- ☐ 75π
- ☐ 100π
- ☐ 400π
6. Paul drives from his apartment to his parents' house and back. On the trip to his parents' house, he travels at an average speed of 60 miles per hour. On the return trip, Paul drives at an average speed of 80 miles per hour. Which of the following is the closest approximation of Paul's average speed, in miles per hour, for the round trip?
- ☐ 60.0
- ☐ 68.6
- ☐ 70.0
- ☐ 71.4
- ☐ 80.0

In general, follow these guidelines:

1. Double-check before you choose an answer that was "too easy" on a difficult question.
2. When you get stuck on a tough question, eliminate the predictable trap answers before you guess.

MATH 1

DATA SUFFICIENCY

Think of statements (1) and (2) as rules to follow when you answer data sufficiency questions.

Many test takers feel that data sufficiency questions are much harder than problem solving questions. However, both questions test the same math topics. So, why do data sufficiency questions seem so much harder? The primary reason is the strange format of the questions and answer choices. It is very important to become familiar with the types of data sufficiency questions, the fact statements, and the meaning of each answer choice. Essentially, a data sufficiency problem asks you to find which fact or combination of facts provides enough information to answer the question. Let's look at the format.

What the Answer Choices Say

Directions: This data sufficiency problem consists of a question and two statements, labeled (1) and (2), in which certain data are given. You have to decide whether the data given in the statements are sufficient for answering the question. Using the data given in the statements plus your knowledge of mathematics and everyday facts (such as the number of days in July or the meaning of counterclockwise), you must indicate whether

- ☐ statement (1) ALONE is sufficient, but statement (2) alone is not sufficient to answer the question asked;
- ☐ statement (2) ALONE is sufficient, but statement (1) alone is not sufficient to answer the question asked;
- ☐ BOTH statements (1) and (2) TOGETHER are sufficient to answer the question asked, but NEITHER statement ALONE is sufficient;
- ☐ EACH statement ALONE is sufficient to answer the question asked;
- ☐ statements (1) and (2) TOGETHER are NOT sufficient to answer the question asked, and additional data specific to the problem are needed.

What The Answer Choices Mean

Here's what each answer choice really means:

- (A) ☐ 1 ✗
- (B) ✗ ☐ 2
- (C) ☒ 1 ☒ 2
- (D) ☐ 1 ☐ 2
- (E) ~~1~~ ~~2~~

What to Do—AD/BCE

Each time you work a data sufficiency question, read the question carefully. Ask yourself, “What do I know?” and determine what information is provided. Next, ask yourself, “What do I need to know?” and determine what additional information is needed to answer the question.

If you think of the five answer choices as A, B, C, D, and E, you can check one fact at a time to eliminate answers. Look at Fact (1) and narrow the answer choices to “AD” or “BCE.” Write these on your scratch paper to facilitate POE.

Try the following problems by first using Fact (1) to eliminate some answers. Then (and only then) look at Fact (2).

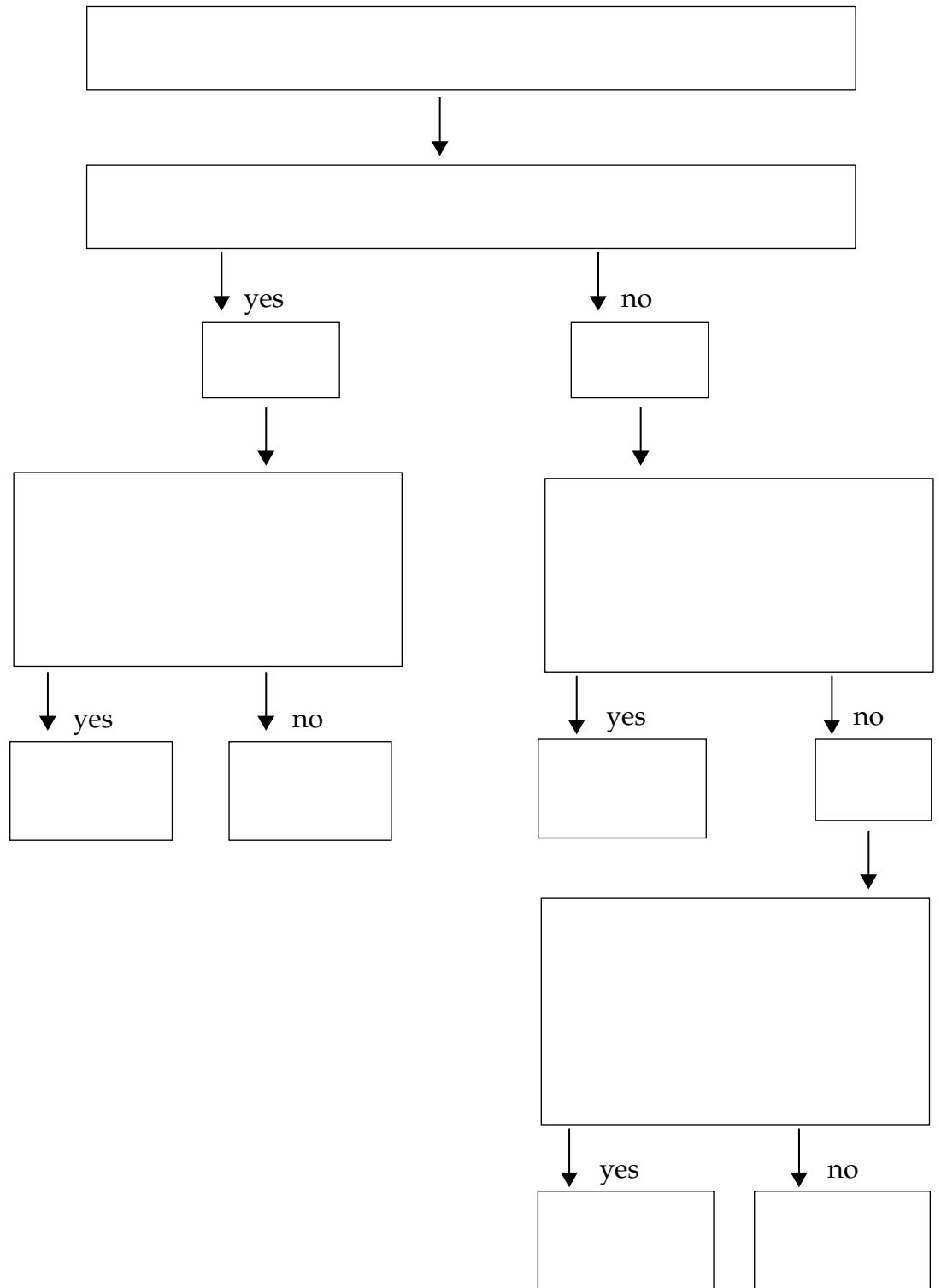
1. What is the value of x ?
 - (1) If x is subtracted from 60, the result is 20.
 - (2) If x is divided by z , the result is 20.

2. If $x + y + z = 180$, what is the value of x ?
 - (1) $y = 75$
 - (2) $y + z = 141$

3. If x and y are positive integers and $\frac{x}{y} = 2$, what is the value of x ?
 - (1) $1 < y < 5$
 - (2) y is odd.

4. If $\frac{3x}{2y} = z$, what is the value of x ?
 - (1) $yz = 30$
 - (2) $y = 10$ and $z = 3$

5. What is the value of x ?
 - (1) $x + y = 20$
 - (2) $z - 3x = 45$



ALGEBRA VS. ARITHMETIC

Which of the following questions would you rather answer?

1. Max has a 10-dollar bill. He goes into a candy store and buys 3 pieces of candy that cost 50 cents each. How much change, in dollars, does Max receive?

- ☐ \$9.50
☐ \$8.50
☐ \$7.00
☐ \$1.50
☐ \$0.50

2. Max has x dollars. He goes into a candy store and buys y pieces of candy that cost z cents each. How much change, in dollars, does Max receive?

☐ $x - yz$

☐ $yz - x$

☐ $\frac{x - yz}{100}$

☐ $100x - yz$

☐ $x - \frac{yz}{100}$

No matter how good you are at algebra, you're better at arithmetic.

What makes algebra complicated? Variables! Calculations with numbers are much easier than calculations with variables. Wouldn't life be wonderful if you could trade algebra problems in for arithmetic problems?

PLUGGING IN

There is a way to turn algebra problems into arithmetic problems. It's called Plugging In. Just follow these steps:

Step 1: Assign a number to each variable in the problem.

Step 2: Work the problem step-by-step using the numbers you chose. You should end up with a numerical answer (with no variables left over) that answers the question in the problem. Circle the answer. This is your target.

Step 3: Plug the number(s) you assigned to the variable(s) into the answer choices. Choose the answer choice that matches your target. Be sure to check all five answer choices.

3. Steven is three times as old as Jean, and Jean is two years older than Ken. If Ken is k years old, then how old is Steven?

☐ $3k + 2$

☐ $3k + 6$

☐ $3k - 3$

☐ $3k - 6$

☐ $\frac{k-2}{3}$

Choose Good Numbers

Make your life easier by choosing good numbers. Good numbers are those that work well with the other numbers in the problem and make your calculations easier. Check the problem to see what sort of calculations you will make. Then, pick numbers that fit smoothly with those calculations. For example, if you know that you will need to find one-tenth of x , choose a number that is divisible by 10.

4. The Amazing soft drink company interviewed c consumers for a market-research study. The study found that $\frac{2}{5}$ of consumers preferred Zing cola to Diet Zing cola. Of those who preferred Diet Zing, $\frac{1}{6}$ preferred Caffeine Free Diet Zing. How many consumers, in terms of c , did not prefer Caffeine Free Diet Zing?

Choose numbers to fit the calculations in the problem.

☐ $\frac{c}{11}$

☐ $\frac{c}{10}$

☐ $\frac{7c}{15}$

☐ $\frac{9c}{10}$

☐ $\frac{10c}{11}$

Depending on the particular number you choose, more than one answer choice may match. For example, try plugging in $y = 60$ in the problem below.

5. A machine working at maximum capacity can produce x radial tires in an hour. How many tires can the machine produce if it works continuously at maximum capacity for y minutes?

☐ $\frac{x}{60y}$

☐ $\frac{xy}{60}$

☐ $\frac{60x}{y}$

☐ $\frac{60}{xy}$

☐ $60xy$

6. If a and b are distinct integers and $x = a + b$ and $y = a - b$, then which of the following expresses $xy + y$, in terms of a and b ?

- ☐ $2b^2 + 2ab$
☐ $a^2 - b^2 + a - b$
☐ $a - b$
☐ $-a - b$
☐ $a^2 + b^2 - a - b$

If more than one answer choice matches your target, pick a different number and try again. You need only test the answer choices that remained after your first attempt. Even better, avoid the problem in the first place by choosing your numbers carefully. Follow the guidelines below when choosing numbers for Plugging In.

Avoid numbers that can make several answer choices match your target.

- Do not use 0 or 1.
- Do not use numbers that appear in the question or in the answer choices.
- Do not use the same number for more than one variable.

Choose numbers to make the calculations easy:

- If there are fractions in the question, choose a number that's a common multiple of the denominators. (Multiplying the denominators together is an easy way to find a good number.)
- If a question involves different units, use a multiple or factor of the conversion number.
- If a question involves percents, use 100 or a multiple of 100.

HIDDEN PLUG INS

In some Plugging In problems, the variable is invisible. You won't see an x , y , or z , even though the problem involves an unknown quantity. The question will ask for a fraction or percent of some unknown amount.

1. Maggie pays $\frac{1}{8}$ of her monthly income for food, $\frac{1}{8}$ for utilities, $\frac{1}{8}$ for student loans, and $\frac{4}{5}$ of the remainder for rent. If at the end of each month Maggie puts $\frac{1}{2}$ of her remaining income into a CD account, what portion of Maggie's monthly income does she put into the account?

What's the invisible variable?

☐ $\frac{1}{8}$

☐ $\frac{1}{10}$

☐ $\frac{7}{80}$

☐ $\frac{1}{16}$

☐ $\frac{1}{20}$

2. If 20 percent of the trees in a certain park are evergreens, and 40 percent of the non-evergreens are maple trees, and there are 75 percent as many oak trees as maple trees in the park, what percent of the trees in the park are not maples, oaks, or evergreens?

What's a good number to plug in on percent problems?

☐ 10%

☐ 12%

☐ 20%

☐ 24%

☐ 25%

PLUGGING IN THE ANSWERS

You've seen how to use the Plugging In approach when the answer choices contain variables. You can also use a variation of that method when the answer choices contain numbers. The goal is the same: turn algebra into arithmetic.

Why start with the middle answer choice?

1. Rob has to make 5 payments on his student loans from college. Each payment will be twice the amount of the previous payment. If the total amount he has to pay back is \$1,550, how much is Rob's first payment?

- ☐ \$10
- ☐ \$20
- ☐ \$25
- ☐ \$50
- ☐ \$75

Follow these three steps:

Step 1: Identify what the question is asking.

Step 2: Plug in the middle answer choice. Work the problem step-by-step and see if everything matches.

Step 3: Eliminate answers that don't work. Keep plugging in answers until you find one that works.

Work the problem in bite-sized pieces. Write down each step as you go.

2. Mike has twice as many stamps as Jean has. After he gives Jean 6 stamps, he still has 8 more stamps than Jean does. How many stamps did Mike have originally?

- ☐ 28
- ☐ 32
- ☐ 36
- ☐ 38
- ☐ 40

Choose Good Numbers

Not all of the answers fit nicely with the calculations. Start with the answers that work well in the problem.

3. Julia is twice as old as her brother Paolo, who is five times as old as their dog Winnie. In 10 years, Julia will be four times as old as Winnie will be then. How old is Paolo?

- ☐ 25
- ☐ 27
- ☐ 30
- ☐ 33
- ☐ 35

4. A dealer charges a price that is 15 percent higher than the manufacturer's price on a computer. During a President's Day sale the dealer then discounts his price by 10 percent. If the dealer earns a gross profit (final price – manufacturer's price) of \$38.50 on a computer purchased during the sale, what was the manufacturer's price for the computer?
- ☐ \$504.25
☐ \$770
☐ \$950
☐ \$1,100
☐ \$1,385
5. Fredrico earns a salary of \$350 in a certain week at an appliance store. In addition, he receives a bonus equal to $\frac{2}{5}$ of the amount of his total sales that exceed \$1500. If he earns \$770, what were Fredrico's total sales for the week?
- ☐ \$1600
☐ \$1750
☐ \$1925
☐ \$2225
☐ \$2550

Identifying Plugging In Problems

Plugging In is a great technique, and you should be on the lookout for opportunities to Plug In.

Plug In

To recognize a basic Plug In problem, look for:

Hidden Plug In

To recognize a Hidden Plugging In problem, look for:

Plugging in the Answers (PITA)

To recognize a Plugging In The Answers problem, look for:

HOMework REVIEW

Use this chart to note any questions you have from the reading or examples in the homework.

[illegible]

PRACTICE

PLUGGING IN

1. If k years from now George will be l years old, how old was George m years ago?

☐ $l - k - m$
☐ $l + (k - m)$
☐ $l + k - m$
☐ $k - (l + m)$
☐ $k + (m - l)$

2. If $2x = 4y = z$, what is $x - y$ in terms of z ?

☐ $\frac{z}{4}$
☐ $\frac{z}{2}$
☐ z
☐ $2z$
☐ $4z$

3. A group of x people are in a room. One-third of the people leave the room, and an additional 2 people enter the room. In terms of x , how many people will now be in the room?

☐ $\left(x - \frac{1}{2}\right) + 2$
☐ $\frac{2}{3}x + 2$
☐ $3x + 2$
☐ $3x + 6$
☐ $x + 2$

4. What must be added to $\frac{x+y}{2}$ to obtain $\frac{x-y}{2}$?

☐ $-y$
☐ $-x$
☐ $2y$
☐ $2x$
☐ $y - x$

5. If $x = 3t - 1$ and $y = 12t^2$, what is y in terms of x ?

☐ $(x + 1)^2$
☐ $4(x + 1)^2$
☐ $\frac{3(x+1)^2}{4}$
☐ $\frac{4(x+1)^2}{3}$
☐ $(x - 1)^2$

6. $\frac{x^2 + 2x - 8}{x^2 - 6x + 8} =$

☐ 1
☐ -1

☐ $\frac{1}{3}x$
☐ $\frac{x+4}{x-4}$
☐ $\frac{x+8}{x-8}$

7. Copper pipe costs x cents per foot in 8-foot lengths, and $x + y$ cents per foot in shorter lengths. What is the lowest possible price, in cents, of 51 feet of pipe in terms of x and y ?

☐ $51(x + y)$
☐ $51x$
☐ $48x + 3y$
☐ $48(x + y)$
☐ $51x + 3y$

HIDDEN PLUGGING IN

1. Twenty-five percent of the residents of City Y are 65 years old or older. Twenty percent of the residents of City Y who are under the age of 65 are age 18 or younger. What percent of the residents of City Y are between the ages of 18 and 65?

☐ 15%
☐ 25%
☐ 45%
☐ 60%
☐ 75%

2. In an election, $\frac{3}{7}$ of the voters voted for Candidate A, and $\frac{3}{5}$ of the remaining voters voted for Candidate B. Of the voters who voted for neither Candidate A nor Candidate B, $\frac{1}{2}$ voted for Candidate C. What fraction of all the votes were cast for Candidate C?

☐ $\frac{1}{70}$
☐ $\frac{4}{35}$
☐ $\frac{9}{35}$
☐ $\frac{13}{35}$
☐ $\frac{9}{10}$

3. A parking garage has places for a certain number

of cars. If $\frac{1}{5}$ of the places are left empty, and $\frac{2}{5}$

of the places are used by compact cars, non-compact cars take up what fraction of the filled spaces in the garage?

☐ $\frac{1}{3}$
☐ $\frac{2}{5}$
☐ $\frac{1}{2}$
☐ $\frac{3}{5}$
☐ $\frac{4}{5}$

4. Fred and Bobbie are book collectors, and Fred has twice as many books as Bobbie does. One-fourth of Fred's books are signed by the authors, and three-fifths of Bobbie's books are signed by the authors. If Fred and Bobbie combine their collections, what fraction of the books are signed by the authors?

☐ $\frac{11}{30}$
☐ $\frac{29}{60}$
☐ $\frac{23}{30}$
☐ $\frac{17}{20}$
☐ It cannot be determined.

5. Computer retailer *A* is selling a computer at a 20 percent discount from its suggested retail price. Computer retailer *B* promises to match this price and then discount it a further 10 percent. Computer retailer *B*'s final price is what percent of the suggested retail price?
- ☐ 75%
☐ 72%
☐ 70%
☐ 68%
☐ 28%
6. At a certain university, 3 out of every 5 students live in an on-campus dormitory. If one out of every 10 students who do not live in an on-campus dormitory lives in a university-owned apartment, what fractional part of the student body does not live in either an on-campus dormitory or a university-owned apartment?
- ☐ $\frac{7}{10}$
☐ $\frac{1}{2}$
☐ $\frac{9}{25}$
☐ $\frac{3}{10}$
☐ $\frac{7}{25}$

PLUGGING IN THE ANSWERS

1. If $2x^2 - 2x - 12 = 0$ and $y^2 - 5y + 6 = 0$
when $x = -y$, then what is the value of x ?
- ☐ -3
☐ -2
☐ 0
☐ 2
☐ 3
2. Several persons rented a car for \$30. If there had been one more person in the group, it would have cost each person \$1 less. How many people were in the group originally?
- ☐ 5
☐ 6
☐ 10
☐ 12
☐ 15
3. This year, half of the clients of a certain consulting firm ended up paying the firm exactly \$22,000 each, while the other half ended up paying the firm exactly \$33,000 each. If the firm received a total of \$275,000 from its clients, how many clients does the consulting firm have?
- ☐ 9
☐ 10
☐ 11
☐ 12
☐ 13
4. A certain bakery produces only chocolate and vanilla cupcakes. If the bakery sells 160 cupcakes per day, and 26 more chocolate cupcakes than vanilla cupcakes are sold per day, how many chocolate cupcakes does the bakery sell per day?
- ☐ 54
☐ 67
☐ 82
☐ 93
☐ 106
5. Judy is 26 years old and Diane is 5 years old. In how many years will Judy be twice as old as Diane?
- ☐ 16
☐ 19
☐ 21
☐ 24
☐ 26
6. Pat has a pocket full of quarters, dimes, and nickels. He takes 6 coins out of his pocket that amount to \$0.70. If there are only two denominations of coins among the 6 coins in Pat's hand, how many nickels is he holding?
- ☐ 2
☐ 3
☐ 4
☐ 5
☐ 6
7. On any given Sunday, 75 percent of the people in a city who own TV sets turn them on. Thirty percent of the people who turn their TV sets on watch football. If 9,900 people watch football on Sunday, how many people in the city own TV sets?
- ☐ 75,000
☐ 44,000
☐ 30,000
☐ 24,000
☐ 20,000

ANSWERS AND EXPLANATIONS

Plugging In

1. **A** Plug in $k = 3$, $l = 10$, and $m = 5$. If George will be 10 in 3 years, he's 7 now. Thus he was 2 (target answer) five years ago.
2. **A** Plug in. Be sure to choose numbers that make the equation in the question true. Let $x = 6$, $y = 3$, and $z = 12$. In that case, $x - y = 6 - 3 = 3$ (target). Only choice (A) works.
3. **B** Use Plugging In. The value of x that you plug in should be a multiple of 3; let $x = 12$. If a third of the people leave, that leaves 8; if 2 come in, your new total is 10 (target answer).
4. **A** If you plug in $x = 3$ and $y = 5$, then $\frac{x+y}{2} = 4$ and $\frac{x-y}{2} = -1$. You have to add -5 (target answer) to 4 in order to get -1 .
5. **D** Plug in for t first. If $t = 2$, then $x = 5$ and $y = 48$. The question wants the value of y , so 48 is your target answer.
6. **D** Plug in $x = 3$, and the fraction becomes $\frac{9+6-8}{9-18+8}$, which reduces to $\frac{7}{-1}$, or -7 (target answer).
7. **E** The 8-foot lengths are less expensive, so you want to get as many as possible: six. If you get six 8-foot lengths, that leaves 3 feet at the more expensive rate. Let $x = 2$ and $y = 3$; the first 48 feet cost \$96, and the remaining 3 feet cost \$5 each, or \$15. Your target answer is \$111.

Hidden Plugging In

1. **D** Plug in 100 for the number of residents. There are 25 people that are 65 or older, and 15 are 18 or younger. There are 60 people in between.
2. **B** Use the denominators to find the ideal number to plug in for the number of voters: 70. From the data given, 30 choose Candidate A and 24 vote for Candidate B. Of the remaining 16, 8 choose Candidate C. The fraction of voters that vote for C is $\frac{8}{70}$, which reduces to $\frac{4}{35}$.
3. **C** If there are 25 places in the garage, then there are 5 empty places and 10 of the places have compact cars in them. If you picked (B), you probably misread the question; the question involves the fraction of non-compact cars in the *filled* spaces. There are 10 non-compact cars and 20 filled spaces, so the fraction is $\frac{1}{2}$.
4. **A** Get rid of (E), which is the Trap Answer. Give Fred 20 books and Bobbie 10 books; 5 of Fred's books are signed and 6 of Bobbie's are signed. The total is 11 out of the 30 books.
5. **B** Use Plugging In. Let the suggested retail price be \$100. Retailer A sells the computer for \$80, and retailer B sells it for \$72. This second price is a 28 percent discount from the original price. The Trap Answer is (C).
6. **C** Use Plugging In. It's important to see here that 3 out of 5 is the same thing as $\frac{3}{5}$. If there are 50 students, then 30 live in a dorm and 2 live in an apartment. There are 18 students remaining, and $\frac{18}{50}$ reduces to $\frac{9}{25}$.

Plugging In the Answers

1. **B** Plug In the Answers. (C) can't be right, because 0 doesn't work in either equation. Plug -2 into the first equation [$2(-2)^2 - 2(-2) - 12 = 0$] and plug 2 into the second equation [$(2)^2 - 5(2) + 6 = 0$].
2. **A** Plug In the Answers. A shrewd eye gravitates to (A) and (B), because $5 \div 6 = 30$ and $6 - 5 = 1$. (C): 10 people pay \$3 each, and 11 people pay \$2.73 (the difference is too small). (A): 5 people pay \$6 each, and 6 people pay \$5 each.
3. **B** Plug In the Answers. Get rid of all the odd numbers (because the clients are split in half). Only (B) and (D) are left; try either one. (B): 10 clients means that 5 pay \$22,000 (\$110,000) and 5 pay \$33,000 (\$165,000).
4. **D** (C) is too small [82 (chocolate) + 56 (vanilla) = 138], but (D) works [$93 + 67 = 160$].
5. **A** In 21 years, Judy is 47 and Diane is 26. Judy is less than twice as old as Diane, so (C) is too big. (A): In 16 years, Judy is 42 and Diane is 21.
6. **C** Pat has to have quarters in his pocket (otherwise, you can't have 70 cents with only 6 coins). (C) If he has 4 nickels, the other two coins must be quarters: $4 \div .05 = .20$, and $2 \div .25 = .50$.
7. **B** If 30,000 own sets, then 22,500 of them are on and 6,750 are tuned to football: too small. (B): If 44,000 people own sets, then 33,000 are on and 9,900 have the football game on.

LESSON 2

SENTENCE CORRECTION 2

The previous lesson covered three of the six most commonly tested errors. This lesson focuses on the remaining three: misplaced modifiers, parallelism, and idioms.

MISPLACED MODIFIERS

You learned about modifiers in the pre-class assignment. Let's look at how they will be tested on the GMAT.

The Rule: A word or phrase that describes something should go right next to thing it modifies.

The modifier is usually an introductory phrase. Make sure that such a phrase describes the noun that follows the comma.

1. Running down the street, a brick fell on my head.

- ☐ a brick fell on my head
- ☐ I was hit by a falling brick
- ☐ I hit a falling brick with my head
- ☐ my head was hit by a falling brick
- ☐ a falling brick hit my head

Who or what was running down the street?

How do the changes in the answers help you spot the error?

2. Eaten in the Mediterranean countries, northern Europeans viewed the tomato with suspicion, for they assumed it had poisonous properties because of its relationship to deadly nightshade.
- ☐ northern Europeans viewed the tomato with suspicion, for they
 - ☐ northern Europeans were suspicious of the tomato, and they
 - ☐ the tomato was viewed with suspicion by northern Europeans, who
 - ☐ the tomato was suspicious to northern Europeans, and it was
 - ☐ the tomato was viewed with suspicion by northern Europeans, it being

Sometimes the introductory phrase is underlined. In this case, you can fix the error in two ways:

- Rewrite the phrase so it describes the subject after the comma.
- Change the phrase into a clause. Since a clause has its own subject, there will be no confusion about what it describes.

3. Sold over the counter at the turn of the century, the government now prohibits the sale of cocaine derivatives.
- ☐ Sold over the counter at the turn of the century
 - ☐ While sold over the counter at the turn of the century
 - ☐ Being sold over the counter at the turn of the century
 - ☐ Although they were selling them over the counter at the turn of the century
 - ☐ Although they were sold over the counter at the turn of the century

4. Controlling most inroads to business ventures in Europe, economists argue that the U.S., with its diminished economic leverage there, now has reason to fear the European Common Market.
- ☐ Controlling most inroads to business ventures in Europe, economists argue that the U.S., with its diminished economic leverage there, now has reason to fear the European Common Market.
 - ☐ Controlling most inroads to business ventures in Europe, the diminished economic leverage of the U.S. there is, according to economists, one reason to fear the European Common Market.
 - ☐ Because it controls most inroads to business ventures in Europe, a place where the U.S. have diminished economic leverage, economists argue that they now have reason to fear the European Common Market.
 - ☐ Because it controls most inroads to business ventures in Europe, economists argue that the U.S.'s diminished economic leverage is a reason for the U.S. to fear the actions of the European Common Market.
 - ☐ Economists argue that the U.S., with its diminished economic leverage in Europe, now has reason to fear the actions of the European Common Market, which controls most inroads to business ventures in Europe.

To spot misplaced modifier errors, look for:

To fix misplaced modifier errors:

Minor Modifier Errors

Occasionally, you will be tested on minor modifier issues such as misplaced adverbs, comparative and superlative adjectives, or quantity words. Although these appear infrequently on the GMAT, it's still a good idea to be familiar with these concepts.

What should "only" emphasize?

5. A swarm of desert honeybees, a phenomenon seen by few of the select scientists and entomologists, occur when a portion of the colony leave the current nest to find a new home.

- ☐ few of the select scientists and entomologists, occur when a portion of the colony leave
- ☐ a select few scientists and entomologists, only occurs when portions of the colony leaves
- ☐ very few selected scientists and entomologists, occur when only portions of the colony leave
- ☐ only very few selected scientists and entomologists, occurs when a portion of the colony leaves
- ☐ only a select few scientists and entomologists, occurs when a portion of the colony leaves

For quantity-word errors, decide if the noun referred to is countable or non-countable.

6. During the State of the Union address every February, the president announced the amount of new programs the administration planned to create and the amounts of money it invested in each one.

- ☐ announced the amount of new programs the administration planned to create and the amounts of money it invested in each one
- ☐ announced the amount of new programs the administration had created and the amount of money that was invested in each one
- ☐ announces the number of new programs the administration plans to create and the amount of money to be invested in each one
- ☐ announces the amount of new programs created by the administration and the amount of money invested in them
- ☐ had announced the administration's plans for a number of new programs and the amounts of money that were to be invested in them

7. Of the numerous decisions facing the Supreme Court this term, the question of an individual's right to die is for certain the more perplexing.

Does the sentence refer to two things or more than two things?

- ☐ the question of an individual's right to die is for certain the more perplexing
- ☐ it seems certain that the more perplexing is the question of an individual's right to die
- ☐ the question of an individual's right to die is certainly the most perplexing
- ☐ it seems certain the question of an individual's right to die is the most perplexing
- ☐ the question of an individual's right to die is certainly the more perplexing

PARALLEL CONSTRUCTION

Sometimes words in a sentence need to be similar in form to other words in that sentence. This similarity is called **parallel construction**. Parallel construction means that words share the same part of speech or that phrases have the same structure.

The Rule: Parallel construction is required of items in a list or comparison.

Lists

Items in a list or series must be parallel. For starters, make sure all the items are all nouns or all verbs. Then, try to match the forms of the nouns or verbs as closely as possible.

1. The two main goals of the Eisenhower presidency were a reduction of taxes and to increase military strength.
- ☐ a reduction of taxes and to increase military strength
 - ☐ to reduce taxes and an increase in military strength
 - ☐ to reduce taxes and to increase military strength
 - ☐ a reduction and an increase in taxes and military strength
 - ☐ taxes being reduced and military strength being increased

Isolate the items in the list and check for parallel construction.

2. In response to losing a significant part of its market share to a competitor, the soft drink company has cut costs by withholding executive bonuses, changed their advertising agency, and have redesigned the company logo.
- ☐ changed their advertising agency, and have
 - ☐ changing their advertising agency, as well as
 - ☐ has changed its advertising agency, and
 - ☐ and changed its advertising agency, and has
 - ☐ changed its advertising agency, and
3. Now even directors of off-Broadway productions, desperate for actors who are talented, charismatic, and aspiring more to a solid career in the theater than to the lucrative rewards and stardom proffered by movie moguls and studios, are turning to independent casting directors for guidance in assembling their casts.
- ☐ are talented, charismatic, and aspiring
 - ☐ are talented, who have charisma, and who are aspiring
 - ☐ have talent and are charismatic and aspire
 - ☐ are talented and have charisma, and whose aspirations are
 - ☐ are talented and charismatic, and who aspire

Comparisons

In comparisons, the items compared must be parallel. Remember, you have to compare “apples to apples and oranges to oranges.”

Use comparison words as a hint to check for parallel construction.

4. The rules of written English are more stringent than spoken English.
- ☐ than spoken English
 - ☐ as spoken English
 - ☐ than those of spoken English
 - ☐ as those of spoken English
 - ☐ so than those of spoken English

5. Scientists now suspect that imperial mammoths of the Pleistocene period were wiped out not by human predation, as were mammoths of earlier periods, but by a sudden and extreme change in climate.

Decide which things or actions the sentence compares.

- ☐ as were mammoths of earlier periods
 - ☐ like that in an earlier period
 - ☐ as in earlier periods
 - ☐ as were earlier periods
 - ☐ like the mammoths in an earlier period
6. The Supreme Court can effectively rule on a case in two ways, either by deciding to hear the case and handing down its judgment, or rather to simply refuse to hear the case at all.
- ☐ either by deciding to hear the case and handing down its judgment, or rather to simply refuse
 - ☐ either by deciding to hear the case and handing down its judgment, or by simply refusing
 - ☐ by either deciding to hear the case and hand down its judgment or to simply refuse
 - ☐ to hear a case and to hand down its judgment or to refuse simply
 - ☐ either by deciding to hear the case and hand down its judgment or by the simple refusing
7. The establishment of land trusts in New England has been praised not because of it restricting overdevelopment but because it allows the inheritance by future generations of the property.
- ☐ not because of it restricting overdevelopment but because it allows the inheritance by future generations of
 - ☐ because it not only is restricting overdevelopment and also allows the inheritance by future generations of
 - ☐ because it does not only restrict overdevelopment, it also allows future generations for inheriting
 - ☐ because of not only the restrictions on overdevelopment but also because future generations can inherit
 - ☐ not only because it restricts overdevelopment but also because it allows future generations to inherit

To spot parallel construction errors, look for:

To fix parallel construction errors:

Idioms

As you learned from the pre-class assignment, idioms are fixed expressions. You must know the correct version of the idiom in order to identify and correct an error.

The Rule: Know your idioms!

1. People who want to be able to consume more calories per day should consider weight lifting; it not only speeds the metabolism, and it increases muscle mass which burns more calories at rest.
 - ☐ it not only speeds the metabolism, and it increases
 - ☐ it not only speeds the metabolism, but also it does increase
 - ☐ not only does it speed the metabolism, but it also increases
 - ☐ not only does it speed the metabolism, as it also is increasing
 - ☐ not only is metabolism speeded, there is an increase in

Fixing idiom errors often involves choosing the right preposition.

2. Some students of literary criticism consider the theories of Blaine to be a huge advance in modern critical thinking and question the need to study the discounted theories of Rauthe and Wilson.
- ☐ to be a huge advance in modern critical thinking and question
 - ☐ as a huge advance in modern critical thinking and question
 - ☐ as being a huge advance in modern critical thinking and questioned
 - ☐ a huge advance in critical thinking and question
 - ☐ are a huge advance in modern critical thinking and questioned
3. Dermatologists are trained to distinguish an irritation, which results from prolonged exposure to the offending substance, from a true allergic reaction, which manifests itself immediately upon contact with the allergen.
- ☐ an irritation, which results from prolonged exposure to the offending substance, from a true allergic reaction, which manifests itself immediately upon contact with the allergen
 - ☐ an irritation, resulting from prolonged exposure to the offending substance, and a true allergic reaction, manifesting itself immediately upon contact with the allergen
 - ☐ between an irritation, which results from prolonged exposure to the offending substance, or a true allergic reaction, which manifests itself immediately upon contact with the allergen
 - ☐ between an irritation, resulting from prolonged exposure to the offending substance, from a true allergic reaction, manifesting itself immediately upon contact with the allergen
 - ☐ a true allergic reaction, which manifests itself immediately upon contact with the allergen, and an irritation, which results from prolonged exposure to the offending substance

4. Unsure of the direction in which the technology of home entertainment is headed, many telecommunications companies now question if people will ever sit down at their computers to watch movies and television.

- ☐ question if people will ever sit down at their computers to watch
- ☐ question if people will ever sit down at their computers and watch
- ☐ question whether people will sit down at their computer and watch
- ☐ wonder if ever a person will sit down at their computer and watch
- ☐ wonder whether people will ever sit down at their computers to watch

To spot idiom errors, look for:

To fix idiom errors:

MATH 2

KEEP PLUGGING AWAY

Some problem solving questions ask which answer choice must be true. Use Plugging In to determine which choices aren't necessarily true. If you know a choice is false, even in one case, eliminate it. Because you need to eliminate the four false choices to find the one that is always true, you'll probably need to plug in more than once. Start by plugging in easy numbers, then try weird numbers. Keep going until only one choice remains.

Find the right answer by disproving and removing the wrong answers.

1. If a and b are distinct positive integers, and a is even, then which of the following must also be even?

- ☐ $2(a + b) - 3$
☐ $(a - b) + 2$
☐ $a + b - 1$
☐ $a - b$
☐ $ab - 2$

For must-be questions:

- Plug in numbers for the variables. Eliminate choices that are false.
- Plug in again using different numbers. Choose numbers that will help you eliminate more choices. Try weird numbers.
- Keep plugging away until only one choice remains.

Consider the "weird" numbers:

Zero

One

Negatives

Extremes

Fractions

2. If p and q are integers, such that $p < 0 < q$, and s is a non-negative integer, which of the following must be true?

- ☐ $p^2 < q^2$
☐ $p + q = 0$
☐ $sp < sq$
☐ $sp \neq sq$
☐ $\frac{p}{q} < s$

Work I/II/III questions by eliminating statements that can be false.

3. If x , y , and z are nonzero integers and $x > yz$, which of the following must be true?

I. $\frac{x}{y} > z$

II. $\frac{x}{z} > 1$

III. $\frac{x}{yz} > 1$

- ☐ None of the above
☐ I only
☐ III only
☐ I and II only
☐ I, II, and III

YES-NO DATA SUFFICIENCY

You've already seen data sufficiency problems that ask about a value. In those problems, you must decide if the statements provide sufficient information to let you solve for the value. Other data sufficiency problems ask questions that require a "yes" or "no" answer. Even though the questions are a little different, you'll still use the AD/BCE method to work these problems.

Let's try a few examples:

1. Does $x = 10$?

- (1) Ten percent of x is 1.
 (2) One-fifth of x is an even prime number.

2. Has Brand X been purchased by more than 50 percent of the 5,000 people in Westport?

- (1) Brand Y has been purchased by 75 percent of the people in Westport.
 (2) Brand X has been purchased by exactly 2,200 people in Westport.

3. Is $x > 0$?

- (1) $x + y > 0$
 (2) $y = 12$

The "issue" of a question is what a "yes" means versus what a "no" means.

Think of the statements as rules to follow when you plug in on Yes-No data sufficiency.

Plugging In for Yes-No Data Sufficiency

When Yes-No data sufficiency questions involve variables, Plugging In can help you determine whether a statement is sufficient. Begin by plugging in a number that fits the statement and see whether you get a “yes” or “no” answer to the question. Next, try to plug in a number that fits the statement but returns the opposite answer. Using weird numbers on Yes-No data sufficiency questions is a good idea. However, you must choose numbers that fit the statement you are considering.

For Yes-No data sufficiency questions:

- Determine what a “yes” answer means and what a “no” answer means before you consider the statements.
- A statement is **sufficient** if it enables you to answer the question with a definite “yes” **OR** a definite “no.”
- A statement is **insufficient** if you can answer the question with a “yes” **AND** a “no” or **if the statement does not provide enough information to determine any answer.**
- When you plug in, you must choose numbers that fit the rules laid down by the statements.

4. Is $x + 7$ an odd integer?

- (1) $2x + 3$ is an odd integer.
- (2) $x + 11$ is an even integer.

The issue is even v. odd.
Try both even and odd numbers.

5. Is x a positive number?

- (1) $x + 6 > 2$
- (2) $x^2 > 25$

Always plug in numbers that fit the statements, not the question.

PERCENT CHANGE

You learned about percent change problems in the pre-class assignment. The percent change formula is:

$$\% \text{ Change} = \frac{\text{difference}}{\text{original}} \times 100$$

Increase or *greater* means that the original number is the _____ number.

Decrease or *less* means that the original number is the _____ number.

Use the percent change formula to solve the following questions. Make sure you correctly identify the original number.

1. By what percent did Company A's revenues increase from 1980 to 1985?
 - (1) In 1980, Company A's revenues were \$600,000; in 1985, its revenues were \$800,000.
 - (2) Each year from 1980 to 1985, Company A's revenues increased by \$40,000.
2. The original price of an article was reduced by 25 percent. During a special sale the new price was decreased by 10 percent. By approximately what percent would the price now have to be increased in order to restore the price of the article to its original amount?
 - ☐ 32.5%
 - ☐ 35%
 - ☐ 48%
 - ☐ 65%
 - ☐ 67.5%

What technique should you use when a question asks for a percentage of an unknown total?

PROBABILITY

As you learned in the pre-class assignment, probability questions test your understanding of part-to-whole relationships.

$$\text{probability} = \frac{\text{number of outcomes you want}}{\text{number of total possible outcomes}}$$

To find the probability of a series of events, multiply the probabilities of the individual events.

1. Alejandro flips a coin four times. What is the probability that he will get heads on the first 2 throws and tails on the last 2 throws?

☐ $\frac{1}{16}$

☐ $\frac{1}{8}$

☐ $\frac{1}{4}$

☐ $\frac{1}{2}$

☐ $\frac{3}{4}$

When calculating the probability of events in a series, assume that each event occurred.

2. Griffin has a bag of marbles that contains only 6 black marbles and 4 red marbles. If he removes three marbles at random without replacing any of the marbles, what is the probability that all three marbles selected will be red?

☐ $\frac{2}{5}$

☐ $\frac{1}{6}$

☐ $\frac{3}{25}$

☐ $\frac{8}{125}$

☐ $\frac{1}{30}$

Remember to change the numbers as objects are removed.

So far, we have seen problems involving the probability of one thing AND another thing happening. Let's look at the probability of one thing OR another thing happening.

3. Ann has a deck of 52 cards, made up of four suits, each with cards numbered 1 through 13. If she selects a card at random, what is the probability that she selects a 2 or 7 or 9?

☐ $\frac{1}{3825}$

☐ $\frac{1}{64}$

☐ $\frac{3}{52}$

☐ $\frac{3}{13}$

☐ $\frac{3}{4}$

$$\text{Probability(A and B)} = \text{Probability(A)} \times \text{Probability(B)}$$

$$\text{Probability(A or B)} = \text{Probability(A)} + \text{Probability(B)}$$

Sometimes you will be asked to find the probability that an event won't happen.

If the probability of your favorite team winning the big game is 70 percent, what is the probability that your favorite team won't win the big game?

4. A six-sided die with faces numbered one through six is rolled twice. What is the probability that the face with the number 2 on it will not be facing upward on either roll?

☐ $\frac{1}{6}$

☐ $\frac{2}{3}$

☐ $\frac{25}{36}$

☐ $\frac{17}{18}$

☐ $\frac{35}{36}$

If the probability it won't rain is $\frac{2}{5}$, the probability that it will rain is _____.

Probability (event happens) + Probability (event does NOT happen) = 1

5. Kevin flips a coin four times. What is the probability that he gets heads on at least one of the four flips?

☐ $\frac{1}{16}$

☐ $\frac{1}{4}$

☐ $\frac{3}{4}$

☐ $\frac{13}{16}$

☐ $\frac{15}{16}$

How many outcomes are there in which Kevin would NOT get a head on every flip?

6. A bowl of fruit contains only apples, oranges, and pears. If a piece of fruit is randomly selected from the bowl, what is the probability that the piece of fruit is an apple?

- (1) Half of the fruit in the bowl is oranges.
(2) The probability of selecting a pear is 30 percent.

TEST SMARTS

When you see a question that seems to demand complex calculations, stop and think for a moment. Look for opportunities to apply good test-taking strategy.

1. $\sqrt{4\sqrt{80} + \frac{4}{9+9\sqrt{5}}} =$

☐ $\frac{4\sqrt{5}}{9}$

☐ 2

☐ 6

☐ $\frac{4\sqrt{5}+2}{3+2\sqrt{5}}$

☐ 36

Ask yourself the following questions:

- Do I know how to do this problem? Do I want to do the tedious calculations?
- What should I do when I don't know what to do? Is there an easier way to do this?
- Are there any obvious traps?
- Could I ballpark?

HOMework REVIEW

Use this chart to note any questions you have from the reading or examples in the homework.

[illegible]

PRACTICE

PART-TO-WHOLE AND YES-NO

1. Does $x + y = xy$?
 - (1) x is neither a positive integer nor a negative integer.
 - (2) y is neither a positive integer nor a negative integer.

2. If $-2 < a < 11$ and $3 < b < 12$, then which of the following is NOT true?

☐ $1 < a + b < 23$

☐ $-14 < a - b < 8$

☐ $-7 < b - a < 14$

☐ $1 < b + a < 23$

☐ $-24 < ab < 132$

3. If $\frac{1}{3} < z < \frac{2}{3}$, then what is the value of z ?
 - (1) When positive integer x is divided by 2, the result is z .
 - (2) When positive even integer y is divided by 12, the result is z .

4. On the first of the year, James invested x dollars at Proudstar bank in an account that yields 2% in interest every quarter year. At the end of the year, during which he made no additional deposits or withdrawals, he had y dollars in the account. If James had invested the same amount in an account which pays interest on a yearly basis, what must the interest rate be for James to have y dollars at the end of the year?

☐ 2.04%

☐ 6.12%

☐ 8%

☐ 8.25%

☐ 10%

5. All votes cast in a recent presidential election were for either the incumbent or the challenger. The challenger received 5.4 million votes and the incumbent received 5 million. If after a recount of the votes and the addition of previously uncounted absentee ballots, the incumbent had 5.2 million votes while the challenger had 5.4 million, then the percentage of the total number of votes that were for the challenger

☐ decreased approximately 10%

☐ decreased approximately 1%

☐ neither increased nor decreased

☐ increased approximately 1%

☐ increased approximately 2%

6. If x and y are greater than zero, then what is the value of x^2y ?
 - (1) $y = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$
 - (2) x has exactly two distinct positive factors, one of which is even.

7. If a , b , and x are integers greater than zero, then which of the following must be greater than

$$\frac{a}{a+b}?$$

- ☐ $\frac{a+x}{a+b+x}$
☐ $\frac{a-x}{a+b+x}$
☐ $\frac{2a}{2a+2b+x}$
☐ $\left(\frac{a}{a+b}\right)^2$
☐ $\frac{a-1}{a+b-1}$

8. Does positive integer a equal 5?

- (1) a is a not factor of 6006.
 (2) 5 is the largest divisor of a .

9. At the same time that Rick opened an account with \$150 at Bank A, Mary Jane opened an account at Bank B with \$150. Mary Jane's account has 10% simple annual interest and Rick's gives 12% annual interest compounded quarterly. If neither Rick nor Mary Jane make any additional deposits or withdrawals, then what percent more does Rick have in his account after a year?

- ☐ $\frac{150(1.03)^4 - 150(1.10)}{150(1.10)}$
☐ $150(.12)(2) - 150(.10)(2)$
☐ $\frac{150(.12)(4) - 150(.10)}{150(.10)}$
☐ $150(1.03)^4 - 150(1.10)$
☐ $\frac{150(1.12)^4 - 150(1.10)}{150(1.10)}$

10. If x is a positive integer, is x a multiple of 5?

- (1) 5 is a factor of $\frac{x}{2}$.
 (2) $3x + 5$ is a multiple of 5.

PROBABILITY

1. Tammie has 10 cards numbered 1 through 10. If she deals two to Tarrell without replacing any of them, what is the probability that Tarrell will get both a 2 and a 3?

- ☐ $\frac{1}{5}$
☐ $\frac{1}{45}$
☐ $\frac{1}{50}$
☐ $\frac{1}{90}$
☐ $\frac{14}{45}$

2. A die with x sides has consecutive integers on its sides. If the probability of NOT getting a 4 on either of two tosses is $\frac{36}{49}$, how many sides does the die have?

- ☐ 4
☐ 5
☐ 7
☐ 8
☐ 13

3. In a drawer of shirts, 8 are blue, 6 are green, and 4 are magenta. If Mason draws 2 shirts at random, what is the probability that at least one of the shirts he draws will be blue?

☐ $\frac{25}{153}$

☐ $\frac{28}{153}$

☐ $\frac{5}{17}$

☐ $\frac{4}{9}$

☐ $\frac{12}{17}$

4. A certain consulting firm employs 8 men and 4 women. In March, 3 employees are selected at random to represent the company at a convention. What is the probability that the representatives will NOT all be men?

☐ $\frac{14}{55}$

☐ $\frac{3}{8}$

☐ $\frac{41}{55}$

☐ $\frac{2}{3}$

☐ $\frac{54}{55}$

5. A jar contains only nickels, dimes, and quarters. If a coin is drawn from the jar at random, what is the probability that it will be either a nickel or a quarter?

(1) The probability the coin will be a nickel or a dime is $\frac{1}{2}$.

(2) The probability the coin will be a dime is $\frac{1}{5}$.

6. Three students, Mark, Peter, and Wanda, are all working on the same math problem. If their individual probabilities of success are $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{3}{8}$, respectively, then what is the probability that at least one of the students will get the problem correct?

☐ $\frac{3}{80}$

☐ $\frac{9}{32}$

☐ $\frac{23}{32}$

☐ $\frac{77}{80}$

☐ $\frac{39}{40}$

7. James and Logan are taking batting practice. If their individual probabilities of hitting a homerun are x and y , respectively, then what is the probability that James will not hit a homerun but Logan will?

☐ $x - xy$

☐ $x - y^2$

☐ $y - xy$

☐ $\left(\frac{y}{x}\right)$

☐ $\frac{1}{x} - y$

ANSWERS AND EXPLANATIONS

Part-to-Whole and Yes-No

1. **E** Statement (1) lets us know that x must either be a fraction or zero. This information is insufficient because without information about y , there is no way to answer the question. Eliminate AD. Statement (2) lets us know that y must either be a fraction or zero. This information is insufficient because without information about x , there is no way to answer the question. Eliminate choice (B), keep CE. Taken together, we know that both x and y can be either fractions or 0; this information is insufficient because if they were each $\frac{1}{2}$ the sum would not equal the product. However, if they were each 0, the sum would equal the product. Eliminate (C).
2. **C** Whenever you are asked to combine two inequalities, you should just do all four possibilities using the math given. In this instance, unfortunately, each answer choice gives us a different calculation to do. That's not hard, just very time-consuming. For example, to check to see if answer (E) is true, we would have to multiply to combine the given ranges. $-2 \times 3 = -6$, $-2 \times 12 = -24$, $11 \times 3 = 33$ and $11 \times 12 = 132$. Take the largest and smallest values (132 and -24), and you can see that answer (E) is true. Move on to the next answer choice and try that one.
3. **D** Statement (1) tells us that z must be $\frac{1}{2}$ since the only fraction between $\frac{1}{3}$ and $\frac{2}{3}$ you can get by dividing an integer by 2 is $\frac{1}{2}$. Statement (1) is sufficient; eliminate BCE. Statement (2): If y is an even integer and z must be a fraction between $\frac{1}{3}$ and $\frac{2}{3}$, then y must be 6, so z must be $\frac{1}{2}$.
4. **D** Just estimate. 2 percent interest per quarter would be 8 percent annually if the account didn't earn interest on the money gained each quarter. But it does. So the correct answer should be a little more than 8 percent.
5. **B** The simplest way to solve this question is just to estimate. The original total number of votes is 10.4 million. It increases to 10.6 million after the recount, but the challenger still has the same number of votes. Since the total increased, but the challenger's number remained the same, the challenger's percentage of the total vote decreased, so the answer must be (A) or (B). Since the change in the total number of votes was very small (only 200,000) out of about 10 million, or a change of about 2 percent), the percent of the total vote that the challenger received must have changed only slightly, so the answer is (B).
6. **C** Statement (1) only tells us about y so it's not sufficient by itself, leaving us choices BCE. Statement (2) tells us only about x , so it's also insufficient alone, thus we eliminate (B). When the two statements are taken together, Statement (1) allows us to calculate the value of y and Statement (2) tells us that the value of x is 2 (2 is the only number with exactly two factors, only one of which is an even positive), so the correct answer is (C).
7. **A** Plug In. Let $a = 2$, $b = 3$, and $x = 4$. Then the question can be read as "which of the following is greater than $\frac{2}{5}$." The next step is to plug those numbers into the answers and see which one gives a number bigger than $\frac{2}{5}$. Remember to check all the choices, even though the first one works.
8. **B** Statement (1) does not resolve the question whether a is 5 or not. The fact that a is not a factor of 6006 tells us that it could be anything except a factor of 6006. Eliminate AD. The largest factor (another word for divisor) of any number is itself, thus Statement (2) tells us that $a = 5$.

9. **A** The problem wants us to find the formula for two different compound interests and *then* do the percent change formula, which would just be time-consuming and brutal. The easier way out is to start to eliminate choices as soon as possible. If we know the compound interest formula: $(\text{principle})(1 + \text{interest rate})^{\# \text{ of compounding periods}}$, then we should know that we need to raise the amount to the *power* of the number of periods, not multiply by the number of periods. This eliminates answer choices (B) and (C). Second, if we pay attention to how often Rick's interest is compounded, we should note that his account compounds every quarter, which means that his 12% yearly is actually a 3 percent quarterly interest rate. This means that we should have (1.03); not (1.12) eliminate (D) and (E). Answer (D) also does not actually do the *percent more* that the problem asks for; it only finds the difference in their amounts after the 2 years.
10. **D** Statement (1) tells us that 5 is a factor of half of x , which means that 5 must also be a factor of x , which is only a slightly different way of saying that x is a multiple of 5. Statement (1) is sufficient; eliminate choices BCE. Statement (2) tells us $3x$ is a multiple of 5 because if you add 5 to any number that is not a multiple of 5, you will not get a multiple of 5. Since $3x$ is a multiple of 5, x must be a multiple of 5 since 3 clearly is not. Since the second statement is also sufficient, the correct choice must be (D).
2. **C** Don't do algebra here; it's a nightmare. Plug In the answers, since what you are given is possibilities for the number of sides. Then find the probability of NOT getting a 4 and see if it matches what you are given. If you plug in answer choice (C), you get seven sides. The odds of NOT getting a number that is on one of those seven sides is $\left(\frac{6}{7}\right)\left(\frac{6}{7}\right) = \frac{36}{49}$. The answer is (C).
3. **E** Remember that *at least one* is a clue, and when you see that phrase, you need to find the probability of getting everything except what you want (in other words, the probability of getting any other color except blue), and then subtract that from 1. The formula for this would be $1 - (\text{the probability of getting the other colors})$.
 $1 - \left(\frac{10}{18} \times \frac{9}{17}\right) = 1 - \frac{5}{17} = \frac{12}{17}$.
4. **C** The question here really translates to "what is the probability that *at least one* woman will go." The easiest way to determine this is to take 1 minus the chances that all the delegates will be men. $1 - \left(\frac{8}{12} \times \frac{7}{11} \times \frac{6}{10}\right) = 1 - \frac{14}{55} = \frac{41}{55}$.
5. **B** We're looking for the chances of selecting either a nickel or a quarter. Statement (1) gives the chances of getting a nickel or a dime, which allows you to infer the chances of getting a quarter. We still don't know the exact chances of getting a nickel, though, so it's not sufficient. Eliminate AD. Statement (2) is sufficient because if you know that the chances of getting a dime are 1 in 5, then that means the chances of getting either a nickel or a quarter are 4 in 5.

Probability

1. **B** Remember probability is expressed as a fraction: (number of possibilities that meet requirement of question) divided by (number of total possibilities). On the first deal, Tammie has 2 cards that would meet the requirement and 10 total possibilities. On the second deal, there is only 1 that would meet the requirement and only 9 possibilities. Thus, we get two fractions:
 $\frac{2}{10} \times \frac{1}{9} = \frac{1}{45}$.

6. **C** The words *at least one* are key here: You need to figure the odds of NONE of the students solving the problem correctly, and then subtract that number from one. The chances of NONE of them answering the question correctly are $\left(\frac{3}{4}\right)\left(\frac{3}{5}\right)\left(\frac{5}{8}\right) = \frac{9}{32}$. Subtract that from 1, and you get your answer: $\frac{23}{32}$.
7. **C** Plug In. If we plug some fraction in for x and y , then we can work the problem with relative ease. For example, if $x = \frac{1}{4}$ and $y = \frac{1}{3}$, then James' probability of *not* hitting a home run is $\frac{3}{4}$ and Logan's probability of hitting a home run is $\frac{1}{3}$. The next step would be to multiply these two fractions: $\frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$. Now plug your values for x and y into the choices to see which one gives you your target answer. Remember to check all five answers.

LESSON 3

CRITICAL REASONING 1

In the pre-class assignment, you learned how to analyze an argument by breaking it into its component parts. Now we'll see how to use that analysis to work critical reasoning questions.

BASIC APPROACH

Follow the same basic approach for all critical reasoning questions.

Step 1: Identify the question.

Begin by reading the question stem. Identify the question type before you read the argument.

Step 2: Work the argument.

For most question types, you will work the argument by finding the point, reasons, and gap.

Step 3: Predict what the answer should do.

Think about what the credited response needs to do. If you have an idea of what the right answer will do, you will be much less likely to be tempted by trap answers.

Step 4: Use POE to find the answer.

It's easier to find wrong answers than right answers. Aggressively eliminate choices that do not fit your prediction of what the answer should do. Slow down if you are having difficulty choosing between two answer choices, and find a reason why one choice is better than another.

We'll show you how to approach all the different types of arguments questions that appear on the GMAT:

- Assumption
- Weaken
- Strengthen
- Identify the Reasoning
- Inference
- Resolve/Explain
- Evaluate the Argument
- Parallel the Reasoning

ASSUMPTION QUESTIONS

Step 1: Read and identify the question.

Look for *assumption*, *assume*, or *presupposition*. Assumption questions commonly ask:

- The argument above assumes which of the following?
- Which of the following is an assumption on which the argument depends?
- The claim made above rests on the questionable presupposition that

Step 2: Work the argument.

Find the point and reasons using the Why Test. Identify the gap in the argument. Pay close attention to language shifts to find the gap, and look for common flaws.

Step 3: Predict what the answer should do.

Think about the gap, and state the assumption in your own words, if possible. The correct answer makes the link between the point and reasons, or it rules out obstacles to that link.

Step 4: Use POE to find the answer.

Ask yourself how each choice affects the argument's point. Eliminate answers that:

- Are irrelevant. Eliminate choices that are outside the scope or bring in unnecessary information.
- Go too far. Eliminate choices that go far beyond what is needed to make the link. Be wary of choices that use extreme words such as *always*, *never*, *everyone*, and *cannot*. Strongly worded answers are usually wrong, unless the conclusion is very strong.
- Weaken the argument. Eliminate choices that hurt the conclusion.

If you have difficulty choosing between two choices, reread the conclusion. Choose the answer that does a better job of linking it to the premises. Using the Negation Test can help you decide whether a choice is relevant to the point.

1. The technological conservatism of tennis racket manufacturers is a reflection of the kinds of demand they are trying to meet. The only tennis players who are seriously interested in innovation and willing to pay for it are professional players. Therefore, innovation in tennis racket technology is limited by what authorities will accept as standard for purposes of competition in professional tennis.

Don't forget to write down the conclusion and premise.

Which of the following is an assumption made in drawing the conclusion above?

- ☐ The market for cheap, traditional rackets cannot expand unless the market for technologically advanced rackets also expands.
- ☐ Professional tennis rackets are likely to improve more as a result of technological innovations developed in small workshops than as a result of technological innovations developed in major manufacturing concerns.
- ☐ Professional tennis players do not generate a strong demand for innovations that fall outside what is officially recognized as standard for the purposes of competition.
- ☐ The technological conservatism of tennis racket manufacturers results primarily from their desire to manufacture a product that can be sold without being altered to suit different national markets.
- ☐ The authorities who set standards for professional tennis rackets do not keep informed about innovative racket design.

2. In an effort to reduce the amount of fat and the number of calories that they consume, many citizens of the United States are making significant changes in their diets. For them, staying in shape and looking fit now take precedence over eating foods that are filling and that taste good. It is likely that if they maintain these new priorities with consistent regard for other health issues, the length and quality of their lives will increase significantly.

Which one of the following is an assumption upon which the argument is based?

- ☐ Eating foods that are filling and that taste good is inconsistent with staying in shape and looking fit.
 - ☐ Controlling the quality of one's life requires more than mere dietary adjustments.
 - ☐ A combination of diet and exercise is necessary if one wishes to stay in shape and look fit.
 - ☐ Many citizens of the United States have only recently discovered the importance of diet to living a longer, healthier life.
 - ☐ Staying in shape and looking good are current goals for most citizens of the United States.
3. Singing in the Rain Umbrella Corp. plans to institute a marketing campaign in which it sells umbrellas at the exits of subway stations during rainy weather. The umbrellas will be sold at a price that is slightly higher than normal. The company thinks the sales of these higher-priced umbrellas will be greater than normal sales of umbrellas, because the purchasers of these umbrellas will be forced to buy them if they do not want to get wet.

The author assumes which of the following about purchasers of umbrellas in predicting the sales of umbrellas?

- ☐ Customers who do not feel immediate pressure to purchase will not do so.
- ☐ Normally priced umbrellas are not profitable for Singing in the Rain Umbrella Corp.
- ☐ Very few people buy Singing in the Rain's normally priced umbrellas.
- ☐ Singing in the Rain Umbrella Corp. will have to stop selling normally priced umbrellas when it starts selling higher-priced umbrellas.
- ☐ Most potential customers of Singing in the Rain's higher priced umbrellas will not have acquired an umbrella previously.

The Negation Test

On some assumption questions, you may have a lot of difficulty spotting the necessary assumption due to the way the answers are worded. This is particularly true with answer choices containing negative words such as *not* and *never*. One way to check whether an answer choice is a necessary assumption is to negate, or reverse, that answer. If the negation would weaken the argument, that answer is a necessary assumption. Remember, the way to weaken an argument is to attack the assumption. To use the Negation Test, simply take each answer choice and reverse it. If the reversed answer choice weakens the argument, that answer may be a necessary assumption.

4. Computers that can be identified and traced are of no use to thieves. So Maguda Inc. has instituted a plan to reduce theft by etching the serial number of each computer both on the frame of the monitor and on the face of the hard drive where it will be impossible to overlook.

Maguda Inc.'s plan assumes that

- ☐ thieves do not steal computers that are of no value to them
- ☐ Maguda Inc.'s competitors also plan to etch serial numbers on their computers
- ☐ thieves are more inclined to steal computers than any other type of office equipment
- ☐ etching the serial number on the computer has no effect on the performance of the computer
- ☐ imposing stiffer jail sentences for thieves would not decrease the number of computers stolen

WEAKEN QUESTIONS

Weaken questions ask you to find an answer that attacks the argument's conclusion. To weaken an argument, you must widen the gap between the conclusion and premise, thereby showing the assumption to be false.

Step 1: Read and identify the question.

Look for key words like *weaken*, *undermine*, *cast doubt*, or *flaw*. Weaken questions typically ask:

- Which of the following, if true, most seriously weakens the conclusion drawn in the passage?
- Which of the following, if true, would cast the most serious doubt on the conclusion above?
- Which of the following indicates a flaw in the reasoning above?

Step 2: Work the argument.

Find the conclusion and premises using the Why Test. Identify the gap in the argument.

Step 3: Predict what the answer should do.

Think about the gap you need to widen. The correct answer invalidates the point by widening the gap between the conclusion and premises. It often introduces a new fact that calls the conclusion into question.

Step 4: Use POE to find the answer.

Ask yourself how each choice affects the argument's point. Look for the choice that attacks the conclusion most strongly and most directly. The answer can bring in new information as long as it's relevant to the conclusion. Eliminate answers that:

- Are irrelevant. Eliminate choices that do nothing to the argument or fall outside the scope.
- Strengthen the argument. Eliminate choices that help the conclusion.
- Don't weaken enough. Eliminate choices that weaken the argument only a little. If you have to think three steps beyond the answer to make it work, it's probably wrong.

If you have difficulty choosing between two choices, reread the conclusion. Choose the answer that does a better job of attacking it.

1. A leading cement manufacturer has been having problems processing manufacturing and delivery records since it expanded its operations. To solve this problem, it plans to install a new platform on its central computing system which will run its tracking program five times faster than the current system does.

Which one of the following casts the most serious doubt on the manufacturer's plan?

- ☐ Not all computer platforms can make the company's tracking program run more quickly.
- ☐ The cost of the new computer platform will require the cement manufacturer to raise prices for its products.
- ☐ The cement company's computer system does not have the capability to run the new platform and cannot be updated.
- ☐ The company has been increasing sales of cement by 1.5 percent a month for the past 18 months.
- ☐ A completely new computer system could make the program run six times faster than it does currently.

2. Certain gemstones are formed by magma, a material found deep inside the Earth, that has pushed toward the surface and solidified; others are created when superheated water and gases cool. When chemical action, heat, or pressure changes the existing structure of rocks, they may recrystallize to form gemstones such as emeralds. A large chemical company plans to create a new source of profits by applying certain of the chemicals it manufactures to specifically chosen rock samples, which would be a much less expensive way to produce emeralds than mining them.

Which one of the following, if true, poses the most serious problem for the success of the company's plan?

- ☐ Man-made emeralds will not be considered as valuable as naturally produced emeralds.
- ☐ Natural emeralds are produced by heat and pressure much more frequently than by chemical action.
- ☐ The types of material from which emeralds are formed are not found on the Earth's surface.
- ☐ Laboratory conditions are difficult to replicate in a large-scale factory.
- ☐ The gases that will mix with superheated water are so toxic that workers could suffer from a number of serious medical conditions if the handling of such gases is not strictly controlled.

3. Critics insist that government regulation of business has made it more difficult for American businesses to operate profitably and has destroyed their ability to compete with foreign producers of consumer goods. This argument is not compelling. Government regulation of business was never as active in this nation as it was between the years 1975 and 1985, and in those years the average net worth of American families increased more than it had in any previous decade.

The author's argument would be most seriously weakened if it were true that

- ☐ Eighty percent of all businesses started during the decade 1975–1985 lost money in their first year of operation.
- ☐ More businesses declared bankruptcy during the period 1975–1985 than in any previous decade.
- ☐ During the decade 1975–1985, America imported more consumer goods than it exported.
- ☐ The increase in net worth of average American families during the years 1975–1985 was largely due to an increase in the value of residential homes.
- ☐ The average interest rate charged on business and home mortgage loans was higher during the decade 1975–1985 than it had been in any previous decade.

STRENGTHEN QUESTIONS

To strengthen an argument, find the gap and then build a bridge across it, making the argument stronger. Strengthen questions go a step beyond assumption questions. For strengthen questions, add a premise that provides extra support for the argument.

Step 1: Identify the question.

Look for key words like *support*, *strengthen*, or *justify*. Strengthen questions typically ask:

- Which of the following, if true, would most support the claims above?
- Which of the following statements, if true, would most strengthen the conclusion?

Step 2: Work the argument.

Find the conclusion and premises using the Why Test. Identify the gap in the argument.

Step 3: Predict what the answer should do.

Think about the gap. The correct answer will add a new consideration that supports to the conclusion or rules out obstacles to the conclusion.

Step 4: Use POE to find the answer.

Ask yourself how each choice affects the argument's point. The answer can bring in new information as long as it's relevant to the conclusion. Eliminate answers that:

- Are irrelevant. Eliminate choices that do nothing to the argument or fall outside the scope.
- Weaken the argument. Eliminate choices that hurt the conclusion.
- Don't strengthen enough. Eliminate choices that strengthen the argument only a little. If you have to think three steps beyond the answer to make it work, it's probably wrong.

If you have difficulty choosing between two choices, reread the conclusion. Choose the answer that does a better job of supporting it.

1. A recession is not caused by any economic force other than a nationwide loss of confidence. If the economy is perceived as being unstable, banks are conservative in lending money, investors take fewer risks, and hence economic growth is slowed.

Which of the following, if true, would most strengthen the argument above?

- ☐ A recession is severely affected by the response of the Federal Reserve's setting of interest rates.
 - ☐ A recession can be brought on by the failure of a major bank that had been loaning money.
 - ☐ Slowed economic growth is not the only result of a recession.
 - ☐ When investors begin taking greater risks it is enough to stimulate economic growth.
 - ☐ It is a fallacy to assume that economic growth is necessary for economic stability.
2. The mayor of City X wishes to increase voter turnout for the next city council election. A recent poll of registered voters in neighboring City Y showed a high level of dissatisfaction with that city's voting facilities. Based on this result, City X's mayor proposed a dramatic update of his city's voting facilities to increase turnout at the next election.

Which of the following, if true, most supports the mayor's plan?

- ☐ A majority of respondents in City Y's poll had voted in favor of a tax cut in the previous election.
- ☐ City Y's voter turnout rates increased as the result of the publicity the recent poll generated.
- ☐ A City X poll of registered voters revealed that their behavior is not strongly influenced by their perception of voting facilities.
- ☐ Even the best polling methods have difficulty capturing voter opinions accurately.
- ☐ The City Y poll showed that voters who are satisfied with voting facilities are more likely to vote.

3. In an effort to increase revenues, Sky Air recently partnered with several airlines to create a joint frequent-flier program. In the new program, miles earned on all partner airlines can be combined for reward tickets valid on any airline in the partnership. Sky Air's CEO cites a recent increase in the number of passengers on all the partner airlines as evidence that the new frequent-flier program is having the desired effect. Industry analysts doubt the CEO's claim.

Which of the following, if true, best supports the analysts' position?

- ☐ Air travel tends to be seasonal, with more passengers traveling during warmer months and holiday periods.
- ☐ The recent increase in passenger rates for all the partner airlines was preceded by the lowest rates of air travel in the last decade.
- ☐ Many travelers achieved reward tickets more quickly under the new frequent-flier program and redeemed them recently for free travel.
- ☐ Travelers redeemed frequent-flier rewards on Sky Air's partners more frequently than on Sky Air.
- ☐ A study of frequent-flier programs showed that they generally result in increased long-term airline revenues.

IDENTIFY-THE-REASONING QUESTIONS

Identify-the-reasoning questions ask about the method used by the author to reach the conclusion or what role a phrase plays in an argument. To answer these questions, find the premises and conclusion, and think about *how* they are related. There's no need to find the gap or flaw in the argument. The credited response will describe the structure of the argument in general terms.

Step 1: Identify the question.

Look for questions that ask what *technique, strategy, or method* the author employs or what role a bolded phrase plays. Identify-the-Reasoning questions typically ask:

- The argument uses which one of the following methods of reasoning?
- The bolded phrases play which of the following roles in the argument above?

Step 2: Work the argument.

Find the conclusion and premises. If you are answering a question about what role a bolded phrase plays in the argument, decide if it is a premise or conclusion. If the argument involves two speakers, find the conclusion and premises for each speaker.

Step 3: Predict what the answer should do.

Ask yourself how the premises lead to the conclusion. Think about the structure of the argument in general terms.

Step 4: Use POE to find the answer.

Match each part of the answer choice to the argument, eliminating choices that do not match. Avoid answers that:

- Are only partial matches.
- Describe something the argument did not do.

1. Poppy: High taxes have a chilling effect on the economy. When individuals and corporations are taxed, they have less money to spend. Demand for products and services is reduced, causing unemployment to increase. Taxes must be lowered.

Lilly: But you must also consider that taxes generate funds for the government. If taxes are lowered, the government will be forced to borrow more money, thus reducing the amount of credit available. Unable to borrow money easily, businesses and individuals will be forced to limit their purchases.

Lilly objects to Poppy's argument by

- ☐ claiming that Poppy has exaggerated the adverse effects of high taxes
 - ☐ indicating that Poppy has based his argument on insufficient evidence about the effects of taxes on the economy
 - ☐ noting that Poppy has failed to adequately define the term "taxes"
 - ☐ demonstrating that the danger of reducing taxes is far more severe than the threat of maintaining them at current levels
 - ☐ suggesting that the economic benefits of easy credit outweigh the danger of unemployment
2. Many people feel that hard work deserves a reward. However, this is not true. **Hard work should be its own reward**, because such work intrinsically gives the worker satisfaction when it is performed satisfactorily.

The bolded phrase plays which of the following roles in the argument?

- ☐ It provides information that undermines the conclusion.
- ☐ It points out the assumption with which the author disagrees.
- ☐ It repeats a previously stated premise.
- ☐ It offers support for the author's conclusion.
- ☐ It is the conclusion the author's argument is designed to reach.

3. Although computer manufacturer *X* has experienced decreasing sales in the last quarter, **stockholders should not sell their shares of the company**. The stock price of a financially troubled company can rise dramatically once those problems are solved. **Last year, the stock price of steel manufacturer *Y* rebounded after the company reduced its accounts receivable backlog.**

The bolded phrases play which of the following roles in the argument above?

- ☐ The first phrase contains the author's conclusion and the second phrase contains unrelated information.
- ☐ The first phrase states a position and the second phrase provides evidence to undermine that position.
- ☐ The first phrase states a premise on which the conclusion is based and the second phrase provides the conclusion.
- ☐ The first phrase states the conclusion and the second phrase supports the conclusion with an analogy.
- ☐ The first phrase offers advice and the second phrase draws a contrast between two companies.

MATH 3

RATIOS

What should you do when there are variables in the answers?

As you learned in the pre-class assignment, ratios are part-to-part relationships. By itself, a ratio tells you only the relative amounts of the quantities. It does not tell you the actual amounts.

1. A can of nuts has almonds and cashews in a ratio of x : y . If there are z almonds in the can, which of the following represents the number of cashews?

☐ $y(x + z)$

☐ $y(z - y)$

☐ $\frac{xy}{z}$

☐ $\frac{yz}{x}$

☐ $\frac{xy}{y}$

You need a ratio and one actual number to use the Ratio Box.

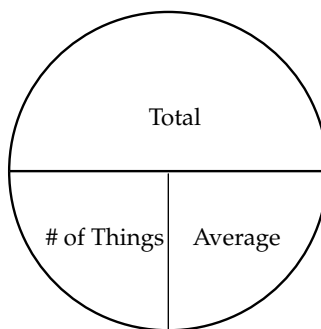
2. A certain jar contained only pennies and nickels. After 3 pennies were removed, how many nickels remained in the jar?

(1) The original ratio of pennies to nickels in the jar was 7: 4.

(2) After the 3 pennies were removed, the ratio of pennies to nickels in the jar was 3: 2.

AVERAGES

The *average* is also called the *arithmetic mean*, or simply the *mean*. Use the Average Pie to organize your information.



Can you eliminate any trap answers?

1. On a geography quiz, Emily and Katherine scored an average of 80 points. If Katherine and Julia scored an average of 85 points, how many more points did Julia earn than Emily?

☐ 15

☐ 10

☐ 7.5

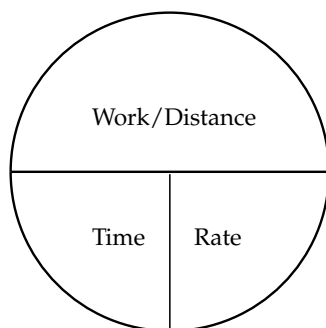
☐ 5

☐ It cannot be determined from the information given.

2. If the average (arithmetic mean) of a , b , and c is 28, is the average of a and c larger than b ?
 - (1) The average of a and b is 30.
 - (2) The average of b and c is 28.
3. If Set Q contains only positive integers, is the average (arithmetic mean) of the numbers in Set Q is equal to the median of the set?
 - (1) The numbers in Set Q are consecutive and odd.
 - (2) Set Q contains an odd number of numbers.

RATES

Rate problems ask you questions about work or distance. Use the Rate Pie to organize your information. Use as many Rate Pies as necessary for all the information in the problem. When people are working together or traveling toward or away from each other, add their rates to find their combined rate.



1. Rob and David live 200 miles apart. Deciding to have a picnic, they both start driving at 9:00 a.m., traveling in a straight line towards each other. Rob drives at a steady speed of 50 miles per hour, and David drives at a steady speed of 30 miles per hour. What is the time when they meet for their picnic?
 - ☐ 11:30 a.m.
 - ☐ 1:00 p.m.
 - ☐ 1:30 p.m.
 - ☐ 2:00 p.m.
 - ☐ 3:40 p.m.

When Rob and David are both traveling, how many total miles do they travel in an hour?

If the task is undefined, it is often helpful to define the task yourself, for example, making widgets. Treat questions with undefined amounts of work as hidden plug in problems and choose a number for the amount of work.

What's a good number to use for the amount of work?

2. Joan can do an entire job in 12 hours. Ken can do an entire job in 6 hours. If Joan and Ken work together for 3 hours, how many hours will it take Ken to finish the rest of the job by himself?

- ☐ 1
☐ $1\frac{1}{2}$
☐ $4\frac{1}{2}$
☐ 6
☐ 9

What information do you have? What information do you need?

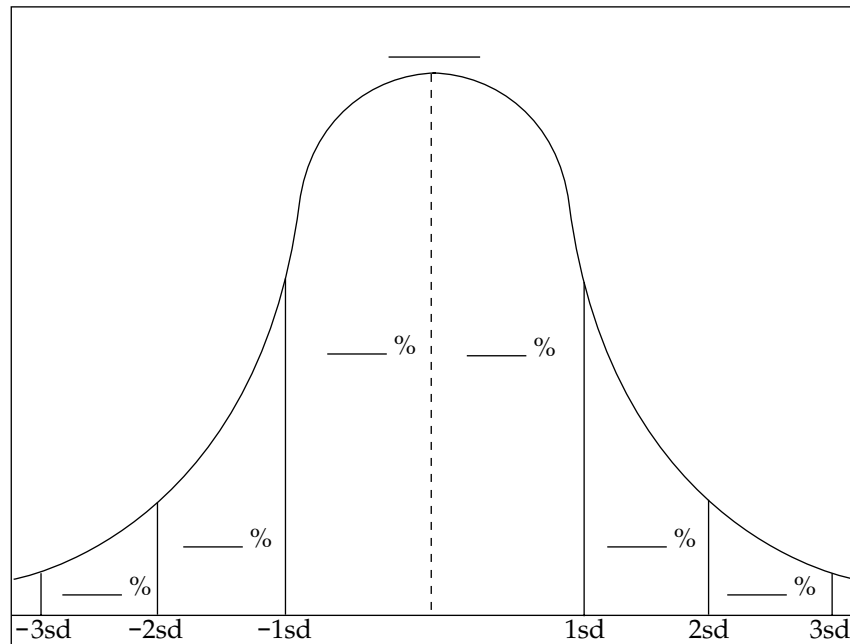
3. Working together without a break, Jermaine and Crystal can do a job in 12 hours. How long will it take Crystal to do the job alone?

- (1) Working without a break, Jermaine takes 16 hours to do the job alone.
 (2) Working alone and without a break, Crystal takes 24 hours to do half the job.

STANDARD DEVIATION

When you're told that a set of numbers has a *normal distribution*, the values fall along a bell curve. Though ETS loves a good bell curve, standard deviation questions are not terribly common on the GMAT.

Let's review what you learned about standard deviation in the pre-class reading. Fill in the missing labels in the chart below.



1. If the mean of a set of data is 27 and the standard deviation is 6, which of the following contains the interval two standard deviations from the mean?

☐ 9 to 15
☐ 9 to 21
☐ 15 to 39
☐ 21 to 33
☐ 27 to 37

Does the information tell you any values on the bell curve?

2. The fourth grade at School X is made up of 300 students who have a total weight of 21,600 pounds. If the weight of these fourth graders has a normal distribution and the standard deviation equals 12 pounds, approximately how many of the fourth graders weigh more than 84 pounds?

☐ 12
☐ 16
☐ 36
☐ 48
☐ 60

3. At Rocktown College, the 400 students taking Psychology 101 received an average score of 76 on the final exam, and the scores had a normal distribution. The bottom 16 percent of scorers will receive a failing grade. What is the score at or below which students fail the course?

- (1) 8 students receive a score of 96 or higher.
(2) The standard deviation for the final exam was 10 points.

MORE DATA SUFFICIENCY

You've seen how to approach data sufficiency questions systematically using the AD/BCE technique. Let's look at some other techniques you can use on data sufficiency questions.

PIECES OF THE PUZZLE

You've already learned that you should determine what information you have and what information you need before you look at the statements in a data sufficiency question. Think of data sufficiency problems as a puzzle. The question stem usually provides some pieces, but one or more puzzle pieces will be missing. Your job is to identify the missing piece, and decide if the statements provide it. This approach lets you avoid working the problem all the way through for each statement.

Problems involving percentages, ratios, and averages lend themselves to the Pieces of the Puzzle approach. Think about the missing piece(s) as you work the following questions.

Which missing piece will help you find the average egg yield?

1. A farmer has 4 henhouses, each containing 6 egg-laying hens. Each henhouse is painted a different color: red, blue, green, and yellow. What was the average egg yield per hen last week?
 - (1) The hens in the red and blue henhouses yielded a total of 36 eggs last week.
 - (2) The hens in the blue and yellow henhouses yielded a total of 48 eggs last week.

Which missing piece will help you find the percent change?

2. If this year there are 2,500 more Brand X consumers than last year, by what percent has the number of Brand X consumers increased from last year to this year?
 - (1) There are 1.5 times as many Brand X consumers this year as last year.
 - (2) The number of Brand X consumers this year is 7,500.

Which missing piece(s) will help you find the ratio?

3. In City Y last year, what was the ratio of the number of commuters who rode buses to the number of commuters who rode trains?
 - (1) In City Y last year, 50,000 commuters rode trains and 48,000 commuters rode buses.
 - (2) In City Y last year, if the number of commuters who rode trains had been 20 percent greater, the number of commuters who rode trains would have been 125 percent of the number of commuters who rode buses.

SIMULTANEOUS EQUATIONS

You've learned how to solve for multiple variables using simultaneous equations. On data sufficiency questions, you don't need to solve for the variables. You only need to decide if it's possible to solve for the variable in the question.

1. What is the value of x ?
 - (1) $5x + 10y = 65$
 - (2) $x + y = 8$
2. Jinhee spent \$65 on hats. If she bought only derbies and fedoras, how many derbies did she buy?
 - (1) Jinhee bought derbies for \$5 each and fedoras for \$10 each.
 - (2) Jinhee bought a total of 8 hats.
3. If $5x + 10y = 65$, what is the value of x ?
 - (1) $x + y = 8$
 - (2) x is an integer.
4. Elizabeth spends a total of 65¢ on postage stamps. She buys only stamps with face values of 5¢ and 10¢. How many 5¢ stamps does she buy?
 - (1) She spends 50¢ on envelopes.
 - (2) She buys a total of 8 stamps.

Are the Equations Really Enough?

Simultaneous equations are a powerful tool for data sufficiency questions. However, you must carefully consider the number of equations and the number of variables.

5. What is the value of x ?
 - (1) $5x + 10y = 65$
 - (2) $130 - 20y = 10x$
6. If $x + 2y = 11$, what is the value of x ?
 - (1) $3x - y = 12$
 - (2) $x^2 - 4y = 21$
7. What is the average (arithmetic mean) of $8a$ and $4b$?
 - (1) $a + b = 5$
 - (2) $2a + b = 11$

Simultaneous equations only work for linear equations (no exponents).

Do you need to know the values for a and b individually?

8. What is the ratio of x to y ?
- (1) $.8x + .5y = 40$
 - (2) $.625y = 50 - x$
9. At a store, Jonah bought only CDs and DVDs. If he bought a total of 12 items, how many CDs did he buy?
- (1) The average price of each DVD was \$10.
 - (2) The average price of each CD was \$16, and Jonah spent \$80 on CDs.
10. If each marble in a set of 50 marbles is either red, black, or white, how many are red?
- (1) 25 marbles are not black and 40 marbles are not white.
 - (2) There are 15 more black marbles than white marbles, and 5 more red marbles than white marbles.

TEST SMARTS

Remember the pacing chart from the first class? Notice that we've allotted more time for the first ten questions in a section than for any other set of questions. These questions determine your scoring range, so work them slowly and carefully.

MATH

Question numbers

| Score | 1–10 | 11–20 | 21–30 | 31–37 |
|----------|---------|---------|---------|---------|
| Under 35 | 30 min. | 25 min. | 15 min. | 5 min. |
| 35–42 | 30 min. | 20 min. | 15 min. | 10 min. |
| Above 42 | 25 min. | 20 min. | 20 min. | 10 min. |

VERBAL

Question numbers

| Score | 1–10 | 11–20 | 21–30 | 31–41 |
|----------|---------|---------|---------|---------|
| Under 28 | 30 min. | 25 min. | 10 min. | 10 min. |
| 28–34 | 27 min. | 20 min. | 18 min. | 10 min. |
| Above 34 | 25 min. | 20 min. | 15 min. | 15 min. |

PACING: THE BEGINNING

Careless mistakes early in the test can really hurt your score. There are a few ways to avoid these mistakes and maximize your score.

Start slowly and carefully.

Be sure to read the problem carefully. Take time to identify the question and determine the best way to approach it.

Reread the question and double check your work.

In the Math section, make sure you finished the problem. Don't fall into the trap of choosing a partial answer. Before you confirm your answer, reread the question. Check to see that your answer matches what the question asked.

In the Verbal section, don't rush through POE. Eliminate choices systematically until only one answer choice remains. Before you confirm your answer, reread the question. Don't fall into the trap of answering the wrong question (i.e., choosing a strengthen answer on a weaken question).

Use your scratch paper.

Making good use of your scratch paper is the best way to stay on track. Do not do calculations in your head; this is a surefire way to make careless mistakes. Work math problems in bite-sized pieces, and write down each step as you go. Take time to write down the conclusion, premise, and gap for critical reasoning questions. Write out “ABCDE” for all verbal questions, and cross out choices as you work POE.

Remain calm and confident.

Do not panic if you have difficulty with a question. While you should strive for a high degree of accuracy, you might encounter a problem you don’t know how to approach. Spending excessive time on such a question won’t help your score. Use the POE criteria to eliminate a few choices, and make a guess. Don’t forget that one in four questions is experimental, so the question might not have counted toward your score anyway. Clear your mind, and move on to the next question.

TEST 2 GOALS

Use the techniques you’ve learned when you take the test. You’re likely to encounter some questions on topics we haven’t covered yet. Use what you’ve learned about eliminating traps and Ballparking to work unfamiliar or difficult questions.

For this test:

- Work on becoming more comfortable with the format of an online CAT.
- Practice setting up and using your scratch paper.
- Focus on the first ten questions in each section. Try to get as many of them right as possible.
- Focus on the test, not your score. Worrying about your score while you take the test distracts you from working the questions.
- Aim for a modest improvement. Improvements come in stages, and you shouldn’t expect to reach your ultimate goal yet.

TEST ANALYSIS**Pacing**

As you take the exam, look at the clock after you complete every 10 questions. Note the time remaining in the spaces below.

Math

#10 _____

#20 _____

#30 _____

Verbal

#10 _____

#20 _____

#30 _____

Accuracy

After you complete the test, look at the score report. Count the number of questions right and wrong for the entire section. Then, count the number wrong for each portion of the exam.

| Test Analysis | |
|---------------------------------|--|
| Overall Score | |
| Math Score | |
| Verbal Score | |
| Math Section | |
| Problem solving % correct | |
| Data sufficiency % correct | |
| Number Right in questions 1–10 | |
| Number Right in questions 11–20 | |
| Number Right in questions 21–30 | |
| Number Right in questions 31–37 | |
| Verbal Section | |
| Sentence Correction % correct | |
| Critical Reasoning % correct | |
| Reading Comprehension % correct | |
| Number Right in questions 1–10 | |
| Number Right in questions 11–20 | |
| Number Right in questions 21–30 | |
| Number Right in questions 31–41 | |

Content

Review the questions you missed and complete the log. First, note the question number and question format (problem solving, data sufficiency, sentence correction, reading comprehension, or critical reasoning). Second, write down the question type/topic, the concepts or skills tested by the question. For example, a math question might test ratios, a sentence correction question might test idioms, or a critical reasoning question might test strengthening an argument. Finally, use the online explanations to determine why you missed the question. You might have made an error in reading the problem, performing calculations, or using a technique. Don't worry if you miss questions that deal with topics we haven't covered yet.

[illegible]

Action Plan

What adjustments will you make to your pacing for the next test?

Which techniques do you need to practice?

Do you need to review any content?

Do you have any questions for your instructor?

Bring your score report to your next class. If there were any problems you couldn't figure out after reviewing the explanations, print them out and bring them to class or extra help.

HOMework REVIEW

[illegible]

PRACTICE

- Frances can complete a job in 12 hours, and Joan can complete the same job in 8 hours. Frances starts the job at 9 a.m., and stops working at 3 p.m. If Joan starts working at 4 p.m. to complete the job, at what time is the job finished?
 - ☐ 6 p.m.
 - ☐ 7 p.m.
 - ☐ 8 p.m.
 - ☐ 10 p.m.
 - ☐ 12 p.m.
- In a mixture of alcohol and water there is $\frac{1}{5}$ more alcohol than water. What is the ratio of alcohol to water in the mixture?
 - ☐ 5: 1
 - ☐ 4: 1
 - ☐ 12: 5
 - ☐ 6: 5
 - ☐ 5: 6
- Susan and Iman are each typing a term paper. Who finishes first?
 - (1) Susan begins typing at 9 a.m. and finishes at 5 p.m.
 - (2) Iman finishes typing 6 hours after she starts.
- In six successive games, a baseball team scored 3 runs once, 6 runs twice, and 9 runs three times. What was the average (arithmetic mean) number of runs the team scored per game over the six-game period?
 - ☐ 8
 - ☐ 7
 - ☐ 6
 - ☐ 5
 - ☐ 4
- If the average (arithmetic mean) of a , b , and c is $2x$, and the average of d and e is $3y$, what is the average of a , b , c , d , and e in terms of x and y ?
 - ☐ $x - y$
 - ☐ $x + y$
 - ☐ $6x + 6y$
 - ☐ $\frac{2x + 3y}{5}$
 - ☐ $\frac{6x + 6y}{5}$
- Working together, Wayne and his son can shovel the entire driveway in three hours. If Wayne can shovel three times as fast as his son can, how many hours would it take for his son to shovel the entire driveway on his own?
 - ☐ 4
 - ☐ 6
 - ☐ 8
 - ☐ 9
 - ☐ 12
- If the range of the set of numbers $\{150, 90, 125, 110, 170, 155, x, 100, 140\}$ is 95, which of the following could be x ?
 - ☐ 80
 - ☐ 85
 - ☐ 95
 - ☐ 125
 - ☐ 185
- Working at their individual rates, Marcus and Latrell can build a certain brick house in 7.5 and 5 hours, respectively. When they work together, they are paid \$35 per hour. If they share their pay in proportion to the amount of work each does, then what is Marcus' hourly pay for building the house?
 - ☐ \$3
 - ☐ \$6
 - ☐ \$7
 - ☐ \$14
 - ☐ \$21
- Working together, Max and Mandy painted the living room in six hours. How long would it have taken for Max to paint the room by himself?
 - (1) Mandy paints twice as fast as Max does.
 - (2) If Max had left when the job was half finished, it would have taken Mandy hours $4\frac{1}{2}$ to finish the job by herself.

10. If two projectiles are launched at the same moment from 1,320 miles apart and travel directly towards each other at 480 miles per hour and 510 miles per hour, respectively, how many minutes will it take for them to meet?
- ☐ 40
☐ 44
☐ 80
☐ 88
☐ 90
11. The mean score on a certain test is 80. If the test scores have a normal distribution and the standard deviation is 7, what percent of the test scores are between 66 and 94?
- ☐ 50%
☐ 68%
☐ 82%
☐ 96%
☐ 98%
12. Both Robert and Alice leave from the same location at 7:00 a.m. driving in the same direction, but in separate cars. Robert drives 30 miles per hour while Alice drives 40 miles per hour. After 6 hours, Alice's car stops. At what time will Robert's car reach Alice's car?
- ☐ 1 p.m.
☐ 3 p.m.
☐ 4 p.m.
☐ 8 p.m.
☐ 9 p.m.
13. Before leaving home for the town of Madison, Pete checks a map which shows that Madison is 5 inches from his current location, Gardensquare. Pete arrives in Madison 2.5 hours later and drove at an average speed of 60 miles per hour. At what scale, in inches per mile, is the map drawn?
- ☐ $\frac{1}{3}$
☐ $\frac{1}{30}$
☐ $\frac{1}{10}$
☐ 2
☐ 30

ANSWERS AND EXPLANATIONS

1. **C** Plug In for number of tasks to be completed before the job is done; let the job involve making 24 widgets. Thus, Frances makes 2 per hour, and Joan makes 3 per hour. Frances works for 6 hours, so she makes 12 widgets. Joan needs 4 hours to make the other 12 widgets; if she starts at 4 p.m., she'll finish at 8 p.m.
2. **D** You have more alcohol than water, so get rid of (E). Plug In: If you have 5 gallons of water, you have 1 more gallon of alcohol. That's 6 gallons of alcohol and 5 gallons of water.
3. **E** Fact (1) doesn't tell you anything about Iman, so eliminate (A) and (D). Fact (2) doesn't mention Susan, so eliminate (B). Even (1) and (2) together don't tell who finishes first. You know that Iman takes less time than Susan does, but she might have started later.
4. **B** The team scored a total of 42 runs over the course of 6 games.
5. **E** It's a Plug In extravaganza! If you plug in $a = 5$, $b = 6$, $c = 7$, then $x = 3$; further, if you plug in $d = 11$ and $e = 13$, you'll find that $y = 4$. The average value of 5, 6, 7, 11, and 13 is $\frac{42}{5}$ (target answer).
6. **E** Let the driveway have an area of 120 square feet. Together, Wayne and his son shovel 40 square feet per hour. You can use a Ratio Box to show that if Wayne shovels three times as fast, Wayne shovels 30 per hour and his son shovels 10 per hour. Thus, it would take his son 12 hours to shovel 120 square feet.
7. **E** They tell us the range of the set {150, 90, 125, 110, 170, 155, x , 100, 140} is 95. Since the present range without x is 80, x has to be either the highest or the lowest number in the set. If x is the lowest number, it would be $170 - 95 = 75$, but that's not an option. Therefore x has to be the highest number. $90 + 95 = 185$.
8. **D** As with the vast majority of work questions, you want to start this by Plugging In for the total size of the job. If the house is 30 units (30 because it's divisible by both 7.5 and 5), then Marcus does 4 units per hour and Latrell does 6 units per hour. At these rates, they would be completing 10 units per hour together and would finish the job in 3 hours. In 3 hours Marcus would have done 12 units ($\frac{2}{5}$ of the job) and Latrell would have done 18 units ($\frac{3}{5}$ of the job). Marcus should therefore get $\frac{2}{5}$ of the money.
9. **D** Statement (1) is tricky, but it's do-able. Say that the paint job involves painting 72 square feet. Together, Max and Mandy paint 12 per hour. If Mandy paints twice as fast, then she paints 8 square feet per hour and Max paints 4. Thus, the job would take Mandy 9 hours to do by herself. (Note: You'll get the same answer no matter what you plug in instead of 72.) Statement (2) offers similar info. If Mandy does half the job in $4\frac{1}{2}$ hours, then she can do the whole job in 9 hours.
10. **C** The projectiles rocket toward each other at a combined speed of 990 m.p.h. (At this point, you know it will take more than an hour; eliminate (A) and (B).) Now use the formula $rt = d$: If $990t = 1,320$, $t = \frac{4}{3}$. Therefore, it takes 1 hour and 20 minutes.
11. **D** If the mean for the test is 80 and the standard deviation is 7, scores between 66 and 94 fall within 2 standard deviations from the mean in either direction. That includes everyone except for those in the third standard deviation from the mean in either direction, or $2\% + 2\%$. Thus, 96% of the examinees score between 66 and 94.
12. **B** Alice travels 10 m.p.h. faster, so she is 60 miles ahead after 6 hours (at 1 p.m.). Robert travels 30 m.p.h., so he needs 2 hours to catch up. He'll be there at 3 p.m.
13. **B** First, use the rate formula to calculate the distance from Madison to Gardensquare, which is 150 miles. Now you know that 5 inches on the map is equal to 150 miles in real life, thus, 1 inch must represent 30 miles. Since the answers are in *inches per mile*, we know that inches must be on the top of any fraction.



Unless your instructor has told you otherwise, you should take an online computer-adaptive test (CAT) before Class 4.

Sign in to your Online Student Center at www.princetonreview.com.

The OSC is a vital resource designed to enhance your GMAT preparation. It will allow you to do the following:

- Take full-length, fully-adaptive CATs
- Complete online drills
- Review course lessons/concepts
- Preview course lessons/concepts
- Make up classes

(If you experience any problems with the site, call our Tech Support department toll-free at 800/546-2102.)

How's the course going? Questions? Problems? Let us help; call your local office or ask your instructor.

LESSON 4

READING COMPREHENSION

THE BASIC APPROACH

Step 1: Work the passage.

Read for the passage's main idea, and get a sense of its structure. Don't read every word and try to memorize all the facts. You may feel safer having spent more time on the passage, but the clock is running. Cut to the chase the first time through and save careful reading for answering the questions.

Step 2: Translate the question.

Figure out what the question is asking, and state it in your own words. If you go to the passage without a clear idea of what you are looking for, you will get mired in the muck of the passage and waste time backtracking.

Step 3: Find the information in the passage and paraphrase it.

Once you find the part of the passage that answers the question, read it carefully and frame an answer in your own words. Avoid heading to the answer choices until you have a good idea of what the correct answer should say. The answers are designed to be confusing, and trap answers will distract you if you're not focused.

Step 4: Use POE to find the answer.

Look at the answer choices and see whether you can match one with the answer you stated in your own words. Use POE to eliminate the wrong answers. If you don't find an answer that matches your self-stated answer, don't panic. Go back to the passage. You might have looked in the wrong place, or you might have misread something.

Find the best answer by eliminating the worst answers.

Most questions have at least one trap answer choice designed to attract you. If you rush through the choices or fail to read carefully, you're likely to fall for a trap answer. If you're stuck between two answer choices, notice the difference between them and think about which one better answers the question. You can waste a great deal of time trying to make a good case for the credited response when it is one of those "lesser of two evils" answers. It is often faster to figure out why four choices are wrong, pick the one that's left, and move on to the next question.

QUESTION TYPES

Most reading comprehension questions fall into one of two categories: specific questions or general questions. Expect to see more specific than general questions. For example, a passage accompanied by three questions might include two specific questions and one general question.

SPECIFIC QUESTIONS

Specific questions ask about a fact or detail from the passage. The question stem may include a line reference or key phrase that helps you locate the information within the passage.

Work the passage:

The wealth of morphological, phonetic, and word similarities among certain languages has lead linguists to recognize the unity of the well-defined family of languages called the Aryan or Indo-European family. Yet even this
 5 latter term is largely a misnomer. This group of languages spreads over an enormous range virtually without interruption, reaching from Central Asia to the fringes of westernmost Europe. The westernmost terminus of the family is Celtic, while its easternmost representatives were
 10 the Tokharian languages, a pair of tongues once spoken by the residents of the Tarim River Basin in Western China and unearthed in documents written more than a thousand years ago.

So remarkable and definite are the similarities among
 15 these languages that linguists are convinced they all derived from an earlier language spoken by some community in the prehistoric past. While we know that Latin began as a rustic dialect in the province of Latium, no one knows where proto-Aryan was first spoken. Some speculate that it was first used in Southern Russia, while still
 20 others point to the Iranian plateau as a potential cradle. Though some philologists believe that the Old Indic and Persian of the *Avesta* contain the most archaic features of Aryan found to date, this does not necessarily fix the
 25 habitat of these early Aryan-speaking peoples closer to Asia than to Europe. Consider Icelandic. Though this language has strayed far from its original source, it preserves many of the characteristics discarded by those who remained behind.

30 From the existing evidence, only one thing seems certain. By the time of Vedic hymns, the first recorded instance of Aryan, those tribes speaking this early language had already begun their widespread dispersal.

Now let's look at different types of specific questions.

Detail Questions

These questions give you a key phrase or line reference that you can use to go back to the passage to find the information.

1. The "misnomer" (line 5) refers to
 - ☐ the fact that not all Indo-Europeans are of Aryan descent
 - ☐ the family of languages goes beyond the boundaries implied by the name
 - ☐ the fact that the family of languages actually originated in Western China
 - ☐ the fact that the languages are no longer spoken
 - ☐ the fact that proto-Aryan was spoken in the area known as Latium

Detail Purpose

These questions are similar to detail questions in that you are asked to locate specific information in the passage. However, this type of question asks why the author mentions something.

2. The author mentions Icelandic in order to
 - ☐ provide support for the contention that early Aryan-speaking peoples did not necessarily live in Asia
 - ☐ provide another example of the archaic features found in the *Avesta*
 - ☐ provide evidence for the inception of proto-Aryan
 - ☐ counter the speculation that proto-Aryan was first spoken in Russia
 - ☐ provide an alternative explanation for the similarities found in early languages

Infer/Imply/Suggest Questions

Although these questions may seem like they ask you to go beyond what the passage says, the correct answer will always be something directly supported by the passage. Treat these as you would any other specific question.

3. Which of the following does the passage imply about the Tokharian languages?
 - ☐ They date to a far earlier period than does Celtic.
 - ☐ They possess many of the same features as did the Old Indic and Persian of the *Avesta*.
 - ☐ They were once spoken over a far greater range than they are today.
 - ☐ They undermine the accuracy of the name "Indo European" for their particular family of languages.
 - ☐ They are remarkably similar to the languages spoken by current residents of the Tarim River Basin.

EXCEPT Questions

Here you have four questions for the price of one. This type of question asks you to determine which answer choice does not belong. Treat the answer choices as a checklist, refer to the passage, and cross off those that the passage supports.

4. According to the passage, all of the following characterize Indo-European languages EXCEPT
- ☐ similarities in the structure of words and in the sounds employed
 - ☐ common origins in some prehistoric community
 - ☐ widespread, intercontinental dispersal
 - ☐ recorded instance of early use in ancient literature
 - ☐ preservation of characteristics discarded by earlier speakers

POE for Specific Questions

As tempting as it is to eliminate answer choices because they don't "sound right," don't fall back on this approach. Eliminate using the main idea and your sense of what the answer should say. Stick to the POE criteria, and read every word of every choice carefully. One word can make or break a correct answer.

On specific questions, eliminate answers that are:

- Beyond the Information Given (B.I.G.). If the answer choice goes beyond the scope of the information provided in the passage, eliminate it. If you cannot find a part of the passage that supports a choice, it's wrong. Be especially wary of answers that make predictions.
- Too strong. If the tone of the answer choice is stronger than that used by the author, eliminate it. Be wary of choices that use absolute words such as *always*, *will*, and *never*.
- Contradictory to information in the passage. Sometimes the test writers construct answer choices that go against information in the passage.
- Are true but don't answer the question. Some answer choices contain information from the passage, but the information doesn't answer the question.
- Half right. Some answer choices are only partially correct. If part of a choice is wrong, the entire choice is wrong.

GENERAL QUESTIONS

General questions ask about the passage as a whole. Use your sense of the passage's main idea and structure to work these questions, and refer to the passage as needed.

- Until recently, corporate ideology in the United States has held that bigger is better. This traditional view of the primacy of big, centralized companies is now being challenged as some of the giants of American business
- 5 are being outperformed by a new generation of smaller, streamlined businesses. If it was the industrial revolution that spawned the era of massive industrialized companies, then perhaps it is the information revolution of the 1990s that is spawning the era of the small company.
- 10 For most of this century, big companies dominated an American business scene that seemed to thrive on its own grandness of scale. The expansion westward, the growth of the railroad and steel industries, an almost limitless supply of cheap raw materials, plus a population boom
- 15 that provided an ever-increasing demand for new products (although not a cheap source of labor) all coincided to encourage the growth of large companies.
- But rapid developments in the marketplace have begun to change the accepted rules of business and have under-
- 20 scored the need for fast reaction times. Small companies, without huge overhead and inventory, can respond quickly to a technologically advanced age in which new products and technologies can become outmoded within a year of their being brought to market.
- 25 Of course, successful emerging small companies face a potential dilemma in that their very success will tend to turn them into copies of the large corporate dinosaurs they are now supplanting. To avoid this trap, small companies may look to the example of several CEOs of large corpora-
- 30 tions who have broken down their sprawling organizations into small, semi-independent divisions capable of making their way into the twenty-first century.

Now let's look at some general questions.

Main Idea/Primary Purpose Questions

Articulating the main idea is one of the most important steps you can take to maximize both your accuracy and your speed in working through the questions. Most passages will be accompanied by a question that asks for the author's main idea or primary purpose.

1. The primary purpose of the passage is to
 - ☐ present evidence that resolves a contradiction in business theory
 - ☐ discuss reasons why an accepted business pattern is changing
 - ☐ describe a theoretical model and a method whereby that model can be tested
 - ☐ argue that a traditional ideology deserves new attention
 - ☐ resolve two conflicting explanations for a phenomenon

Tone/Attitude Questions

If a question asks about the author's tone or attitude, look for words that provide clues about the author's opinion. Decide if the author is positive, negative, or neutral toward the subject.

2. The author's attitude toward emerging small companies is one of
 - ☐ disappointment
 - ☐ optimism
 - ☐ uncertainty
 - ☐ criticism
 - ☐ elation

Structure Questions

Structure questions ask about the organization of the passage or paragraphs within the passage. When you read for the main idea, pay close attention to trigger words that help you understand how the passage and paragraphs are organized.

3. Which of the following best describes the organization of the first paragraph of the passage?
 - ☐ A conventional model is described and an alternative is introduced.
 - ☐ An assertion is made and a general supporting example is given.
 - ☐ Two contradictory points of view are presented and evaluated.
 - ☐ A historical overview is given to explain a phenomenon.
 - ☐ A new theory is described and then qualified.

POE for General Questions

Good answers to a general question fit with the scope of the passage as a whole. Eliminate choices that are:

- Too broad. Answers should not go beyond the scope of the passage.
- Too narrow. Answers that match only one part of the passage are wrong.
- Too strong. Answers that make stronger points or express a stronger opinion than those in the passage are wrong. Be very wary of extreme answers when you work tone/attitude questions. The authors of GMAT passages rarely have passionately positive or negative opinions.
- Half right. Some answer choices are only partially correct. If part of a choice is wrong, the entire choice is wrong.

PACE YOURSELF WISELY

Adjust your approach to reading comprehension passages and questions according to where the passage appears in the Verbal section. Spend the time you need in order to get the early questions right, and spend less time on each successive passage.

Passage #1

- Passages early in the test matter most. Make sure you spend enough time to get the questions right.
- Always go back to the passage. Make sure you can support your answer with proof or evidence. Spend as much time here as you need.

Passage #2

- The second passage counts less than the first in determining your final score. However, it is still important to spend a good amount of time backing up your answers with evidence from the passage.
- Do not get stuck on a killer question. Pay attention to your timing in relation to the suggested time allotted for each part of the section.

Passage #3

- In order to establish a plan for the third passage, evaluate your strengths and weaknesses.
- If you excel at reading comprehension, invest the time to get these questions right. They will be getting more difficult, so it may take you just as long to get through these questions as it did to get through those in the first two passages.
- If reading comprehension is your weakest area, pick up your pace on the third passage. Push through these questions to give yourself more time to spend on your stronger areas.

Passage #4

- The final passage affects your score the least. Spend more time here if this is your strongest area of the test and less if it is your weakest.
- If time is running out, dispense with working the passage. Go directly to the questions, and read only what you need to find the answers.
- If you only have a few minutes left in the section, eliminate answers with extreme wording or those that make predictions, and pick an answer choice. The few seconds it takes to eliminate these traps can help you get questions right and can make a difference in your score.

MATH 4**MORE ROOTS AND EXPONENTS**

Harder problems test the same concepts as easier problems. No matter how difficult the problem sounds, try to relate it back to the basic concepts.

1. $\frac{4^{12} - 4^{11}}{64^3} =$

☐ $\frac{4}{64^3}$

☐ $\frac{1}{16^3}$

☐ 16

☐ 48

☐ 64

What technique should
you use?

2. If $y = (m^x)^2$, then $y^2m =$

☐ m^{2x}

☐ m^{2x+1}

☐ m^{4x+1}

☐ m^{4x+2}

☐ m^{4x+y}

3. Which of the following contain reciprocal expressions?

I. $x^{\frac{1}{2}}$ and $\frac{\sqrt{x}}{x}$

II. $\sqrt{x^{-2}}$ and x

III. x^3 and $x^{\frac{1}{3}}$

☐ I only

☐ II only

☐ I and II only

☐ II & III only

☐ I & III only

MORE FACTORS AND FACTORIALS

ETS sometimes throws factorials into questions to try to make them appear to be more work than they actually are. Don't be thrown off. Most factorial questions can be solved either by reducing or by breaking the question down into simple parts.

1. If $\frac{15!}{3^m}$ is an integer, what is the greatest possible value of m ?

- ☐ 4
☐ 5
☐ 6
☐ 7
☐ 8

2. If x is an integer, is 2^x a factor of $12!$?
- (1) x is the sum of two distinct single-digit prime numbers.
 (2) $0 < x < 11$

Use the distributive property to factor expressions.

3. If k is a multiple of 11, is $k!$ a multiple of 154?
- (1) $!$ has all the same factors as 42.
 (2) k is divisible by 21.

4. If $\sqrt[3]{x}$ is a positive integer, does $\sqrt[3]{x}$ have more than two distinct integer factors?
- (1) $64 < x < 216$
 (2) x is divisible by 5.

MORE QUADRATICS

Recall that a quadratic equation is any equation of the form $ax^2 + bx + c = 0$. A **root** of an equation is just another word for a solution of an equation.

1. If in the equation $x^2 - 2x - c = 10$, x is a variable and c is a constant, what is the value of c ?
- (1) $(x + 3)$ is a factor of the equation $x^2 - 2x - c = 10$.
 (2) 5 is a root of the equation $x^2 - 2x - c = 10$.

Look for the common
quadratics.

2. Which of the following is equivalent to $\frac{(x^2 - y^2)^2}{x^2 + 2xy + y^2}$?

☐ $x^2 - y^2$

☐ x

☐ $\frac{1}{x^2 - y^2}$

☐ $x^2 - 2xy + y^2$

☐ $x^2 + 2xy + y^2$

MORE YES-NO DATA SUFFICIENCY

Plugging In can be very helpful on harder Yes-No Data Sufficiency questions. When you plug in, make sure that your number satisfies the statement. Then, use that number to answer the question. Next, try another number to see if you can get a different answer to the question.

1. Are v and w integers?

(1) $\sqrt{v+w}$ is an integer.

(2) $\frac{v}{w}$ is an integer.

2. If $abc \neq 0$ and $\frac{a}{b} = \frac{b}{c}$, does $b = c$?

(1) $\frac{b}{c} = \frac{c}{a}$

(2) $a = 1$

3. If x is an integer, is x even?

(1) $x^2 - y^2 = 0$

(2) $x^2 + y^2 = 18$

4. Is x^{-y} positive?

(1) x is positive.

(2) y is negative.

Try different sorts of
numbers the second time
you plug in.

PERMUTATIONS AND COMBINATIONS

Some GMAT problems deal with calculating the number of ways to choose or arrange a group of objects. The mathematical terms for these concepts are permutations (counting arrangements) and combinations (counting different groups).

Choosing From Different Sources

In these problems, you are choosing a number of items to fill specific spots, and each spot is filled from a different source. To find the total number of possible combinations, just multiply the number of choices for each of the spots.

- Each night before he goes to bed, Jordan likes to pick out an outfit to wear the next day. He has 12 different shirts, 10 different pairs of jeans, and 8 pairs of sneakers. If an outfit consists of 1 shirt, 1 pair of jeans, and 1 pair of sneakers, how many different outfits does Jordan have?

- ☐ 30
☐ 90
☐ 240
☐ 480
☐ 960

Same Source—Order Matters

Some problems involve choosing objects from a single source. These problems ask for the number of possible **arrangements** or possible **orders** for the selected objects. These problems work a lot like the different source problems. You multiply the number of possible choices for the first item by the number of choices left for the second item, and so forth. Just remember that the size of the source group gets smaller as you go. Problems like this are called **permutation** problems.

- Five people are running in a race. The first one to finish wins a gold medal, the second wins a silver medal and the third wins a bronze medal. How many different arrangements of medal winners, in order from first to third, are possible?

- ☐ 5
☐ 10
☐ 60
☐ 120
☐ 125

A single source gets smaller as each choice is made.

$$\frac{\text{choices for gold}}{\text{choices for gold}} \times \frac{\text{choices left for silver}}{\text{choices left for silver}} \times \frac{\text{choices left for bronze}}{\text{choices left for bronze}} = \frac{\text{different arrangements}}{\text{different arrangements}}$$

Same Source—Order Doesn't Matter

What if you don't care about the exact order of the top three finishers? For example, if you already had a group consisting of (a, b, c) you would not want to also count the group (b, a, c) . To find the number of distinct groups, start by finding all the possible arrangements. Then you need to divide that by the number of ways you can arrange three people. Problems like this are called **combination** problems. You can recognize them because they ask about **different** or **distinct** groups.

When the order doesn't matter, always divide by the number of ways to arrange the group you chose.

3. Five people are running in a race. The first three to finish win gift certificates. How many different groups of people could win the gift certificates?

- ☐ 5
☐ 10
☐ 60
☐ 120
☐ 125

The Formulas

You can use some formulas to help you calculate these numbers in simple situations. These formulas apply only to “same source” problems.

Keep in mind:

- **Permutation** questions ask about order or arrangements. When order matters, use the permutation formula.
- Order doesn't matter in **combination** questions. Instead, you must count the number of distinct groups. When order doesn't matter, use the combination formula.

$$\text{Number of arrangements (permutations)} = \frac{n!}{(n-r)!}$$

$$\text{Number of different groups (combinations)} = \frac{n!}{r!(n-r)!}$$

n = Number of objects in the source

r = Number of objects selected

These formulas won't always be easy to apply in some of the tougher problems. For those, you need to understand the concepts underlying the formulas.

There are two key questions to ask when doing one of these problems:

- 1) Is there a single source, or are there multiple sources?
- 2) If there is a single source, do you want to count distinct groups or different arrangements/orders?

Combining Concepts

Some problems combine the concepts of same source and different source. Calculate everything for one source and then multiply all the different sources together.

4. Kris is purchasing gear to ride his bike this winter. He wants to buy 2 pairs of gloves, 1 parka, 2 hats, and 3 pairs of boots. If the catalog from which he will order offers 5 types of gloves, 3 different parkas, 4 hats, and 6 pairs of boots, how many different orders could he place?

☐ 360
☐ 720
☐ 3,600
☐ 7,200
☐ 36,000

Drill—Permutations and Combinations

1. Greg is renting movies from the video store. He must choose 3 videos from a list of 10 videos and decide in which order to watch them. How many different schedules of videos can he create?

☐ 30
☐ 70
☐ 700
☐ 720
☐ 1,000

2. Doug is selecting books to read on his vacation. He must choose 1 poetry book, 1 novel, 1 self-help book, and 1 pop-up story book from his collection. If his collection includes 5 poetry books, 3 novels, 4 self-help books, and 10 pop-up story books, how many different selections are possible?

☐ 22
☐ 128
☐ 500
☐ 600
☐ 1,040

3. Ben needs to form a committee of 3 from a group of 8 engineers to study design improvements for a product. If two of the engineers are too inexperienced to serve together on the committee, how many different committees can Ben form?

☐ 20
☐ 30
☐ 50
☐ 56
☐ 336

4. Carol is buying birthday gifts for her niece. She has a list of 10 potential gifts from which to choose, but she can only afford to buy 2 gifts. How many different pairs of gifts can Carol buy?

- ☐ 10
☐ 20
☐ 45
☐ 90
☐ 200

TEST SMARTS

No matter where you are on the test, eliminating traps is important. Strategic guessing increases your chance of getting difficult questions right.

1. If two sets of numbers, P and Q, have the same number of elements, is the mean of set Q lower than the mean of set P?
- (1) Set Q consists of consecutive even integers and set P of consecutive odd integers.
(2) The median of set Q is higher than the mean of set P.

Ask yourself the following questions.

- Does the first statement seem to be enough?
- Does the second statement seem to be enough?
- Do the two statements combined seem to be enough? What would the average test taker choose at this point?
- What am I left with?

When you guess, move on! Concentrate on the question in front of you, not on the questions behind you.

HOMWORK REVIEW

Use this chart to note any questions you have from the reading or examples in the homework.

[illegible]

PRACTICE

ROOTS AND EXPONENTS

1. If $a \neq 3$ and $\frac{a^4 - 3a^3 - a + 3}{a - 3} = -126$, then what is the value of a ?

☐ -5
☐ 5
☐ $\sqrt[3]{126}$
☐ $\sqrt{126}$
☐ 25

2. If a and b are distinct integers greater than -1 , then what is the value of a^2 ?

- (1) a^{-1} is undefined.
 (2) $b = 2$

3. Is integer n greater than 3?

(1) $\left(\frac{1}{100}\right)^n > .0001$

(2) $\left(\frac{1}{100}\right)^{n-1} > .001$

4. If the speed of sound in air is 3.316×10^2 meters per second and the speed of sound in water is 1,500 meters per second, then approximately how many times faster does sound travel in water than in air?

☐ 2
☐ 3
☐ 5
☐ 10
☐ 20

5. If a , b , and c are positive integers, is $\left(\frac{a}{b}\right)^c > 1$?

- (1) $b - a = 9$
 (2) $c > 1$

6. $\frac{1}{2^{12}} + \frac{2}{2^{13}} + \frac{4}{2^{14}} + \frac{8}{2^{15}} =$

☐ $\frac{1}{2^{10}}$
☐ $\frac{1}{2^{12}}$
☐ $\frac{15}{2^{15}}$
☐ $\frac{2}{2^{10}}$
☐ $\frac{23}{2^{16}}$

7. Is x^{-y} positive?

- (1) x is positive.
 (2) y is negative.

8. $\frac{48\left(\frac{1}{2^4} + \frac{1}{3^2} + \frac{1}{2^2}\right)}{3^2} =$

☐ $\frac{16}{27}$
☐ $\frac{61}{27}$
☐ $\frac{61}{3}$
☐ 129
☐ 183

9. Is m^2 an integer?

- (1) m^2 is an integer.
 (2) \sqrt{m} is an integer.

10. $\frac{50(5^{-2} - 2^{-2})}{5^2}$

☐ $\frac{3}{2}$

☐ $-\frac{3}{2}$

☐ $-\frac{21}{50}$

☐ $\frac{21}{50}$

☐ $-\frac{21}{25}$

11. If a is not equal to zero, is a^{-3} a number greater than 1?

(1) $0 < a \leq 2$

(2) $ab = a$

4. Is m a multiple of 6?

(1) More than 2 of the first 5 positive integer multiples of m are multiples of 3.

(2) Fewer than 2 of the first 5 positive integer multiples of m are multiples of 12.

5. If 6 is a factor of a and 21 is a factor of b , is ab a multiple of 70?

(1) a is a multiple of 4.

(2) b is a multiple of 15.

6. Does $s = t$?

(1) $\sqrt{s} = t$

(2) s is both a factor and multiple of t .

7. If n is an integer greater than 0, what is the remainder when 9^{12n+3} is divided by 10?

☐ 0

☐ 1

☐ 2

☐ 7

☐ 9

FACTORS, DIVISIBILITY, AND MULTIPLES

1. What is the sum of positive integers x and y ?

(1) $x^2 + 2xy + y = 16$

(2) $x^2 - y^2 = 8$

2. Which of the following are roots of the equation

$$\frac{x(x+5)(x^2-4)}{x+12} = 0?$$

☐ -2, 0, 5, -12

☐ 0, -5, 2, 12

☐ -2, 0, 2, 5, -12

☐ -5, -2, 0, 2

☐ 0, 4, 5

3. What is the remainder when integer n is divided by 10?

(1) When n is divided by 110 the remainder is 75.

(2) When n is divided by 100 the remainder is 25.

FACTORIALS

1. If n is an integer greater than 5.3, then $n!$ must be divisible by which of the following numbers?

☐ 7

☐ 11

☐ 12

☐ 13

☐ 14

2. If for all positive integers x and y , $y > x$, then is

$\frac{y!}{x!}$ even?

(1) $y = 13$

(2) $y - x = 2$

3. The fraction $\frac{11!}{77}$ is equivalent to which of the following?

☐ $2^7 3^3 5^2$

☐ $2^9 3^4 5^2$

☐ $2^8 3^4 5^2$

☐ $2^8 3^4 5^2 7^2$

☐ $2^8 3^4 5^2 11$

4. Is a prime?
 (1) $x! = a$
 (2) $x > 2$
5. What is the value of xyz ?
 (1) $y! = 6$ and $x! > 720$
 (2) z is the least even integer greater than -1 .
6. If x and n are positive integers, is $n! + x$ divisible by x ?
 (1) $n > x$
 (2) n is not a prime number.
4. The co-op board of a certain residential building must consist of two men and three women. If there are six men and seven women who want to be on the committee, how many different make-ups of the committee exist?

- ☐ 65
☐ 525
☐ 1,050
☐ 1,287
☐ 100,800

PERMUTATIONS AND COMBINATIONS

1. Flippy's Flowers is designing a special prom corsage that consists of one rose, one orchid, and one gardenia. If Flippy carries four types of roses, three types of orchids, and five types of gardenias, how many different corsages can Flippy design?
- ☐ 12
☐ 24
☐ 30
☐ 60
☐ 120
2. At a prestigious dog show, six dogs of different breeds are to be displayed on six adjacent podiums. If the Springer spaniel must be displayed on the leftmost podium, how many display arrangements of the six dogs are possible?
- ☐ 5
☐ 6
☐ 30
☐ 120
☐ 240
3. On Random Omelet Monday, a chef creates omelets by randomly choosing three out of a possible six fillings. How many different omelets can the chef possibly make?
- ☐ 9
☐ 18
☐ 20
☐ 120
☐ 720
5. Eight Alaskan Huskies are split into pairs to pull one of four sleds in a race. How many different assignments of Huskies to sleds are possible?
- ☐ 32
☐ 64
☐ 420
☐ 1680
☐ 2520
6. In a group of 8 semifinalists, all but 2 will advance to the final round. If in the final round only the top 3 will be awarded medals, then how many groups of medal winners are possible?
- ☐ 20
☐ 56
☐ 120
☐ 560
☐ 720
7. Alan has a flock of sheep from which he will choose 4 to take with him to the livestock show in Houston. If Alan has 15 distinct possible groups of sheep he could take to the show, then which of the following is the number of sheep in his flock?
- ☐ 30
☐ 15
☐ 7
☐ 6
☐ 5

ANSWERS AND EXPLANATIONS

Roots and Exponents

1. **A** If you are asked a specific question to which you are given specific numerical answer choices, you should always Plug In the Answers. Though we typically start with answer choice (C), here we want to start with the choice that will be easiest to calculate, which is (B). If we plug in choice (B), we get the answer $124 = -126$. Clearly this answer is too large, so we need to try something smaller. Try choice (A) since it's the only smaller choice. It works.
2. **A** Statement (1) tells us that $a = 0$, since only division by 0 is undefined. Since b is distinct from a , it cannot be 0, and this statement is sufficient to determine that a^b is 0. Eliminate BCE and keep AD. Statement (2) tells us nothing about a , and so is insufficient; eliminate choice (D).
3. **D** Plug In possible values for n . In both statements, you cannot plug in a value for n that is greater than 3 without contradicting the statement, so each statement is sufficient, and the answer is (D).
4. **C** Convert the speed that's in scientific notation into a "regular" number to avoid confusion. $3.316 \times 10^2 = 331.6$. 1,500 is about 5 times as big, so the answer is (C).
5. **A** Statement (1) tells us that $\frac{a}{b}$ is a fraction less than one, and any fraction less than one, when raised to a positive power, will remain a fraction less than one. Statement (1) is sufficient. Eliminate BCE. Statement (2) does not tell us anything about $\frac{a}{b}$; if $\frac{a}{b}$ is an integer greater than one, then $\left(\frac{a}{b}\right)^c$ will also be greater than one, but if $\frac{a}{b}$ is a fraction less than one, then $\left(\frac{a}{b}\right)^c$ will not be greater than one. Statement (2) is insufficient; eliminate (D).
6. **A** The first thing you should recognize here is that the intention cannot be to get you to just do the math; to find a common denominator here would require ugly calculations. So, look for a simpler way to solve the question. The presence of the exponents and all the multiples of two should give you a hint that expressing things as powers of two might be a good way to get a handle on the problem. If we express the numerators as powers of two, then we would get $\frac{1}{2^{12}} + \frac{2}{2^{13}} + \frac{2^2}{2^{14}} + \frac{2^3}{2^{15}}$. Next, reduce each of the fractions that can be reduced, and you have $\frac{1}{2^{12}} + \frac{1}{2^{12}} + \frac{1}{2^{12}} + \frac{1}{2^{12}}$. Now adding our fractions is really easy, and we get $\frac{4}{2^{12}}$. Unfortunately, they aren't done making us work. Since our answer doesn't show up in the answer choices, we have to reduce again: $\frac{4}{2^{12}} = \frac{2^2}{2^{12}} = \frac{1}{2^{10}}$.
7. **A** This question begs to be translated a little bit before even looking at the statements. $x^{-y} = \frac{1}{x^y}$. The only way $\frac{1}{x^y}$ can be negative is if x is negative and y is odd. Statement (1) tells us that x is not negative, and since it is not negative, there is no way for x^{-y} to be negative. Eliminate BCE. Statement (2) doesn't tell us whether y is even or odd so that doesn't help answer the question.

8. **B** This question calls for straight math, covering fractions and exponents. The easiest way to start is to re-express 48 as powers of 2 and 3 and then distribute. Since $48 = 3 \times 16$, we can write 48 as 3×2^4 . Then we can multiply that by each of the fractions in the parentheses and then reduce: $\frac{3 \times 2^4}{2^4} = 3$, $\frac{3 \times 2^4}{3^2} = \frac{2^4}{3}$, and $\frac{3 \times 2^4}{2^2} = 3 \times 2^2 = 12$. Now add these all together:

$$3 + \frac{2^4}{3} + 12 = 15 + \frac{16}{3} = \frac{61}{3}.$$

Now, you're dividing the whole thing by 3^2 , which is the same as multiplying by $\frac{1}{3^2}$ or $\frac{1}{9}$. $\frac{61}{3} \times \frac{1}{9} = \frac{61}{27}$.

9. **B** Start by translating the question and understanding the pieces of the puzzle given and the pieces needed. To answer this question, we need to know whether m is an integer. Statement (1) is insufficient because m could equal 2 or m could equal $\sqrt{2}$. Eliminate AD. Statement (2) tells us that m must be an integer because it must be the perfect square of an integer, and any integer squared is also an integer.

10. **C** This question tests basic math in a somewhat complex manner; it combines exponent, fraction, and distribution rules. First, we should probably re-express the numbers with negative exponents as fractions in order to multiply them by 50. Remember, a negative exponent is really just 1 over a positive exponent (e.g. 5^{-2} is the same as

$\frac{1}{5^2}$). You should now have the inside of the parentheses as $\left(\frac{1}{25} - \frac{1}{4}\right)$. Now distribute the 50 and reduce, and you get $2 - \frac{25}{2}$. The next step is to deal with the denominator. Dividing by 5^2 is the same as multiplying by $\frac{1}{5^2}$. We can then distribute, which gives us $\frac{25}{2} - \frac{25}{2 \times 25}$. The last step is to reduce and then subtract the fractions. $\frac{2}{25} - \frac{1}{2} = \frac{4 - 25}{25 \times 2} = -\frac{21}{50}$. Don't forget to bowtie to make the subtraction easier.

11. **E** This question begs for a little translation and simplification; it is another way to say $\frac{1}{a^3}$. For $\frac{1}{a^3}$ to be greater than 1, a must be a positive fraction less than 1. Statement (1) does not resolve whether a is a fraction or not. Eliminate AD. Statement (2) only tells you that b is 1; it tells us nothing about a . Eliminate (B). When we look at the statements together, we know nothing more about a than we knew in Statement (1), so together they are still not sufficient.

Factors, Divisibility, and Multiples

1. **D** Start with Statement (1). If we factor the equation given, it yields $(x + y)^2 = 16$, so $x + y = 4$ (note that we're told that x and y are positive), so Statement (1) is sufficient. Eliminate BCE. Statement (2) can also be factored, and yields $(x + y)(x - y) = 8$. This tells us that $x + y$ and $x - y$ must be factors of 8. Eight only has four factors, 1, 2, 4, 8. If we consider each possible factor in turn, and if x and y are positive integers and must equal one of these factors, there is no way that $x + y$ can equal 1. If $x + y$ must equal 2, then x and y must both be 1, but in that instance $x - y$ would not equal 4, thus $x + y$ cannot be 1. If we continue to try each factor, the only factor of 8 that $x + y$ could be is 4, thus this statement is also sufficient.

2. **D** The roots of an equation are those values that make the equation equal 0. So all we have to do is find what values will make the equation equal 0. If you don't see anything that will make the equation equal 0, then just plug in the answers. If you do, then look for your numbers in the answer choices, and eliminate anything without your numbers. The roots are -5 , -2 , 0 , and 2 .
3. **D** Look at Statement (1). Any multiple of 110 will have a units digit of 0. Add 75, and you'll have a units digit of 5. The remainder of any integer divided by 10 is its units digit, so Statement (1) is sufficient. The same process will reveal that Statement (2) is sufficient. The answer is (D).
4. **B** Start with Statement (1). Multiples of 6 (6, 12, 18, 24, and 30) would yield an answer of "yes." Multiples of 3 (3, 6, 9, 12, 15) would yield a "no". Thus Statement (1) is insufficient. Eliminate AD. Approach Statement (2) the same way. The information we are given in this statement doesn't allow us to use 6 or any multiple of 6 for m , thus answering the question with a definitive "no!".
5. **B** This question is all about factoring. We need to determine whether 70 is a factor of ab , and the easiest way to do that is to break 70 down into its prime factors. $7 \times 5 \times 2 = 70$. So if ab is divisible by 7, 5, and 2, then it's divisible by 70. The question itself lets us know that 70 is divisible by 2 (since 6 is a factor of a) and by 7 (since 21 is a factor of b), so all we need is proof that it is divisible by 5. Statement (1) does nothing to help, but Statement (2) shows that b is divisible by 5, and so is sufficient.
6. **B** Statement (1) is insufficient because s and t could both be 1, which would be equal, or s could be 4 and t could be 2. Eliminate AD. What we are given in Statement (2) answers the question because the only number that can be both a factor and a multiple of t is t , thus s must be equal to t .
7. **E** When something looks like an insane amount of work, start looking for a shortcut. In this case, the shortcut is a pattern: $9^1 = 9$. $9^2 = 81$. $9^3 = 81 \times 9 = 729$. Multiply that by another 9? You'll get a number ending in 1. Then one ending in 9. And so forth and so on. So the bottom line is that whenever 9 is raised to an odd power, the units digit is 9. When it's raised to an even power, the units digit is 1. Adding 10 to a number won't do

anything to change the units digit, and when you divide a number by 10, its remainder will always be its units digit. No matter what value you plug in for n , we're going to be raising 9 to an odd power, so the units digit and the remainder will both be 9.

Factorials

1. **C** If n is greater than 5.3, then the smallest $n!$ can be is $6!$. Since $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$, it is definitely divisible by 12, because any $n!$ bigger than 6 will include both a 6 and a 2, thus making it a multiple of 12. Also, $n!$ does not have to be divisible by anything greater than 6, so 7, 11, and 13 are eliminated as are any multiples of those numbers, like 14.
2. **B** Statement (1) only tells us what y is. Without knowing something about x , we cannot find out if $\frac{y!}{x!}$ is even or odd, because if x is 11, then $\frac{13!}{11!}$ would give us 13×2 , which is an even number, while if x were 12, $\frac{13!}{12!}$ would give us 13, which is an odd number. Eliminate (A) and (D). Statement (2) tells us the relationship between x and y by telling us that they are 2 apart. We now know that no matter what y is, x will be two less. So when you divide $y!$ by $x!$, you will always be left with the two highest numbers, one even and one odd, and an even times an odd will always be even.

3. **C** Write out the factorial and then cancel everything that you can. Since $11!$ includes both a 7 and an 11 we can cancel those numbers with the 77 in the denominator leaving us with $10 \times 9 \times 8 \times 6 \times 5 \times 4 \times 3 \times 2$ and making answers (D) and (E) incorrect. Next, a glance at the answers shows that we don't need to solve the equation, but rather just put it in an exponential form, so the next step is to express the remaining numbers as products of 2's, 3's, and 5's. 10 can be expressed as 2×5 , 9 as 3×3 , and so on. Your final step would be to apply your exponent rule that tells you when you multiply exponents with the same base, you *add* the exponents.
4. **C** Statement (1) does not tell us anything about the value of x or a . We can't say whether it's prime or not, because if x is 2, then a is prime, but if x is anything other than 2, then a is not prime. Statement (1) alone is insufficient. Eliminate AD and keep BCE. Statement (2) says nothing about a , thus Statement (2) is insufficient alone. Eliminate B and keep CE. Taken together, we know that x is greater than 2, and so a is the product of at least 3 integers ($3!$). Since a prime number has only 2 factors, a cannot be prime, and the correct answer is (C).
5. **B** Statement (1) tells us what the value of y is, but does not give us the exact value of x or tell us anything about the value of z , and so is insufficient. Eliminate AD. Statement (2) tells us that z is 0, and thus we don't need to know anything about the value of any of the other variables.
6. **A** This one is tough. To understand the relevance of Statement (1), you have to recognize the following:
- A factorial is divisible by all positive integers less than or equal to the integer you are taking a factorial of. For example, $x!$ is divisible by all positive integers smaller than x .
 - If b is a multiple of y , then if you add y to b , the result will still be divisible by y . For example, 12 is divisible by 3. If you add $12 + 3$ it will still be divisible by 3.
- Alternatively, plug in values for x and n and you will find out the facts mentioned above, but that's a lot of messy work. Statement (1) is sufficient. Keep AD, eliminate BCE. Statement

(2) tells us nothing about x nor its relationship to n . Stating that n is NOT prime means it could be a vast number of values. Thus Statement (2) is not sufficient.

Permutations and Combinations

1. **D** If Flippy chooses one of each flower, he has $4 \times 3 \times 5$, or 60, different corsages.
2. **D** The Springer spaniel must be on the far left. After that, 5 dogs could be on the second podium, 4 on the third, and so forth. The total number of arrangements is $1 \times 5 \times 4 \times 3 \times 2 \times 1$, which equals 120.
3. **C** This is a combination problem. To choose 3 out of 6 fillings, the formula is $\frac{6 \times 5 \times 4}{3 \times 2 \times 1} = \frac{120}{6} = 20$.
4. **B** First, find the number of ways you can choose 3 of 7 women: $\left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35 \right)$ and the number of ways you can choose 2 of 6 men: $\left(\frac{6 \times 5}{2 \times 1} = 15 \right)$. Multiply the numbers together: $35 \times 15 = 525$.
5. **E** Here we have another combination problem; you must find out how many ways you can create the four teams. For the first team, you have $\frac{8 \times 7}{2 \times 1} = 28$ possibilities. For the second team, you have $\frac{6 \times 5}{2 \times 1} = 15$ possibilities (you have only 6 options because 2 dogs were assigned to the first sled). For the third sled, you have $\frac{4 \times 3}{2 \times 1} = 6$ possible groups. For the final team, you have $\frac{2 \times 1}{2 \times 1} = 1$ group. To arrive at your final answer, just multiply the numbers together.

6. **B** The entire discussion of rounds is a red herring.

The question is asking for possible combinations of the final three, and it is possible for any of the original 8 contestants to have advanced to the final round, thus we need to pick 3 out of 8, and order doesn't matter. $\frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$.

7. **D** This is a combination question that is somewhat tougher because we aren't given the number of sheep that were initially in the flock, or so it seems. Plug In the Answers! In the answers we are given choices for the number of sheep in the flock; all we have to do is try them out until one gives us the right number of distinct groups when choosing 4 out of the flock. Start with choice (C). If there are 7 sheep in the flock, then we need to find out how many ways we can choose 4 of 7 sheep when order does not matter. The math would look like: $\frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1}$. Since this yields 35, there must be fewer than 7 sheep. Try a smaller number. Six works.

LESSON 5

CRITICAL REASONING 2

There are a few types of critical reasoning questions that do not require you to identify the conclusion, premise, and gap. It is particularly important to identify these question types before you begin working the question.

INFERENCE QUESTIONS

Inference questions ask you to infer or conclude something based on the passage. In other words, you have to assume that the information in the passage is true and find the answer that is a true statement. You will rely heavily on POE when you work inference questions. For each answer choice, ask yourself, “Must this be true?” If the answer is no, eliminate that choice.

Let’s review what you learned about inference questions in the pre-class assignment.

Step 1: Read and identify the question.

Inference questions typically ask:

- Which of the following can be inferred from the information above?
- Which of the following conclusions is best supported by the passage?
- Which of the following conclusions could most properly be drawn from the information above?
- Which of the following must be true on the basis of the statements above?

Step 2: Work the argument.

Read the passage. Don’t look for the conclusion and premises.

Step 3: Predict what the answer should do.

Most of the time, you will not be able to come up with an answer in your own words. Simply keep in mind that you want the answer best supported by the facts.

Step 4: Use POE to find the answer.

Look for answer that **must be true**. Avoid answers that:

- Go Beyond the Information Given (B.I.G). Eliminate choices that bring in new information, require outside knowledge, or need additional assumptions.
- Go against information in the passage.
- Are broader or more extreme than the passage. Correct inference answers frequently use words such as *may*, *might*, or *sometimes*. Avoid answers that include strong words such as *all*, *must*, or *never*, unless they are clearly supported by the argument.
- Could be true but cannot be proven using the facts in the passage.

1. According to a recent study, fifteen corporations in the United States that follow a credo of social responsibility are also very profitable. Because of their credos, these fifteen corporations give generously to charity, follow stringent environmental protection policies, and have vigorous affirmative action programs.

Which of the following can be correctly inferred from the statements above?

- ☐ Following a credo of social responsibility helps to make a corporation very profitable.
- ☐ It is possible for a corporation that follows a credo of social responsibility to be very profitable.
- ☐ A corporation that gives generously to charity must be doing so because of its credo of social responsibility.
- ☐ Corporations that are very profitable tend to give generously to charity.
- ☐ Corporations that have vigorous affirmative action programs also tend to follow stringent environmental protection policies.

2. The two divisions of a high-tech company have performed quite consistently over the past five years. In each year, the telecommunications equipment division accounted for 35 percent of profits and 15 percent of revenues, and the high-speed internet division made up the balance.

Which of the following can properly be inferred regarding the past five years from the information above?

- ☐ The telecommunications equipment division has made higher profits per dollar than the high-speed internet division.
 - ☐ Sales for both divisions have remained flat over the five years.
 - ☐ The high-speed-internet market involved tougher competition than the telecommunications equipment market during the past five years.
 - ☐ Management devoted a greater number of company resources to the telecommunications equipment division than to the high-speed internet division over the past five years.
 - ☐ More profitable products made up a higher percentage of the products offered by the telecommunications division.
3. A combination of anxiety and external pressure leads to nausea. All the auditioners for the new reality show *Wanna Be's* suffer from external pressure. Some of the auditioners feel anxiety about performing well for the producers, but others do not feel anxious. The producers of *Wanna Be's* like to choose contestants who feel anxiety.

Which of the following conclusions is most strongly supported by the passage above?

- ☐ The auditions for *Wanna Be's* cause more performers to feel external pressure than do auditions for other shows.
- ☐ Most of the people who audition become contestants on *Wanna Be's*.
- ☐ There is more nausea among auditioners for *Wanna Be's* than among producers of *Wanna Be's*.
- ☐ No auditioner who doesn't feel anxiety has nausea.
- ☐ Most of the auditioners for *Wanna Be's* who become contestants have nausea.

4. In broad thermodynamic terms, the distinction between solar energy and energy derived from fossil fuel is artificial. Fossil fuel molecules represent the decayed remains of plants. All of the energy these fossil fuels contain once resided in the sun and was, so to speak, trapped by plants here on Earth through the process of photosynthesis, whereupon it was housed, principally, within the carbohydrate molecules of which the plants were composed. The process of burning unleashes that energy, and when we run our lights, factories, and automobiles by burning fossil fuels _____.

Which of the following is the most logical completion of the passage above?

- ☐ we deplete our stores of an ever more precious resource
- ☐ we use energy that is, in fact, derived from the sun
- ☐ we spend the legacy left to us by our prehistoric ancestors
- ☐ we mimic the process of energy generation that exists in the sun
- ☐ we return to the sun that which originally resided there

RESOLVE/EXPLAIN QUESTIONS

Some questions ask you to resolve an apparent paradox or explain a discrepancy. On the GMAT, a paradox is a seemingly contradictory pair of facts that is explained by one of the answer choices.

Let's review what you learned about resolve/explain questions in the pre-class assignment:

Step 1: Identify the question.

Look for key words like *resolve* and *explain*.

Step 2: Work the argument.

Read the passage. Identify the facts in conflict. Look for words like *but*, *yet*, and *however* to find the paradox.

Step 3: Predict what the answer should do.

Phrase the question that the correct choice will answer. Ask, "Why X but also Y"?

Step 4: Use POE to find the answer.

The correct answer will provide additional information that allows both facts to be true and clears up the paradox. Avoid answers that:

- Do nothing to clear up the conflict.
- Make the conflict worse.
- Address only one side of the conflict.

1. In September of last year, the number of people attending movies in theaters dropped precipitously. During the next few weeks after this initial drop the number of filmgoers remained well below what had been the weekly average for the preceding year. However, the total number of filmgoers for the entire year was not appreciably different from the preceding year's volume.

Which of the following, if true, resolves the apparent contradiction presented in the passage above?

- ☐ People under the age of 25 usually attend films in groups, rather than singly.
- ☐ The gross income from box office receipts remained about the same as it had been the preceding year.
- ☐ For some portion of last year, the number of people attending movies in theaters was higher than it had been during the previous year.
- ☐ The number of people attending movies in theaters rises and falls in predictable cycles.
- ☐ The quality of films released in September and October of last year was particularly poor.

2. Although the mathematical validity of the laws of probability is indisputable, most people do not trust the dictates of these laws. Even among people who claim to have studied probability theory, for instance, a majority express a greater fear of flying on commercial airlines than of driving an automobile on our nation's highways, despite the fact that the probability that one would suffer an automobile-related death or injury by choosing to drive is more than twenty times the probability of an airline-related death or injury if one chooses to fly.

Which one of the following, if true, provides the best explanation for people's mistrust of the laws of probability in the case described above?

- ☐ A complete understanding of the laws of probability requires a thorough knowledge of advanced statistical analysis techniques.
- ☐ People who studied probability theory in an academic environment may be ill equipped to apply that knowledge to real-world situations.
- ☐ People tend to suspend their belief in probability when they feel somewhat in control of their own fate.
- ☐ The probability of automobile-related injury or death is not significant enough to dissuade many people from driving.
- ☐ The greatest risk to the individual driver in terms of automobile-related injuries or fatalities are the actions of the other drivers on the road.

3. In France, the nuclear-generated electricity potential—the approximate peak output ability of France’s nuclear power plant network—is twice what it was fifteen years ago. Yet during this period no new nuclear power plants were built in France; in fact, five plants were decommissioned, due to various spending and safety concerns.

Which one of the following, if true, best explains the discrepancy described above?

- ☐ Over the last fifteen years, the demand for electricity in France has outstripped the supply available from domestic sources.
- ☐ The current rate of increase in domestic demand for electricity in France is not as great as it was fifteen years ago.
- ☐ The planned expansion of the nuclear power plant network in France has been held up due to increased concerns over the safety of such facilities.
- ☐ The price of domestically generated electricity in France has declined significantly over the last fifteen years.
- ☐ The average French power plant has the potential to generate more electricity than it did earlier, due to advances in nuclear power-generation technology over the last fifteen years.

MINOR QUESTION TYPES

A few other question types occasionally appear on the GMAT. To answer these questions, you'll need to break down the arguments and understand their structures.

Evaluate-the-Argument Questions

These questions deal with the assumptions in an argument. Here's an example of an evaluate-the-argument question:

1. Ergonomically designed computer keyboards tend to lose their "play"—the responsiveness of the keys—more quickly than do traditional keyboards. A software designer has suggested that it is in fact the curvature of the key rows and not increased typing speed that is to blame. Due to the bent shape of the board, it is more difficult for the average user to clean between the keys, resulting in a gradual deadening of the spring mechanisms.

The answer to which of the following questions will most likely yield significant information that would help to evaluate the software designer's hypothesis?

- ☐ Do traditional keyboards and ergonomically designed keyboards utilize the same plastics?
- ☐ Does sprinkling a keyboard with dust impede the spring action beneath the keys?
- ☐ Does a keyboard with deadened play make typing more difficult?
- ☐ Do computer manufacturers receive more complaints about ergonomically designed keyboards than about traditional keyboards?
- ☐ Are software designers more likely than other users to utilize their keyboards when working?

Here are the steps for working evaluate-the-argument questions:

Step 1: Read and identify the question.

Evaluate-the-argument questions often use words such as *evaluate* or *assess*. Also, the answer choices are often phrased as questions.

Step 2: Work the argument.

Identify the conclusion, premises, and gap.

Step 3: Predict what the answer should do.

Think about what you would need to know to fill in the gap. The correct answer will deal with one of the assumptions.

Step 4: Use POE to find the answer.

The format of the answer choices can be confusing. To alleviate this problem, try turning the questions into answers. The correct answer will either weaken or strengthen the argument. Eliminate the obviously wrong answers and choose the best answer from the ones that are left.

2. The recent surge in fear over the virulence of the Ebola virus is irrational and unfounded. While in 1996 only 66 deaths worldwide were directly attributed to Ebola, some 603 deaths were caused by the influenza virus in the United States alone in that same year. Yet no such hysteria has surrounded influenza, despite the significantly higher number of fatalities.

Which of the following pieces of information would be most useful in evaluating the logic of the argument presented above?

- ☐ The geographical distribution of deaths directly attributable to the Ebola virus
- ☐ The incubation periods for a range of tropical viral diseases
- ☐ The probable cause of an outbreak of infection by the influenza virus in selected regions of the United States
- ☐ The relative survival rates for individuals infected with the Ebola virus and for those infected with the influenza virus
- ☐ The numbers of deaths attributable to other, non-viral diseases with similar origins

Parallel-the-Reasoning Questions

A parallel-the-reasoning question is really six questions in one. You not only need to identify the reasoning in the argument, but you must also identify the reasoning in each of the five answer choices. Fortunately, these questions rarely appear on the GMAT.

Here's an example:

1. If we reduce the salaries of our employees, then profits will increase by 35 percent. Because we must increase our profits, it is clear that employee salaries must be reduced.

Which of the following most closely parallels the reasoning used in the argument above?

- ☐ If I eat less food, I will lose weight. Since I started skipping breakfast, I have lost ten pounds.
- ☐ If I work four more hours each week, I will earn enough money to afford a new hobby. Because I would like a new hobby, I will collect coins.
- ☐ If God does not exist, then there is no basis for morality. Because some actions are morally wrong, God must exist.
- ☐ If there is an economic recession, then salaries will be reduced. Because salaries are not decreasing, there is not an economic recession.
- ☐ If there were more commercials, the number of television watchers would decline. Because the number of television watchers should be reduced, the number of commercials should be increased.

MATH 5

Geometry on the GMAT is mostly a matter of knowing some facts and formulas and learning how to apply them. Even the toughest geometry problems test fairly basic ideas, but they layer a lot of them together and combine them in unusual ways.

RIGHT TRIANGLES

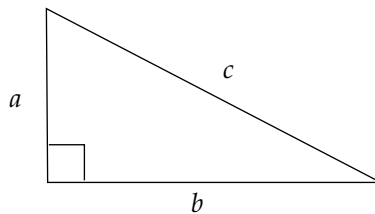
As you learned, a right triangle is a triangle that contains a 90° angle. Right triangles obey the same rules that all triangles obey, but they also have some special properties.

The relationship among the sides of any right triangle is expressed by the Pythagorean Theorem. If you know two of the sides of any right triangle, you can always find the third with the Pythagorean Theorem.

If a and b are the lengths of the legs, and c is the length of the hypotenuse, then:

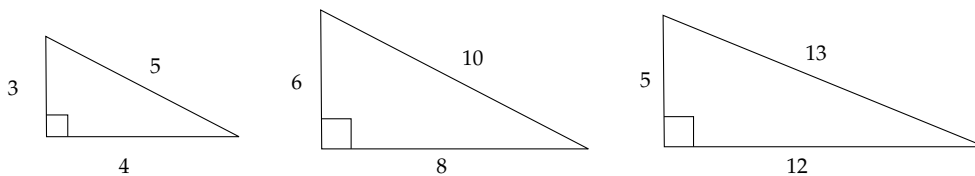
$$a^2 + b^2 = c^2$$

The Pythagorean Theorem applies only to right triangles.

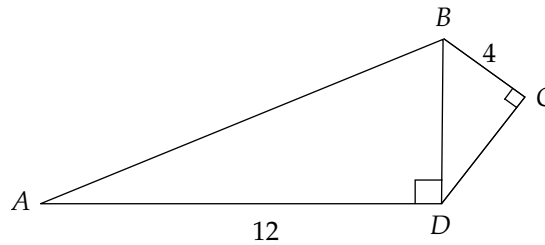


Common Right Triangles

Certain right triangles show up quite frequently in GMAT problems. Any set of three numbers that satisfy the Pythagorean Theorem—and can therefore be the lengths of the sides of a right triangle—is called a *Pythagorean triple*. So, 3: 4: 5, 6: 8: 10, and 5: 12: 13 are Pythagorean triples.

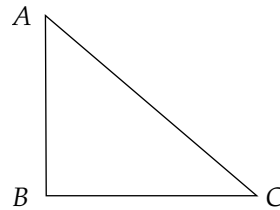


Memorize these triangles and keep an eye out for them on the test. Recognizing them will save you time and effort. Notice that the 6: 8: 10 triangle is just a multiple of the 3: 4: 5. Pythagorean triples are ratios, so other multiples of these triangles will work too.



1. In the figure above, if the area of triangle ABD is 30, what is the sum of AB and BC ?

- ☐ 5
☐ 8
☐ 12
☐ 16
☐ 19

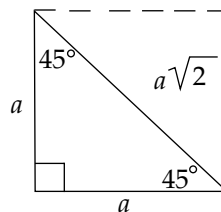


2. What is the area of triangle ABC ?
- (1) $AB = 3$ and $AC = 5$
- (2) B is a right angle.

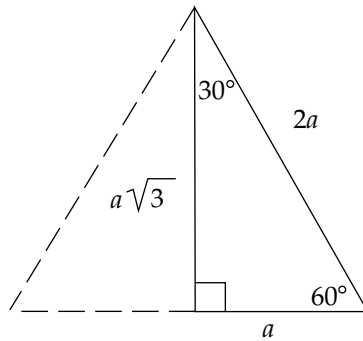
There are two other common right triangles you should know:

- The $45^\circ: 45^\circ: 90^\circ$ right triangle. The ratio of the sides is $a: a: a\sqrt{2}$.
- The $30^\circ: 60^\circ: 90^\circ$ right triangle. The ratio of the sides is $a: a\sqrt{3}: 2a$.

Memorize the relationships among the sides:



A $45^\circ: 45^\circ: 90^\circ$ triangle is also known as an isosceles right triangle. If you split a square diagonally, you create two $45^\circ: 45^\circ: 90^\circ$ triangles.



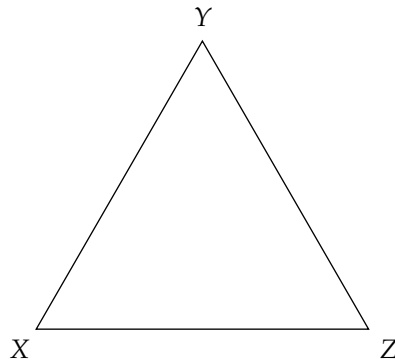
Splitting an equilateral triangle in half creates two $30^\circ: 60^\circ: 90^\circ$ triangles.

To apply the Pythagorean Theorem, you need to know the lengths of two sides. If you know the angle measures of a $45^\circ: 45^\circ: 90^\circ$ or $30^\circ: 60^\circ: 90^\circ$ triangle, you only need to know *one* side to figure everything else out.

Sometimes it's helpful to use the approximate values of $\sqrt{2}$ and $\sqrt{3}$ to estimate answers:

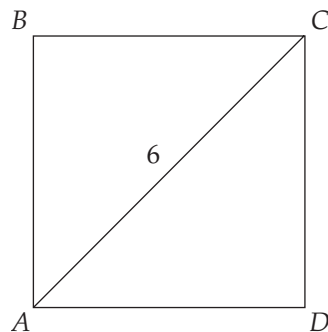
- $\sqrt{2} \approx 1.4$
- $\sqrt{3} \approx 1.7$

Hint to remember the ratios: The $45^\circ: 45^\circ: 90^\circ$ triangle has 2 distinct angles, so it goes with $\sqrt{2}$. The $30^\circ: 60^\circ: 90^\circ$ triangle has 3 distinct angles, so it goes with $\sqrt{3}$.



3. Triangle XYZ in the figure above is an equilateral triangle. If the perimeter of the triangle is 12, what is its area?

- ☐ 4
☐ $4\sqrt{3}$
☐ 8
☐ 12
☐ $8\sqrt{3}$

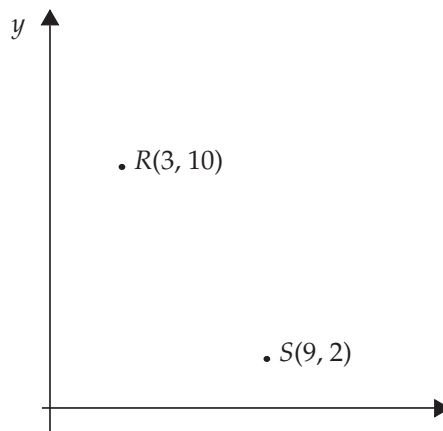


4. In the figure above, what is the area of square $ABCD$?

☐ 4
☐ 10
☐ 12
☐ 18
☐ 24

COORDINATE GEOMETRY

Many coordinate geometry problems are really about right triangles. If you need to determine the length of a line that's not parallel to the x -axis or the y -axis, turn it into a right triangle problem.



1. In the coordinate grid above, what is the distance between point R and point S ?

☐ 10
☐ $6\sqrt{3}$
☐ 12
☐ $9\sqrt{2}$
☐ 20

2. In a coordinate grid, if the points $D(-1, -1)$, $E(-1, 1)$, $F(a, 1)$, and $G(a, -1)$ are the vertices of a rectangle with a diagonal length of $2\sqrt{5}$, then what is the value of a ?

- ☐ 3
- ☐ 5
- ☐ $3\sqrt{5}$
- ☐ 9
- ☐ $7\sqrt{3}$

Slope

Slope is the measure of the steepness of a line. The steeper the line, the greater the absolute value of the slope. Slope is calculated by taking the vertical change between any two points on the line, and dividing it by the horizontal change between those two points. This is often called putting the *rise* over the *run*.

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Vertical Change}}{\text{Horizontal Change}} = \frac{y_2 - y_1}{x_2 - x_1}$$

where (x_1, y_1) and (x_2, y_2) are points on the line

It doesn't matter which coordinate you call y_2 or y_1 as long as you use the corresponding x_2 and x_1 .

3. What is the slope of the line that passes through the points $(1, 4)$ and $(5, 2)$?

- ☐ -2
- ☐ $-\frac{1}{2}$
- ☐ $\frac{1}{2}$
- ☐ $\frac{2}{3}$
- ☐ 2

Line Equations

For some problems you need to know the general equation of a line. All the points on any non-vertical line must fit the *line equation*, which is usually expressed in the following form:

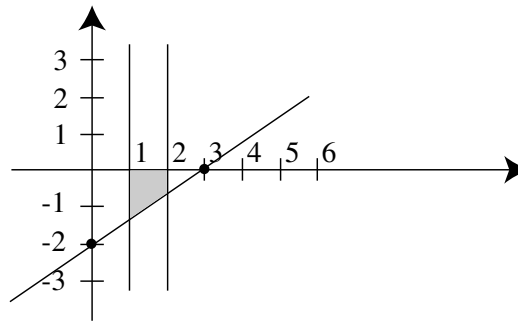
$$y = mx + b$$

x, y = variables that stand for the coordinates of any point on the line

m = slope of the line

b = y -intercept = y -coordinate of the point $(0, b)$ where the line crosses the y -axis

For example, the line with the equation $y = \frac{4}{3}x + 5$ crosses the y -axis 5 units above the origin and has slope $\frac{4}{3}$.



4. In the coordinate system shown above, the shaded region is bounded by straight lines. Which of the following is an equation of one of the boundary lines?

☐ $y = 1$

☐ $y = 2$

☐ $x = 0$

☐ $y = \frac{2}{3}x - 2$

☐ $y = \frac{3}{2}x - 2$

5. What is the x -intercept of the line defined by the equation $y = 4x + 5$?

☐ (5, -4)

☐ $(4, \frac{5}{4})$

☐ $(-\frac{5}{4}, 0)$

☐ $(\frac{4}{5}, 0)$

☐ $(0, -\frac{5}{4})$

At the point where a line crosses the y -axis, the x -coordinate is 0. At the point where a line crosses the x -axis, the y -coordinate is 0.

SOLID GEOMETRY

Let's look at a few ways the GMAT might test you on three-dimensional figures.

1. What is the greatest distance (in inches) between any two corners of a rectangular box with dimensions of 6 inches, 8 inches, and 10 inches?

☐ 10 inches

☐ 12 inches

☐ $10\sqrt{2}$ inches

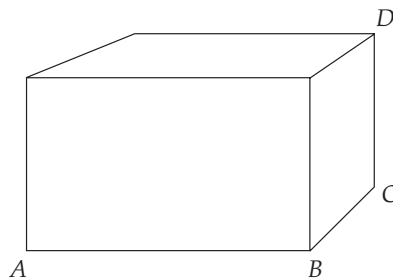
☐ $10\sqrt{3}$ inches

☐ 24 inches

The diagonal (i.e., the longest distance between any two corners of a rectangular solid) can be found using the following formula:

$$a^2 + b^2 + c^2 = d^2$$

where a , b , and c are the dimensions (length, width, and height) of the figure.

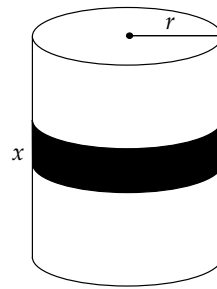


2. In the rectangular solid above, if $AB = 5$, what is the surface area of the solid?

(1) $BC = 8$

(2) The volume of the solid is 80.

Remember that figures may not be drawn to scale in data sufficiency problems.

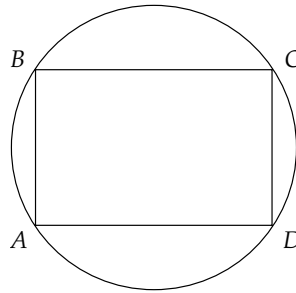


3. A rectangular label of width x has been wrapped around the cylinder above, encircling the cylinder without overlap. If the radius of the cylinder is 6, and the label has the same area as the base of the cylinder, then what is the value of x ?

☐ 3
☐ 5
☐ 6
☐ 6π
☐ 9π

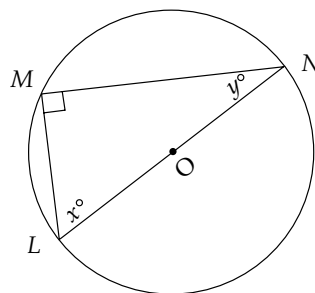
OVERLAPPING FIGURES

Some geometry problems involve multiple geometric shapes that overlap or are inscribed in one another. The question typically tells you something about one of the shapes and then asks you something about the other. The key to these problems is to figure out what the two shapes have in common.



1. Rectangle $ABCD$ has length 8 and width 6. What is the area of the circle?

☐ 10π
☐ 25π
☐ 50π
☐ 64π
☐ 100π

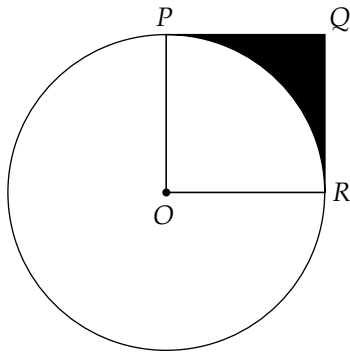


2. If the area of the circle above with center O is 64π , what is the area of triangle LMN ?

(1) $x = 2y$
 (2) $OL = LM$

SHADED REGIONS

Sometimes the GMAT asks you to find the area of a weird-looking region. The key to shaded-region questions is to focus on the non-shaded part. By subtracting the non-shaded portion from the total, you will arrive at the area of the shaded portion.



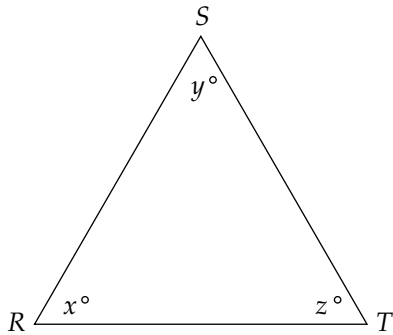
Weird shapes and figures are usually constructed by combining ordinary shapes in odd ways. Look for the shapes with which you're familiar.

1. In the figure above, $OPQR$ is a square. If O is the center of the circle, and the distance between point P and point R is $4\sqrt{2}$, what is the area of the shaded region?

- ☐ $16 - 8\pi$
- ☐ $16 - 4\pi$
- ☐ $32 - 8\pi$
- ☐ $32 - 2\pi$
- ☐ $64 - 12\pi$

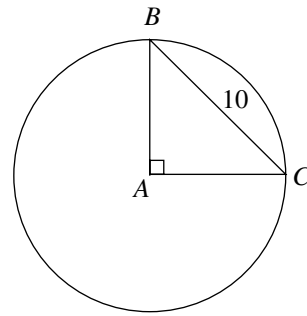
Shaded Area = Total Area - Unshaded Area

DRILL



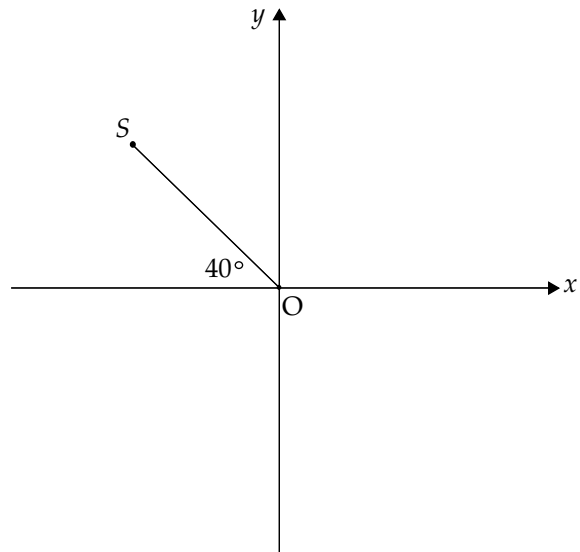
1. In triangle RST in the figure above, does $RS = RT$?
 - (1) $y = 180 - 2x$
 - (2) $x + y = y + z$
2. A rectangular room is 12 feet long, 8 feet wide, x feet high, and has a volume of 1152 cubic feet. What is the greatest distance (in feet) between the center of the ceiling and any other point in the room?

- ☐ $8\sqrt{2}$
- ☐ 12
- ☐ 14
- ☐ $12\sqrt{2}$
- ☐ 20



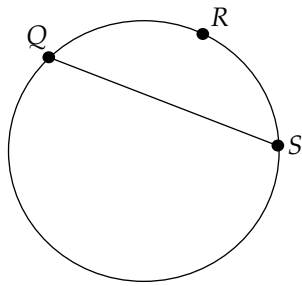
3. In the figure above, $\angle A = 90$ and the length of chord $\overline{BC} = 10$. Point A is the center of the circle. What is the area of the circle?

- ☐ 10π
- ☐ $10\sqrt{3}\pi$
- ☐ 25π
- ☐ 50π
- ☐ 100π



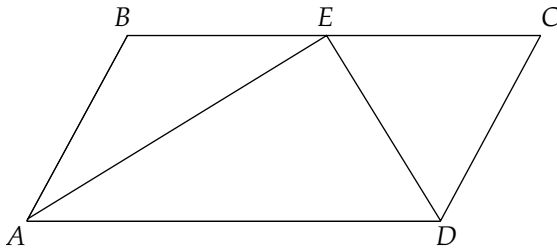
4. In the coordinate system above, the length of OS is 6. Which of the following must be true?
 - I. The x -coordinate of point S is greater than -6.
 - II. The slope of OS is greater than -1.
 - III. The y -coordinate of point S is greater than -6.

- ☐ I only
- ☐ I and II only
- ☐ I and III only
- ☐ II and III only
- ☐ I, II, and III



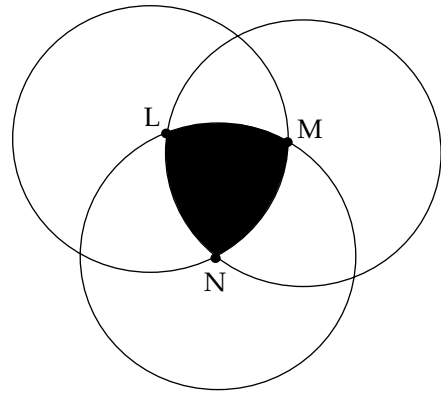
5. What is the circumference of the circle above?
- (1) The length of arc QRS is one-sixth of the circumference.
- (2) The length of chord QS is 6.
6. The points (p, q) and $(p - 2, q + c)$ are both on the line expressed by the equation $y = 2x + 5$. What is the value of c ?

- ☐ -4
- ☐ -1
- ☐ 1
- ☐ 2
- ☐ 4



7. In the parallelogram $ABCD$ above, $AD = 12$. If the area of triangle ABE is $\frac{3}{8}$ the area of parallelogram $ABCD$, then what is the length of EC ?

- ☐ $\frac{3}{2}$
- ☐ 2
- ☐ $\frac{8}{3}$
- ☐ 3
- ☐ 4



8. In the figure above, L , M , and N are the centers of three circles, each with radius 4. What is the perimeter of the shaded region?

- ☐ π
- ☐ $\frac{4\pi}{3}$
- ☐ $\frac{8\pi}{3}$
- ☐ 4π
- ☐ 6π

ANALYSIS OF AN ARGUMENT

Remember the basic approach discussed in the pre-class assignment:

- Step 1:** Brainstorm
- Step 2:** Outline
- Step 3:** Write
- Step 4:** Finish

Your task is to discuss how convincing you find the argument's line of reasoning and the evidence supporting it. You also have the opportunity to suggest how the argument could be made convincing. You'll receive a high score if you do the following:

1. Clearly identify the important features of the argument: conclusion, premises, and assumptions.
2. Critique the author's logic and assumptions, but not the conclusion. Show that the assumptions could be wrong or that some key term is not adequately defined.
3. Suggest ways that the author could improve the argument. Explain the type of evidence that is needed to support the assumptions and fill in the gaps.
3. Do not give your opinion on the truth of the conclusion; that's what the Analysis of an Issue essay is all about. Stick to the logic of the argument or you won't get a good score.

What does that really mean to you? Well, you finally have a chance to respond to their bogus arguments with your own thoughts and suggestions about what makes the argument bad.

Now, let's take a look at what the Argument essay is supposed to look like.

THE TEMPLATE

All Argument essay topics are constructed in similar fashion. All Argument essays should follow a standard format as well. Let's look at a good way to write an Analysis of an Argument essay.

Introduction

Your introduction should quickly summarize the conclusion of the argument and present your position on the logical soundness of the argument. For example:

The argument that (summary of argument) omits some important concerns that must be addressed to substantiate the argument.

The author states that (restate argument). The conclusion is based upon assumptions that seem to be unsupported.

(Restate argument). This premise does not constitute a strong logical argument with reasonable support or proof for the main argument.

It's just that straightforward. This kind of writing may not look fancy, but at the same time, remember: You're not trying to get published, you're trying to express yourself clearly enough that you improve your chances of getting into the B-school of your choice, not someone else's.

Body Paragraphs

Now take the two–four strongest points you have brainstormed, and turn each into a paragraph. Make sure to use standard essay format and transition words. Each point you make should have about two sentences to support it. Assess the plausibility of the author's assumption. Give some details or an example to support your assessment. You can also show how the author could improve the argument. Try using constructions such as these:

The argument does not address the possibility that...

The argument assumes that (assumption). It is possible, however, that...

Conclusion

Write a strong conclusion paragraph. It should definitely begin with something like:

In conclusion, ...

The above essay clearly demonstrates that...

Summarize your critiques, and suggest ways for improvement. On this latter part, keep in mind that you need to have a certain "diplomatic tone" about your criticism. Be constructive, not destructive, in your criticism. Note how the essay in the pre-work points out not only what's wrong with the author's argument, but also how the author could improve the argument. It's important that your essay come across this way as well if you want to improve your chances for a better score in the AWA.

TEST SMARTS

Adjust your approach to your place in the exam.

1. A circle is inscribed in triangle ABC such that point D lies on the circle and on line segment AC , point E lies on the circle and on line segment AB , and point F lies on the circle and on line segment BC . If line segment $AB = 6$, what is the area of the figure created by line segments AD , AE , and minor arc DE ?

☐ $3\sqrt{3} - \frac{9}{4}\pi$

☐ $3\sqrt{3} - \pi$

☐ $6\sqrt{3} - \pi$

☐ $9\sqrt{3} - 3\pi$

☐ It cannot be determined from the information given.

Before you dive into a question, ask yourself:

- Where am I on this test?
- How should I approach this question?

Questions 1–10

Questions 11–30

Questions 31–37

PACING: THE MIDDLE

Work the middle questions a little more quickly than the first ten questions, but not as quickly as those at the end of the exam.

MATH

Question numbers

| Score | 1–10 | 11–20 | 21–30 | 31–37 |
|-----------------|---------|---------|---------|---------|
| Under 35 | 30 min. | 25 min. | 15 min. | 5 min. |
| 35–42 | 30 min. | 20 min. | 15 min. | 10 min. |
| Above 42 | 25 min. | 20 min. | 20 min. | 10 min. |

VERBAL

Question numbers

| Score | 1–10 | 11–20 | 21–30 | 31–41 |
|-----------------|---------|---------|---------|---------|
| Under 28 | 30 min. | 25 min. | 10 min. | 10 min. |
| 28–34 | 27 min. | 20 min. | 18 min. | 10 min. |
| Above 34 | 25 min. | 20 min. | 15 min. | 15 min. |

Your accuracy during the middle of a section still plays a large part in determining your final score. It will not affect your score as much as the first part of the test, but questions here have a bigger impact than do those in the last part of the test.

Your pace should be similar to the pace at which you complete your homework. Move at a pace that allows you to read and work the questions carefully, but don't spend too long on any one question.

Most likely, you won't need to pick up your pace on every question you do. Instead, avoid time sucks. Focus on the questions you know how to do, not the questions you don't.

Finally, think about your pacing for the entire group of questions. Focusing on an ideal time per question is a surefire way to increase anxiety, not raise your score. Some questions take more time to solve and some take less. That's why we provide guidelines for how long sets of questions should take rather than time per question.

TEST 3 GOALS

Now that you're becoming comfortable with the CAT format, work to increase your accuracy. Remedy any problems from your first CAT test.

Here are a few goals:

- Reduce the number of careless mistakes in the first 10 questions in each section.
- Increase accuracy in the middle of the exam. Don't get stuck on killer questions. Instead, focus on what you know. Try to do so within the suggested time frame.
- Make sure to eliminate traps and ballpark rather than blindly guessing.
- Based on your analysis of your last test, make any other necessary adjustments.

TEST ANALYSIS

Pacing

As you take the exam, look at the clock after you complete every 10 questions. Note the time remaining in the spaces below.

Math

#10 _____

#20 _____

#30 _____

Verbal

#10 _____

#20 _____

#30 _____

Accuracy

After you complete the test, look at the score report. Count the number of questions right and wrong for the entire section. Then, count the number wrong for each portion of the exam.

| Test Analysis | |
|---------------------------------|--|
| Overall Score | |
| Math Score | |
| Verbal Score | |
| Math Section | |
| Problem solving % correct | |
| Data sufficiency % correct | |
| Number Right in questions 1–10 | |
| Number Right in questions 11–20 | |
| Number Right in questions 21–30 | |
| Number Right in questions 31–37 | |
| Verbal Section | |
| Sentence Correction % correct | |
| Critical Reasoning % correct | |
| Reading Comprehension % correct | |
| Number Right in questions 1–10 | |
| Number Right in questions 11–20 | |
| Number Right in questions 21–30 | |
| Number Right in questions 31–41 | |

Content

Review the questions you missed and complete the log below. First, note the question number and question format (problem solving, data sufficiency, sentence correction, reading comprehension, or critical reasoning). Second, write down the question type/topic, the concepts or skills tested by the question. For example, a math question might test ratios, a sentence correction question might test idioms, or a critical reasoning question might test strengthening an argument. Finally, use the online explanations to determine why you missed the question. You might have made an error in reading the problem, performing calculations, or using a technique.

[illegible]

Action Plan

What adjustments will you make to your pacing for the next test?

Which techniques do you need to practice?

Do you need to review any content?

Do you have any questions for your instructor?

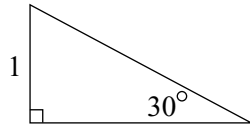
Bring your score report to your next class. If there were any problems you couldn't figure out after reviewing the explanations, print them out and bring them to class or extra help.

HOMework REVIEW

Use this chart to note any questions you have from the reading or examples in the homework.

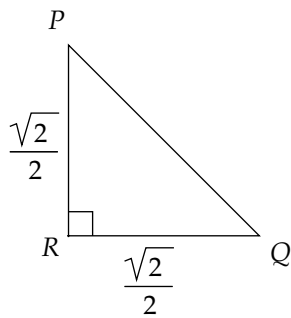
[illegible]

PRACTICE



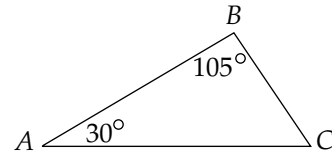
1. What is the perimeter of the triangle above?

- ☐ $\frac{3+\sqrt{3}}{2}$
☐ 3
☐ $2+\sqrt{2}$
☐ $3+\sqrt{3}$
☐ $3+\sqrt{5}$



2. In $\triangle PQR$ above, $PQ =$

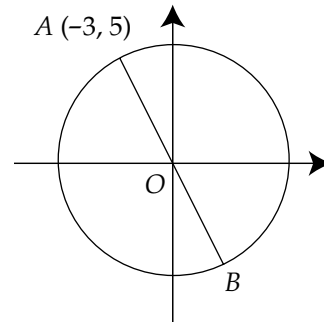
- ☐ $\frac{1}{2}$
☐ $\frac{\sqrt{2}}{2}$
☐ 1
☐ $\sqrt{2}$
☐ 2



Note: Figure not drawn to scale.

3. In the triangle above, if $BC = 4\sqrt{2}$, then what is the area of $\triangle ABC$?

- ☐ 64
☐ $16+16\sqrt{3}$
☐ $8+8\sqrt{3}$
☐ $8+4\sqrt{2}$
☐ $8\sqrt{2}$



4. In the figure above, AOB is the diameter of a circle centered at O . If the coordinates of A are $(-3, 5)$, then the coordinates of B are

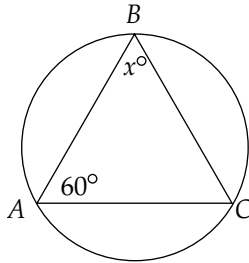
- ☐ $(-3, -5)$
☐ $(-3, 5)$
☐ $(3, -5)$
☐ $(3, 5)$
☐ $(5, 3)$

5. A circle with center $(0, 0)$ and radius 8 will pass through all of the following points EXCEPT

- ☐ $(-8, 0)$
☐ $(0, -8)$
☐ $(0, 8)$
☐ $(8, 0)$
☐ $(8, 8)$

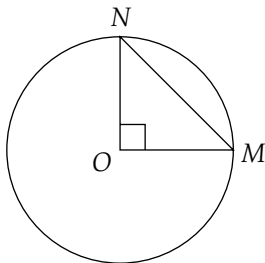
6. The slope of a line containing points $(2, -3)$ and $(4, p)$ is -1 . What is the value of p ?

☐ -6
☐ -5
☐ 1
☐ 5
☐ 12



7. In the figure above, if A , B , and C are points on the circle, and if $AB = AC$, then what is the value of x ?

☐ 40
☐ 45
☐ 50
☐ 55
☐ 60



8. In the figure above, if $MN = 2$, then the area of the circle with the center O is

☐ $\frac{\pi}{4}$
☐ $\frac{\pi}{2}$
☐ π
☐ 2π
☐ 4π

9. A square is inscribed in a circle with area πr^2 . What is the area of the square?

☐ $\frac{r^2}{2}$
☐ $\sqrt{2}r^2$
☐ r^2
☐ $2r^2$
☐ $4r^2$

10. The length of an edge of cube A is 5% greater than the length of an edge of cube B. If the volume of cube B is 27 cubic centimeters, then what is nearest to the volume of cube A?

☐ 23.1
☐ 25.65
☐ 27.125
☐ 28.35
☐ 31.25

11. For the line whose equation is $\frac{y+2-b}{x} = m + \frac{2}{x}$, m is not zero. If the line is rotated 90° , then the slope of that line would be

☐ $\frac{1}{m}$
☐ $-\frac{1}{m}$
☐ m
☐ $-m$
☐ $m - 2$

12. A sphere with a radius of 5 is hollowed out at the center. The part removed from the sphere has the same center, and a radius of 3. What fractional part of the original sphere remained? (The formula

for the volume of a sphere is $\frac{2}{5}V = \frac{4}{3}\pi r^3$.)

☐ $\frac{2}{5}$
☐ $\frac{16}{25}$
☐ $\frac{27}{125}$
☐ $\frac{98}{125}$
☐ $\frac{3}{5}$

13. A certain cube floating in a bucket of water has between 80 and 85 percent of its volume below the surface of the water. If between 12 and 16 cubic centimeters of the cube's volume is above the surface of the water, then the length of a side of the cube is approximately

☐ 4
☐ 5
☐ 7
☐ 8
☐ 9

14. In the rectangle coordinate system, triangle ABC has a vertex at point $(0, 56)$. If point B is at the origin, then how many points on line AC have integer values for both their x and y values?

- (1) The third vertex of triangle ABC lies on the x -axis, and the triangle has an area of 196.
 (2) Point A has a positive x -coordinate and a y -coordinate of zero.

ANSWERS AND EXPLANATIONS

1. **D** In the 30 : 60 : 90 triangle, the hypotenuse equals 2 and the long leg (the base) equals $\sqrt{3}$.
2. **C** It's a 45 : 45 : 90 triangle. The hypotenuse equals $\frac{\sqrt{2}}{2} \times \sqrt{2}$, or 1.
3. **C** When you drop a perpendicular from point *B* to side *AC* (call that point *D*), you create a 30:60:90 triangle ($\triangle ABD$) and a 45:45:90 triangle ($\triangle BCD$). Since $BC = 4\sqrt{2}$, $BD = AC = 4$. Now look at the 30 : 60 : 90 triangle: Since $BD = 4$, $AD = 4\sqrt{3}$. The base $AC = 4 + 4\sqrt{3}$, and the height $BD = 4$. Use the area formula: $\frac{1}{2}(4 + 4\sqrt{3})(4) = 8 + 8\sqrt{3}$.
4. **C** *B* is in the quadrant where *x* is positive and *y* is negative. The only choice with a positive *x*-coordinate and a negative *y*-coordinate is (C).
5. **E** Each of the points in the wrong answer choices is 8 units from the origin. However, the point (8, 8) is $8\sqrt{2}$ units from the origin. (Draw the 45 : 45 : 90 triangle.)
6. **B** Set up the slope formula, and solve: $\frac{p - (-3)}{4 - 2} = -1$, which becomes $\frac{p + 3}{2} = -1$, making $p = -5$.
7. **E** Since $AB = AC$, $\angle B = \angle C$. The measure of $\angle A$ is 60° , so $\angle B + \angle C = 120^\circ$. Thus, $\triangle ABC$ is equilateral, and each angle measures 60° .
8. **D** $\triangle ONM$ is a 45:45:90 triangle. Since $MN = 2$, you can calculate that $ON = OM = \sqrt{2}$. The area of the circle = $\pi(\sqrt{2})^2$, or 2π .
9. **D** Draw the diagram, and make sure the square is inside the circle. Now, plug in $r = 4$. The area of the circle is 16π . The diameter of the circle is 8, which is also the diagonal of the square. Each side measures $4\sqrt{2}$, so the area of the square is **32** (target answer).
10. **E** This is a very work-intensive problem, but there is no way around the work. First, find the side for cube B, which is 3 since its volume is 27. The length of the edge of cube A is 5 percent greater than the length of the edge of cube B. Thus, the length of the edge of cube A is 105 percent of cube B. We can find the length of the side of cube A by multiplying 3 times 1.05. To find the volume of cube A, we must raise the side of cube A to the power of 3. $(3.15)^3 = 31.25$.
11. **B** Since we are given the equation of a line in a strange form, it's probably a good idea to rewrite it in the more familiar form of $y = mx + b$. Rewriting the equation given reveals that we have been given the standard line equation. Now we know the slope is *m*, and the only question is what happens to it when it is rotated 90° . The best way to determine that might be to draw the picture of a line and then rotate it 90 degrees, which would reveal that the slope has become negative, which means we can eliminate choices (A) and (C). You can also eliminate choice (E) because for that choice to be correct the slope would not only rotate but would also change its value, and that does not happen here. The last two choices have only one difference: whether rotating the line makes the slope merely negative or the negative reciprocal. Plugging In should resolve that.

12. **D** This one sounds weird, but don't let yourself be thrown off by the inclusion of the sphere. Remember, any time ETS asks you about a strange figure, they have to supply you with the relevant formula, and here they do. From here, it's a lot like a shaded region question. Start by figuring the area of the whole sphere, using the formula: It's $\frac{500}{3}\pi$. Now, figure out the volume of the smaller sphere. It's 36π . Now, look carefully at the question: We need to determine what fractional part of the original sphere remains. First, determine the volume of the remaining portion of the sphere:
- $$\frac{500}{3}\pi - 36\pi = \frac{392}{3}\pi.$$
- To determine what fractional part remains, we need to take the remaining portion and divide it by the original volume:
- $$\frac{\frac{392}{3}\pi}{\frac{500}{3}\pi} = \frac{392}{500} = \frac{98}{125}.$$
13. **A** The best way to approach this problem is to Plug In the Answers since the answers give us the side of the cube. If we start with the middle choice, (C), then we have a cube with side 7. If the cube has a side of 7, then it will have a volume of 343. We are told that between 80% and 85% of the volume is below the surface of the water, which means that between 15% and 20% of the volume is above the surface. If the volume of the cube is 343, then 20% is about 68 and 15% is about 51. Neither of these numbers is between the 12 and 16 cubic centimeters that are supposed to be above water, so clearly this can't be the answer. Since the numbers are too large, we need to try something smaller. Pick one of the smaller choices and try again.
14. **A** Start by translating the question and understanding the pieces of the puzzle given and the pieces needed. The question tells us where two vertices are, point B at $(0, 0)$ and another point at $(0, 56)$. To answer the question, we need the coordinates of the last point. Statement (1) gives us the area of the triangle, which allows us to calculate the last side of the triangle, and also states that the triangle is a right triangle because the other leg lies on the x -axis. With this information we can find the slope of AC , which allows us to answer the question. Eliminate choices BCE, and keep choices AD. Statement (2) only tells us that the other leg of the triangle is on the x -axis but not how long it is, so it's not sufficient.



Unless your instructor has told you otherwise, you should take an online CAT before Class 6.

Go to the Online Student Center at www.princetonreview.com.

If you're having trouble with any of the material you've covered so far, contact your instructor about an extra-help session.

Feedback is a good thing. Call or e-mail us with any questions or comments you may have.

LESSON 6

SENTENCE CORRECTION REVISITED

STYLE POINTS

In addition to errors of grammar, stylistic mistakes sometimes appear in sentence correction questions. These style problems are not as important as the grammatical errors, so check for them only after you have eliminated all of the grammatical errors. The correct answer *can* contain a style mistake.

Short and Sweet

If two answers both are grammatically correct, choose the shorter and simpler one.

1. The distribution of mass within the core of the Earth, like the mantle that surrounds the core, has been deduced from the orbital behavior of the Earth and the motions of satellites controlled by the Earth's gravity.
 - ☐ the mantle that surrounds the core
 - ☐ that within the mantle surrounding the core
 - ☐ that of the mantle surrounding the core
 - ☐ the mantle the core surrounds
 - ☐ the distribution of mass within the mantle that surrounds the core

Passive vs. Active Voice

If you have the choice, active voice is better than passive voice.

2. After Nixon spent months trying to counter mounting bad press and pressure from his own party, resignation was chosen by him instead of facing the impeachment process.
- ☐ resignation was chosen by him instead of facing the impeachment process
 - ☐ resignation was chosen instead of impeachment
 - ☐ resignation was the choice made by him rather than facing the impeachment process
 - ☐ he chose to resign rather than face impeachment
 - ☐ he chose resignation rather than being impeached

Redundancy

Don't be repetitive. Don't be redundant. Don't be repetitive. The reason for this is because you shouldn't say the same thing more than once.

3. If the depletion of the ozone in the upper portion of the Earth's atmosphere were to continue at its present rate, by the year 2000 the hole in the ozone layer would be at least one thousand miles wide or wider.

- ☐ If the depletion of the ozone in the upper portion of the Earth's atmosphere were to continue at its present rate, by the year 2000 the hole in the ozone layer would be at least one thousand miles wide or wider.
- ☐ Were the depletion of ozone in the upper portion of the Earth's atmosphere to continue at its present rate, by the year 2000 the hole in the ozone layer would be at least one thousand miles wide.
- ☐ Was the depletion of ozone in the upper portion of the Earth's atmosphere to continue at its present rate, by the year 2000 the hole in the ozone layer would be at least one thousand miles wide or wider.
- ☐ If the depletion of ozone in the upper portion of the Earth's atmosphere were continuing at its present rate, by the year 2000 the hole in the ozone layer would be at least one thousand miles wide.
- ☐ Should the depletion of ozone in the upper portion of the Earth's atmosphere continue at its present rate, by the year 2000 the hole in the ozone layer would be at least one thousand miles wide or wider.

SUBJUNCTIVE

Use what you learned in the pre-class assignment to work the following questions.

1. The treaty specifies that the economy of the member nations have inflation rates of less than three percent and resist external tariffs.
 - ☐ economy of the member nations have inflation rates of less than three percent and resist
 - ☐ economy of each member nation have an inflation rate of less than three percent and be resistant to
 - ☐ every economy of the member nations have an inflation rate of less than three percent and be resistant to
 - ☐ economies of each member nation have inflation rates of three percent or less and are resistant to
 - ☐ economies of every member nation have three percent or less of an inflation rate and resist
2. Implementation of the Worldwide Monetary Unit would begin as scheduled if the member nations will settle the dispute over each of the nations' voting power in monetary policy issues.
 - ☐ would begin as scheduled if the member nations will settle the dispute over each of the nations'
 - ☐ were to begin as scheduled if the member nations will settle the dispute about the nations'
 - ☐ would begin as scheduled if the member nations were to settle the dispute over each nation's
 - ☐ will begin as scheduled as the member nations settle the dispute about each nation's
 - ☐ would begin as scheduled when the member nations were settling the dispute over each nation's

HARDER SENTENCE CORRECTION QUESTIONS

The pre-class assignment discussed some common patterns that make questions difficult. Let's use what you learned about those patterns to work some tricky sentence correction questions.

- If the sentence is *hard to follow*, look for specific errors and focus on only the relevant parts of the sentence.
- If there are *several good answers*, look for subtle differences that make some answers worse than others.
- If there appear to be *no good answers*, eliminate answers for breaking known rules, not because they sound bad.

Use those concepts to work the following questions.

1. Since 1999, when Congress repealed the Glass-Steagall Act, which prohibited commercial banks that engaged in investment banking, financial services companies began to spread their activities into all areas of banking, insurance, and securities operations.
 - ☐ which prohibited commercial banks that engaged in investment banking, financial services companies began
 - ☐ forbidding commercial banks from engaging in investment banking, financial services companies have begun
 - ☐ which forbid commercial banks to engage in investment banking, financial services companies began
 - ☐ that prohibited commercial banks from engaging in investment banking, financial services companies began
 - ☐ which prohibited commercial banks from engaging in investment banking, financial services companies have begun

2. *The Gatherers*, a lecture series which discusses life among early human tribes, suggests that hunting and foraging groups be viewed as distinct from agricultural communities, rather than as part of a continuum.
- ☐ which discusses life among early human tribes, suggests that hunting and foraging groups be viewed as distinct from agricultural communities, rather than as part of a continuum
 - ☐ discussing life in early human tribes, suggests hunting and foraging groups should be viewed distinctly from agricultural communities, rather than continuously
 - ☐ that discusses life in early human tribes, suggest that hunting and foraging groups are distinct from agricultural communities, rather than as part of a continuum
 - ☐ discussing life in early human tribes, suggests that early hunting and foraging groups and agricultural communities be viewed as distinct, rather than as parts of a continuum
 - ☐ discussing life among early human tribes, suggested to view hunting and gathering groups as distinct from agricultural communities, rather than in a continuum
3. Agricultural scientists have estimated that the annual loss by erosion of arable land caused by heavy rainfall and inadequate flood controls approaches two million acres per year.
- ☐ the annual loss by erosion of arable land caused by heavy rainfall and inadequate flood controls approaches two million acres per year
 - ☐ the erosion of heavy rainfall and inadequate flood controls causes a loss of arable land approaching two million acres per year
 - ☐ erosion caused by heavy rainfall and inadequate flood controls results in a loss of arable land approaching two million acres per year
 - ☐ an annual loss approaching two million acres of arable land per year results from erosion caused by heavy rainfall and inadequate flood controls
 - ☐ annually a loss of arable land approaching two million acres per year is caused by erosion due to heavy rainfall and inadequate flood controls

4. In the United States, less than \$200 per capita is spent by the government each year to support arts and cultural institutions, such as the National Endowment for the Arts, although the amount in European countries is much greater.
- ☐ In the United States, less than \$200 per capita is spent by the government each year to support arts and cultural institutions, such as the National Endowment for the Arts, although the amount in European countries is much greater.
 - ☐ The United States government spends less than \$200 per capita annually supporting arts and cultural institutions, like the National Endowment for the Arts, although European countries spend a much greater amount.
 - ☐ Although European countries spend a much greater amount, in the United States, the government spends annually fewer than \$200 per capita to support arts and cultural institutions, including the National Endowment for the Arts.
 - ☐ Arts and cultural institutions, like the National Endowment for the Arts, are supported by the United States government spending less than \$200 per capita each year, although the amount in European countries is much greater.
 - ☐ Although European countries spend a greater amount, the United States government annually spends fewer than \$200 per capita to support arts and cultural institutions, such as the National Endowment for the Arts.

SENTENCE CORRECTION REVIEW

To maximize your score on sentence correction questions, you must be extremely well versed in identifying common errors and 2/3 splits. If you're running short on time or working on a killer question, take a few seconds to identify one error and eliminate based on it before you guess.

Make sure you know how to identify the following common errors. You should also be familiar with the corresponding rules.

Verb Tense

ID:

Rule:

Parallel Construction

ID:

Rule:

Idioms

ID:

Rule:

Misplaced Modifiers

ID:

Rule:

Pronouns

ID:

Rule:

Subject-Verb Agreement

ID:

Rule:

Also, eliminate any words that have the word "being," and if you are really in a crunch, choose the shortest answer.

MATH 6**FUNCTIONS**

As you learned in the pre-class assignment, function questions require you to follow the directions and plug the numbers into the equation.

1. The "superprime" of a number is defined as the sum of its distinct prime factors. What is the "superprime" of 40?

☐ 7
☐ 8
☐ 11
☐ 13
☐ 22

2. For every integer n , n^* is defined as the sum of all the distinct factors of n . Which of the following is equal to 10^* ?

☐ 5^*
☐ 15^*
☐ 17^*
☐ 18^*
☐ 100^*

SEQUENCES

Sequence problems are often just repeating functions. You have to pull a number through the same function over and over. The directions can be confusing, and it's tedious work, but keep track of your terms, and work through it.

Determine what place in the sequence you know and what place in the sequence you want to find.

3. A sequence of numbers satisfies the equation $A_n = 2(A_{n-1}) + 1$. If $A_4 = 10$, what is the value of A_1 ?

☐ 0.375
☐ 1.375
☐ 1.75
☐ 4.5
☐ 9

Probability (at least one x)
 $= 1 - \text{Probability (no } x \text{ at all)}$

GROUPS

For group problems, decide whether to use the group equation or the group grid.

1. A group of fourth-graders and fifth-graders are going on a field trip to the zoo. Only 20% of the students remembered to bring written permission from their parents. If 40% of the students are fourth-graders and 25% of the fourth-graders remembered to bring written permission, what percentage of the students are fifth-graders who forgot to bring written permission?

- ☐ 10%
☐ 30%
☐ 50%
☐ 60%
☐ 80%

HARDER PROBABILITY QUESTIONS

With "A or B" probabilities,
 don't double-count things
 that are both A and B.

Previously, you learned a number of basic probability concepts:

- All probabilities are between 0 and 1.
- Probabilities are part-to-whole relationships that represent
$$\frac{\text{number of outcomes you want}}{\text{number of total possible outcomes}}.$$
- Probabilities can be expressed as fractions, decimals, or percents.
- Probability (A and B) = Probability (A) \times Probability (B)
- Probability (A or B) = Probability (A) + Probability (B)

1. The integers from 1 to 100 inclusive are each written on a single slip of paper and dropped into a jar. If one slip of paper is removed at random, approximately what is the probability that the number on it is neither even nor a multiple of 3?

- ☐ 83%
☐ 67%
☐ 50%
☐ 33%
☐ 17%

The probability that something does not happen is a useful concept. Remember that the probability that x happens and the probability that x doesn't happen must add to 100%.

$$\text{Probability (not } x) = 1 - \text{Probability (} x)$$

2. The probability of rain on each of five days is $\frac{1}{6}$, except on the first day, when it is $\frac{2}{5}$, and on the last day, when it is $\frac{4}{5}$. What is the probability that rain will occur on at least one of the five days?

☐ $\frac{1}{675}$

☐ $\frac{5}{72}$

☐ $\frac{5}{27}$

☐ $\frac{22}{27}$

☐ $\frac{67}{72}$

In some cases, drawing a probability tree can help you solve the problem. It shows all the possible outcomes.

3. Marco tosses a coin four times. What is the probability that he gets heads exactly twice?

☐ $\frac{1}{4}$

☐ $\frac{3}{8}$

☐ $\frac{7}{16}$

☐ $\frac{1}{2}$

☐ $\frac{5}{8}$

HARDER PERMUTATIONS AND COMBINATIONS

Remember the permutation and combination formulas you learned earlier:

$$P = \frac{n!}{(n-r)!}$$

$$C = \frac{n!}{r!(n-r)!}$$

n = number of candidates from which to choose

r = number of items to be chosen

Tougher math problems mix permutations and combinations in a single problem or require you to adjust the formulas to account for the specifics of the problem.

If some items are restricted, deal with them first.

1. Kyle is making a CD containing 10 of his favorite songs. He has chosen two blues songs and 8 rock-n-roll songs to be on the CD. The blues songs will be the first and last songs. How many different orderings of songs on the CD are possible?

☐ 3,628,800
☐ 161,280
☐ 80,640
☐ 20,160
☐ 160

2. Katie has 9 employees that she must assign to 3 different projects. If 3 employees are assigned to each project and no one is assigned to multiple projects, how many different combinations of project assignments are possible?

☐ 252
☐ 1,680
☐ 2,340
☐ 362,880
☐ 592,704

3. Coach Miller is filling out the starting lineup for his indoor soccer team. There are 10 boys on the team, and he must assign 6 starters to the following positions: 1 goalkeeper, 2 on defense, 2 in midfield, and 1 forward. Only 2 of the boys can play goalkeeper, and they cannot play any other positions. The other boys can each play any of the other positions. How many different groupings are possible?

☐ 60
☐ 210
☐ 2,580
☐ 3,360
☐ 151,200

4. Judges will select 5 finalists from the 7 contestants entered in a singing competition. The judges will then rank the contestants and award prizes to the 3 highest ranked contestants: a blue ribbon for first place, a red ribbon for second place, and a yellow ribbon for third place. How many different arrangements of prize-winners are possible?

- ☐ 10
☐ 21
☐ 210
☐ 420
☐ 1,260

ANALYSIS OF AN ISSUE

Use the basic approach as you write your essay:

- Step 1:** Brainstorm
Step 2: Outline
Step 3: Write
Step 4: Finish

In this essay, your task is to take a position on a topic. You will either agree or disagree with the statement made by the author. Use examples and detailed explanations to support your position. You'll receive a high score if you do the following:

1. **Pick a side.** In real life, you may like to take the middle of the road and accept some of both sides, but that's not the best way to approach an Issue essay. Pick one side and argue forcefully for it.
2. **Back it up.** Come up with two to four specific reasons why your position is correct. Make each one of these reasons a separate paragraph.
3. **Get specific.** Use concrete examples to support your points. Make them sound as relevant and factual as possible. Provide some detail.
4. **Explain thoroughly.** Don't toss out a point and expect the reader to know exactly what you mean. Don't assume any prior knowledge on the part of the reader. In fact, the grader will regard your paper more highly if he or she learns something from it.

Now, let's take a look at what the Issue essay is supposed to look like.

THE TEMPLATE

All Issue essay topics are constructed in a similar fashion. All Issue essays should follow a standard format as well. Let's look at a good way to write an Analysis of an Issue essay.

Introduction

Your introduction should quickly summarize the statement and present your position (for or against). For example:

While many people feel that (summarize statement), the opposite is true. This essay will demonstrate why the latter position is correct.

The author states that (summarize statement). This position is well justified. There are several reasons that this is true.

It's just that straightforward. This kind of writing may not look fancy, but at the same time, remember: You're not trying to get published; you're trying to express yourself clearly enough that you improve the chances of getting into the B-School of your choice, not someone else's.

Body Paragraphs

Now take the two to four strongest points you have brainstormed, and turn each into a paragraph. Make sure to use standard essay format and transition words. This is all about writing an essay the way you learned in the sixth grade. Each point you make should have about two sentences to support it. You'll create a strong essay if you use specific, well-developed examples.

Conclusion

Write a strong conclusion paragraph. It should definitely begin with something like:

In conclusion, ...

The above essay clearly demonstrates that...

Then, summarize your points and your position. That's all there is to it.

TEST SMARTS

You've developed a pacing plan for the beginning and middle parts of a section. Now it's time to focus on the end of each section.

PACING: THE END

MATH

Question numbers

| Score | 1–10 | 11–20 | 21–30 | 31–37 |
|-----------------|---------|---------|---------|---------|
| Under 35 | 30 min. | 25 min. | 15 min. | 5 min. |
| 35–42 | 30 min. | 20 min. | 15 min. | 10 min. |
| Above 42 | 25 min. | 20 min. | 20 min. | 10 min. |

VERBAL

Question numbers

| Score | 1–10 | 11–20 | 21–30 | 31–41 |
|-----------------|---------|---------|---------|---------|
| Under 28 | 30 min. | 25 min. | 10 min. | 10 min. |
| 28–34 | 27 min. | 20 min. | 18 min. | 10 min. |
| Above 34 | 25 min. | 20 min. | 15 min. | 15 min. |

Our pacing plan allots very little time to the final questions in each section. You may be wondering how you are going to do so much in so little time. Don't worry. The final questions have the smallest effect on your score. The range of possible scores at this point in the exam is much smaller than the range of scores at the beginning of the exam.

Though the final questions have a limited effect on your final score, they do have some effect. Maximize your final score by using the last few minutes wisely.

Choose your battles.

You probably won't have time to attempt every question that you see. Don't use up the remaining time on a time-intensive question. For example, you might see a must-be math problem with three statements. This is not the wisest place to spend your remaining time. Eliminate what you can, guess, and move on to a question that takes less time to solve.

Guess aggressively.

Eliminate as many choices as possible before guessing. Every choice you eliminate increases your odds of getting a question right. Every additional question you get right helps your final score. Use what you've learned about recognizing trap answers.

Answer every question.

Do not let time run out without answering every question. You may not have time to work all the questions, but you must leave enough time to indicate an answer for every question. Leaving even one question unanswered can damage your score.

TEST 4 GOALS

At this point, try to pull together all you have learned in this course. Your focus here should be not only on accuracy, but also on pacing. Put into practice all of the great strategies you have learned!

Here are a few goals:

- ELIMINATE careless mistakes in the beginning of the exam.
- Reduce the number of careless mistakes in the middle of the exam.
- Eliminate traps and ballpark at the end of the exam.
- Don't get stuck on killer questions. Remember, one in four questions doesn't count anyway.
- Keep track of your pacing; try to finish each chunk of questions within the recommended guidelines.

TEST ANALYSIS**Pacing**

As you take the exam, look at the clock after you complete every 10 questions. Note the time remaining in the spaces below.

Math

#10 _____

#20 _____

#30 _____

Verbal

#10 _____

#20 _____

#30 _____

Accuracy

After you complete the test, look at the score report. Count the number of questions right and wrong for the entire section. Then, count the number wrong for each portion of the exam.

| Test Analysis | |
|---------------------------------|--|
| Overall Score | |
| Math Score | |
| Verbal Score | |
| Math Section | |
| Problem solving % correct | |
| Data sufficiency % correct | |
| Number Right in questions 1–10 | |
| Number Right in questions 11–20 | |
| Number Right in questions 21–30 | |
| Number Right in questions 31–37 | |
| Verbal Section | |
| Sentence Correction % correct | |
| Critical Reasoning % correct | |
| Reading Comprehension % correct | |
| Number Right in questions 1–10 | |
| Number Right in questions 11–20 | |
| Number Right in questions 21–30 | |
| Number Right in questions 31–41 | |

Content

Review the questions you missed and complete the log. First, note the question number and question format (problem solving, data sufficiency, sentence correction, reading comprehension, or critical reasoning). Second, write down the question type/topic, the concepts or skills tested by the question. For example, a math question might test ratios, a sentence correction question might test idioms, or a critical reasoning question might test strengthening an argument. Finally, use the online explanations to determine why you missed the question. You might have made an error in reading the problem, performing calculations, or using a technique.

| Question Number | Question Format | Question Type | Diagnosis |
|-----------------|-----------------|---------------|-----------|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

Action Plan

What final adjustments will you make to your pacing?

Which techniques do you need to practice?

Do you need to review any content?

Do you have any questions for your instructor?

Bring your score report to your next class. If there were any problems you couldn't figure out after reviewing the explanations, print them out and bring them to class or extra help.

HOMework REVIEW

Use this chart to note any questions you have from the reading or examples in the homework.

[illegible]

PRACTICE

- Of the 150 students at Hunter High, 45 are in the glee club and 72 are in the key club. If the number who are in neither group is twice the number who are in both groups, how many are in both groups?
 - ☐ 22
 - ☐ 33
 - ☐ 44
 - ☐ 55
 - ☐ 66
- If set N contains only consecutive positive integers, what is the sum of the numbers in set N ?
 - (1) Nineteen times the sum of the first number in the set and the last number in the set is 1729.
 - (2) There are 38 numbers in the set.
- If P is a set of consecutive integers, is there an even number of integers in set P ?
 - (1) The sum of the integers in set P is 0.
 - (2) The product of the integers in set P is 0.
- Alfred, ever hungry, decides to order 4 desserts after his meal. If there are 7 types of pie and 8 types of ice cream from which to choose, and Alfred will have at most two types of ice cream, how many distinct groups of desserts could he consume in his post-prandial frenzy?
 - ☐ 588
 - ☐ 868
 - ☐ 903
 - ☐ 1806
 - ☐ 2010
- Ten telegenic contestants with a variety of personality disorders are to be divided into two "tribes" of five members each, tribe A and tribe B, for a competition. How many distinct groupings of two tribes are possible?
 - ☐ 120
 - ☐ 126
 - ☐ 252
 - ☐ 1200
 - ☐ 1260
- How many 4 digit numbers begin with a digit that is prime and end with a digit that is prime?
 - ☐ 16
 - ☐ 80
 - ☐ 800
 - ☐ 1440
 - ☐ 1600
- Chef Gundy is making a new "style" of salad which will contain two kinds of lettuce, one kind of tomato, one kind of pepper, and two kinds of squash. If Chef Gundy has 8 kinds of lettuce, 4 kinds of tomatoes, 5 types of peppers, and 4 kinds of squash from which to choose, then how many different "styles" of salad can he make?
 - ☐ 640
 - ☐ 1120
 - ☐ 2240
 - ☐ 3360
 - ☐ 13440
- Jean drew a gumball at random from a jar of pink and blue gumballs. Since the gumball she selected was blue and she wanted a pink one, she replaced it and drew another. The second gumball also happened to be blue and she replaced it as well. If the probability of her drawing the two blue gumballs was $\frac{9}{49}$, what is the probability that the next one she draws will be pink?
 - ☐ $\frac{1}{49}$
 - ☐ $\frac{4}{7}$
 - ☐ $\frac{3}{7}$
 - ☐ $\frac{16}{49}$
 - ☐ $\frac{40}{49}$

9. Kurt, a painter, has 9 jars of paint: 4 are yellow, 2 are red, and the remaining jars are brown. Kurt will combine 3 jars of paint into a new container to make a new color, which he will name according to the following conditions:

Brun Y if the paint contains 2 jars of brown paint and no yellow.

Brun X if the paint contains 3 jars of brown paint.

Jaune X if the paint contains at least 2 jars of yellow.

Jaune Y if the paint contains exactly 1 jar of yellow.

What is the probability that the new color will be Jaune?

☐ $\frac{5}{42}$

☐ $\frac{37}{42}$

☐ $\frac{1}{21}$

☐ $\frac{4}{9}$

☐ $\frac{5}{9}$

10. When a die that has one of six consecutive integers on each of its sides is rolled twice, what is the probability of getting the number 1 on both rolls?

(1) The probability of NOT getting an eight is 1.

(2) The probability of NOT getting a seven is $\frac{25}{36}$.

ANSWERS AND EXPLANATIONS

1. **B** Four words: Plug In the Answers. Of course, first you need to set up your group formula: Total = Group 1 + Group 2 – Both + Neither, so $150 = 45 + 72 - B + N$. Start with C. If 44 students are in both groups, 88 would be in Neither. Does $150 = 45 + 72 - 44 + 88$? No. That comes out to $150 = 161$, clearly an incorrect formulation. You need a smaller number. Try 33, which would mean 66 would be in Neither group. Does $150 = 45 + 72 - 33 + 66$? Absolutely.
2. **C** Statement (1) just lets you know that the sum of the first and last numbers in the set is 91, but that allows the first number to be 1 and the last to be 90 or the first to be 45 and the last to be 46, so it's not sufficient. Statement (2) tells you how many numbers are in the set, but gives you no notion of the values of any of those numbers. Together, we know that there are 38 consecutive numbers, and that the sum of the smallest of the numbers and the largest is 91. This information is sufficient, since we now can determine exactly which two numbers are the smallest and largest in the set.
3. **A** Statement (1) lets us know that the sum of the consecutive integers in the set is 0. Since we're dealing with consecutive integers here, we know some have to be positive and some have to be negative, and the positive and negative integers have to balance out (e.g., -1, 0, 1 would be a set that would work). The only way to have this balance of positive and negative integers is to have an odd number of integers, since 0 must also be included. Hence, Statement (1) is sufficient, and the answer must be (A) or (D). Statement (2) only lets us know that one of the numbers in the set is 0, but sheds no light on whether there is an odd or an even number of integers in the set.
4. **C** Figure all the distinct combinations: PPPP (35), PPPI (280), and PPII (588). Add them together, and the total is 903.
5. **C** You have to select five out of ten for tribe A, yielding 252 possible tribe A's. Tribe B must consist of the remaining contestants, so there is just one possible tribe B.
6. **E** This problem is a hidden combination problem. To find out how many different numbers will have a prime as the first digit and a prime as the last digit, we only need find out how many different choices there are for each digit in the four digits. For the first and last digits, we have 4 different possible numbers (prime digits 2, 3, 5 and, 7). For the second and third digits, we have 10 possibilities (0–9, inclusive). If we multiply the possibilities for each digit ($4 \times 10 \times 10 \times 4$), then we get the total number of combinations possible for the four-digit number.
7. **D** Just your average killer combination question.

Find the number of ways you can choose 2 out of 8 lettuces, then 1 out of 4 tomatoes, then 1 out of 5 peppers, and lastly 2 out of 4 squash. Multiply them all together. The math would look like this: $\left(\frac{8 \times 7}{2 \times 1}\right)(4)(5)\left(\frac{4 \times 3}{2 \times 1}\right) = 3,360$.
8. **B** It's all about plugging in the answers here. Just be clear on what the answers represent: the chances of getting pink. Consider answer choice C. If you have a $\frac{3}{7}$ chance of getting pink, that means you have a $\frac{4}{7}$ chance of getting blue, which would give you a $\frac{16}{49}$ chance of getting two blue, which is too big. We need a *smaller* chance of getting blue, which actually means we need a *larger* chance of getting pink. The correct answer is (B).

9. **B** Don't let the weird names confuse you. The question is really asking what the chances are of getting at least one yellow. That's 1 minus the chances of getting no yellow. The probability of using three jars, none of which is yellow, is

$$\left(\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7}\right) = \frac{5}{42}. \text{ Subtracting } \frac{5}{42} \text{ from 1 leaves } \frac{37}{42}, \text{ and the answer is (B).}$$

10. **B** Taking a minute to translate and understand the pieces of the puzzle needed to answer the question reveals that to find out this probability, all we need to know is whether the number 1 is on the die. Statement (1) tells us that 8 is not on the die. The six integers could be either 0-5 or 2-7, so it's insufficient. Eliminate AD. Statement (2) tells us that seven is on the die (since the probability is not 100% there is some chance of getting a 7) thus since all six integers are consecutive one cannot be on the die, thus statement 2 is sufficient.



Your course is wrapping up. Be sure to take another online CAT before Class 7.

If you're having trouble balancing your business school applications work with your business school applications work with your GMAT preparation, we can help. Got to www.princetonreview.com/business for useful information.

As always, let us know how things are going. Call or e-mail your local office with any questions or comments.

LESSON 7

ARGUMENTS AND RC REVISITED

CRITICAL REASONING REVIEW

By now you should be a pro at identifying the different question types and the common wrong answer choices on critical reasoning questions. When running short on time or working on a killer question, be prepared to quickly identify the question type and eliminate the appropriate trap answers. Make sure you know the identification words and POE criteria for the types below.

Weaken

ID:

POE:

Strengthen

ID:

POE:

Assumption

ID:

POE:

Inference

ID:

POE:

Resolve/Explain

ID:

POE:

Identify the Reasoning

ID:

POE:

HARDER ARGUMENTS QUESTIONS

Remember the factors that make verbal questions hard and the methods for coping with them.

- If the argument is *hard to follow*, read carefully to find the important elements of the argument.
- If there are *several good answers*, look for subtle reasons to eliminate answers, not reasons to keep them.
- If there appear to be *no good answers*, double-check each answer for relevance to the conclusion.

Use those concepts to work the following questions.

1. A recent study suggests that regular exercise improves the health of a person's heart and cardiovascular system. Five years ago, people under the age of 60 accounted for 50 percent of the people who had suffered one or more heart attacks. Today, people under the age of 60 account for only 40 percent of the people who have suffered one or more heart attacks. The same study shows that people under the age of 60 exercise more regularly today than they did 5 years ago, while the exercise habits of people aged 60 and over have remained the same.

Which one of the following most strengthens the argument?

- ☐ Some people over the age of 60 exercise as much or more than do people under the age of 60.
- ☐ The proportion of the population aged 60 and over has remained constant over the last five years.
- ☐ The use of cholesterol-lowering drugs has reduced the frequency of heart attacks among all age groups.
- ☐ People aged 60 and over are generally less capable of strenuous exercise than are people under the age of 60.
- ☐ A number of factors, such as nutrition and stress levels, affect the incidence of heart attacks.

2. Bands that are signed to recording contracts with major record labels generate higher average record sales than do bands who sign contracts with independent labels. The characteristics of the record label, such as marketing expertise and promotional budgets, must be a more important causative factor in record sales than are characteristics of the band, such as musical talent and ambition.

Which of the following, if true, most seriously weakens the argument above?

- ☐ A band's musical talent is a more important factor in selling records than its ambition.
 - ☐ Some independent record labels have as much marketing expertise as do major labels.
 - ☐ Consumers state that musical talent is their top criterion in determining which records to buy.
 - ☐ Some bands signed to contracts with independent labels produce top-selling records.
 - ☐ Major record labels base their decisions to sign bands on characteristics of the band, including musical talent and ambition.
3. Every driver in the United States is legally required to purchase liability insurance that protects other individuals in the event that the driver causes property damage or bodily injury. Some politicians argue that this insurance is partly responsible for the high rate of automobile collisions, because it reduces the drivers' financial incentives to operate their automobiles in a safe and responsible manner. If drivers were required to pay directly for any damage they cause, they would drive more carefully.

The politicians' argument makes which of the following assumptions?

- ☐ If drivers were not required to carry liability insurance, individuals would be unprotected from uninsured drivers with little money to pay for damages they cause.
- ☐ Drivers who cause bodily injury to another feel little or no regret for their actions.
- ☐ Responsible drivers and reckless drivers pay similar premiums for liability insurance.
- ☐ The cost of liability insurance is more than some drivers can afford.
- ☐ Most drivers would purchase liability insurance even if they were not required to do so by law.

4. In 1990, the number of new students admitted to Ph.D. programs each year averaged 1,250 students per university, while the total number of Ph.D. students enrolled averaged 7,500 students per university. By 2000, the number of new students admitted to Ph.D. programs each year had fallen to an average of 900 students per university, while the total number of Ph.D. students enrolled averaged 8,100 students per university.

Which of the following conclusions is most strongly supported by the statements above?

- ☐ The total number of students enrolled in Ph.D. programs increased from 1990 to 2000.
- ☐ The average length of time a student remained enrolled in a Ph.D. program increased between 1990 and 2000.
- ☐ The percentage of applicants accepted by Ph.D. programs declined from 1990 to 2000.
- ☐ The number of universities remained constant from 1990 to 2000.
- ☐ The demand for Ph.D. degrees declined from 1990 to 2000.

HARDER READING COMPREHENSION

Remember the factors that can make reading comprehension difficult and your strategies for attacking hard questions.

- If the passage is *hard to follow*, identify the major elements, but don't get lost in the details.
- If there are *several good answers*, look for subtle reasons to eliminate answers, not reasons to keep them.
- If there appear to be *no good answers*, read carefully to find answers that paraphrase information from the passage.

Apply these concepts to the following passage and questions.

The tropical leaf-cutter, or attine, ant provides a remarkable example of complex symbiosis involving several species. The ants cultivate a fungus in underground caverns to serve as a source of food. DNA analysis reveals

(5) that the fungi in attine gardens around the world are clones of a single source. The ants do not allow their fungus crop to develop fruiting bodies, the means by which plants engage in sexual reproduction. Instead, a queen ant starting a new nest takes a sample of the

(10) fungus from the old nest to start the new garden, spreading the fungus vegetatively, or asexually.

For many years, the phenomenon of sexual reproduction puzzled biologists, as it passes only half of the parents' genes to the succeeding generation and requires a more

(15) complex mechanism than does asexual reproduction. What benefit of sexual reproduction would outweigh these limitations? One likely answer is that it provides a defense against parasitic attack. Simple parasites, such as bacteria or molds, mutate rapidly and pose a challenge to other

(20) species developing defense mechanisms. The rapid evolution of the attacking species provides an advantage in this biological arms race. Sexual reproduction allows the more complex species to shuffle its genes between generations and, as a result, evolve quickly enough to

(25) match the parasites' mutations.

A clonal monoculture, such as that of the attine, should be highly susceptible to parasites, yet preliminary research suggested no evidence of such a problem in the ants' gardens. More comprehensive studies showed that the

(30) *Escovopsis* mold, a parasite related to the "green mold" known to commercial mushroom farmers, is present in the ants' crops and poses a serious threat to the fungus. However, the attine ants provide the defense mechanism lacking in the fungus's asexual reproduction by means of a

(35) bacterium that grows in patches on their skin. This actinomycete bacterium produces an antibiotic used to control the mold and limit its destructive effects on the ants' food source.

1. The phrase "clonal monoculture" (line 26) refers to which of the following?

- ☐ The *Escovopsis* mold
- ☐ A species of leaf-cutter ant
- ☐ Sexual reproduction
- ☐ A fungus
- ☐ An antibiotic bacterium

2. Which of the following can be most reasonably inferred from the passage?
- ☐ The fruiting bodies found on numerous fungi are a means of vegetative reproduction.
 - ☐ Plants that rely on sexual reproduction are highly susceptible to parasitic attack by bacteria and molds.
 - ☐ Sexual reproduction is an inefficient method for transmitting a parent's genes to its offspring.
 - ☐ The bacterium found on patches of green mold produces an antibiotic substance used by commercial mushroom farmers.
 - ☐ Parasitic bacteria use sexual reproduction as a means to achieve rapid mutation.
3. The author describes the interaction between a parasite and a sexually reproducing organism as an "arms race" (line 22) in order to
- ☐ emphasize the aggressive nature of the parasitic organism
 - ☐ warn of the dangers of biological weapons
 - ☐ underscore the need for the attine ants to defend their gardens
 - ☐ point out the devastating effects of the *Escovopsis* mold
 - ☐ depict the result of rapid evolution by both organisms
4. The passage suggests which of the following about the fungus grown by the attine ants?
- ☐ By shuffling its genes, it is able to mutate rapidly in response to parasitic threats.
 - ☐ Due to its asexual reproduction, it is susceptible to attack by the actinomycete bacterium.
 - ☐ It could potentially engage in sexual reproduction.
 - ☐ It is closely related to the crops grown by commercial mushroom farmers.
 - ☐ Without the care of the attine ants, it would quickly become extinct.

MATH 7

In the pre-class assignment, you learned about some of the factors that can make math problems more difficult:

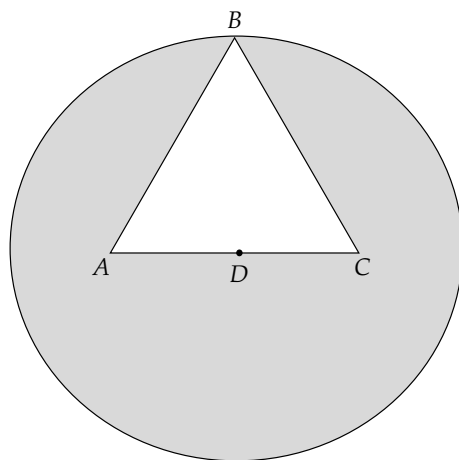
- Complexity
- Difficult phrasing
- Tricks and Traps
- Difficult topics

Let's take a closer look at how to use building blocks to solve complex problems.

COMPLEX PROBLEMS

When dealing with complex math problems, simplify the task and break it down into manageable pieces.

- Use building blocks. Tackle the problem one step at a time.
- Know your goal. Identify the missing piece needed to answer the question.



1. In the figure above, ABC is an equilateral triangle with an area of $9\sqrt{3}$. B is a point on the circle with center D , and D is the midpoint of line segment AC . What is the area of the shaded region?

- ☐ $18\pi - 9\sqrt{3}$
☐ $9\sqrt{3}\pi$
☐ $27\pi - 9\sqrt{3}$
☐ $36\pi - 9\sqrt{3}$
☐ 243π

Geometry problems with multiple shapes are prime candidates for the building blocks approach.

2. What is the product of the average (arithmetic mean) and the median of the set composed of the distinct prime factors of 4,095?

☐ 25
☐ 28
☐ 31
☐ 35
☐ 42

3. Al can complete a particular job in 8 hours. Boris can complete the same job in 5 hours. Cody can complete a second job, which requires twice as much work as the first, in 8 hours. If all three people work together on the larger job for 2 hours, how long, in hours, would it take Al, working alone, to finish the job?

☐ 0.8
☐ 3.0
☐ 6.8
☐ 8.0
☐ 8.8

4. If $6^6 \times 10^3 \times 15^2 = 2^a \times 3^b \times 5^c$, then which of the following is the value of

$$\sqrt{a \times b^{\frac{2}{3}} \times c^{-2}}?$$

☐ -120

☐ $-\frac{2}{3}$

☐ $\frac{12}{25}$

☐ $\frac{72}{5}$

☐ 360

PRACTICE

- This year at the Massachusetts Academy the boys are all 2, 3, 5, or 7 years of age. If the product of the ages of the boys in a given class is 10,500, then how many 5-year-olds are in that class?
 - ☐ 0
 - ☐ 3
 - ☐ 5
 - ☐ 125
 - ☐ 2100
- How many integers between 0 and 1570 have a prime tens digit and a prime units digit?
 - ☐ 295
 - ☐ 252
 - ☐ 236
 - ☐ 96
 - ☐ 76
- In March, Kurt ran an average of 1.5 miles an hour. If by June he had increased his pace by 10 seconds per mile, then which of the following expresses the number of hours it would take Kurt to complete one mile in June?
 - ☐ $\frac{3590}{60^2}$
 - ☐ $\frac{2410}{60^2}$
 - ☐ $\frac{3890}{60^2}$
 - ☐ $\frac{3585}{60}$
 - ☐ $\frac{60^2}{3590}$
- A teacher is assigning 6 students to one of three tasks. She will assign students in teams of at least one student, and all students will be assigned to teams. If each task will have exactly one team assigned to it, then which of the following are possible combinations of teams to tasks?
 - I. 90
 - II. 60
 - III. 45
 - ☐ I only
 - ☐ I and II only
 - ☐ I and III only
 - ☐ II and III only
 - ☐ I, II, and III
- On March 15th, the population of the city of Madrigoon was .15 billion people. On May 1st, an earthquake struck Madrigoon and destroyed .01% of the 30 million homes. If an equal number of people lived in each home and 50% of the people whose homes were destroyed moved to another city, then how many people moved to another city?
 - ☐ 7.5×10^4
 - ☐ 1.5×10^4
 - ☐ 7.5×10^3
 - ☐ $.15 \times 10^4$
 - ☐ $.15 \times 10^3$
- If x is an integer greater than zero but less than integer n , is x a factor of n ?
 - (1) n is divisible by all integers less than 10.
 - (2) x is not a multiple of a prime number.
- Dr. McCoy designed a space shuttle that can theoretically travel at a maximum velocity of 8 times the speed of light. If the speed of light is 300 million meters per second, then which of the following is the theoretical maximum speed, in meters per second, of Dr. McCoy's shuttle?
 - ☐ 2.4×10^3
 - ☐ 2.4×10^8
 - ☐ 2.4×10^9
 - ☐ 3×10^6
 - ☐ 3×10^9

8. The product of all prime numbers less than 29 is approximately equal to which of the following?

☐ 2×10^4
☐ 2×10^6
☐ 2×10^8
☐ 2×10^9
☐ 2×10^{10}

9. An ice cube is floating in a glass of water with between $\frac{1}{6}$ and $\frac{1}{7}$ of its mass above water and the rest submerged below the water's surface. The ratio of the part of the mass above water to the part of the mass below water is between

☐ $\frac{1}{5}$ and $\frac{1}{6}$
☐ $\frac{1}{6}$ and $\frac{1}{7}$
☐ $\frac{5}{6}$ and $\frac{5}{7}$
☐ 6 and 7
☐ $\frac{6}{7}$ and $\frac{7}{6}$

10. Is $(9^x)^{3-2x} = 1$?

- (1) The product of x and positive integer y is not x .
 (2) x is a integer.

11. Of the 600 residents of Clermontville, 35 percent watch the television show *Island Survival*, 40 percent watch *Lovelost Lawyers*, and 50 percent watch *Medical Emergency*. If all residents watch at least one of these three shows and 18 percent watch exactly 2 of these shows, then how many Clermontville residents watch all of the shows?

☐ 150
☐ 108
☐ 42
☐ 21
☐ -21

12. A quarterly interest rate of 5 percent over a 12-month period is equal to an annual interest rate of approximately

☐ 60%
☐ 33%
☐ 22%
☐ 20%
☐ 15%

13. $\left(\frac{45^{-1} + 5^{-1}}{10} \right)^{-1} a$

☐ $\frac{1}{45}$
☐ $\frac{1}{40}$
☐ $\frac{2}{9}$
☐ 5
☐ 45

14. Which of the following is NOT equal to $\frac{(.009)^3}{(.0003)^3}$?

☐ 2.7×10^4
☐ $2^3 \times 2^3 \times 5^3$
☐ $.00027 \times 10^8$
☐ $.03^3 \times 10^8$
☐ $\frac{1}{3^{-3}} \times \frac{1}{10^{-3}}$

15. $\frac{3a^{-1} - 3^{-1}a}{3+a} =$

- ☐ $\frac{3-a}{3a}$
- ☐ $\frac{3+a}{3a}$
- ☐ $3a^{-2} + 1$
- ☐ $\frac{3a}{3-a}a$
- ☐ $\frac{a}{1+a}$

16. On this year's Westchester basketball team, the players are all either 5, 7, or 11 years of age. If the product of the ages of the players on the team is 18,865, then what is the probability that a randomly selected team member will NOT be 7?

- ☐ $\frac{3}{7}$
- ☐ $\frac{2}{5}$
- ☐ $\frac{16}{37}$
- ☐ $\frac{3}{5}$
- ☐ $\frac{49}{55}$

17. Scott, Jean, and Warren are all building wooden models for an architectural presentation at noon tomorrow. If their individual probabilities of finishing on time are x , $\frac{1}{3}$, and $\frac{1}{7}$, respectively, then what is the probability that Warren will finish on time but Jean and Scott will not?

- ☐ $\frac{21x - 38}{21}$
- ☐ $\frac{12x}{21}$
- ☐ $\frac{2-2x}{21}$
- ☐ $\frac{2x-2}{21}$
- ☐ $\frac{x}{21}$

18. If x is greater than 0 but less than 10 and $k = x^9$, what is the value of integer k ?

(1) x^2 has a units digit of 1.

(2) $x^{-2} < \frac{1}{50}$

19. If a is a positive integer, is $a + b$ an even integer?

(1) $x^a x^b = 1$

(2) $x \neq 1$

20. If $a = (2^3)(4^3)(5^9)$ and $b = (4^6)(5^6)(6^9)$, then which of the following values is less than $\sqrt[3]{ab}$?

- ☐ $(2^7)(5^5)(6^3)$
- ☐ $(2)(4^3)(5^5)(6^3)$
- ☐ $(2^{10})(3^3)(5^5)$
- ☐ $(2^{12})(5^5)(6)$
- ☐ $(2^6)(5^3)(6^7)$

21. Is the tens digit of two-digit positive integer p divisible by 3?

(1) $p - 5$ is a multiple of 3.

(2) $p - 11$ is a multiple of 3.

22. Entries in a particular lottery game are made up of three digits, each 0 through 9. If the order of digits in the entries matters, how many different possible entries exist in which all three digits are not equal?

☐ 516
☐ 720
☐ 989
☐ 990
☐ 1321

23. A baseball team consists of 20 players, 5 of whom are pitchers and 15 of whom are position players. If the batting order consists of 8 different position players and 1 pitcher, and if the pitcher always bats last in the order, then which of the following expressions gives the number of possible different batting orders for this baseball team?

☐ $\frac{(15!)(5)}{8!}$

☐ $\frac{(15!)(5)}{7!}$

☐ $\frac{(15!)(5!)}{7!}$

☐ $(15!)(5)$

☐ $20!$

ANSWERS AND EXPLANATIONS

1. **B** The phrase “product of the ages . . . is 10,500” tells us that we need to factor 10,500 to see what its prime factors are. The fact that all the ages given are prime numbers tells us that we want to use a factor tree. If we break 10,500 down to its prime factorization, we get $2 \times 2 \times 3 \times 5 \times 5 \times 7$. There are three five-year-olds.
2. **B** First, there are only 4 prime digits: 2, 3, 5, and 7. Next, if you start writing down the numbers that meet the question’s criteria, you will see a pattern emerge. Between 0 and 99, the only numbers that will work are: 22, 23, 25, 27, 32, 33, 35, 37, 52, 53, 55, 57, 72, 73, 75, and 77, for a total of 16 numbers. Between 100 and 199, the only numbers that will work are: 122, 123, 125, 127, 132, 133, 135, 137, 152, 153, 155, 157, 172, 173, 175, and 177; a total of 16 numbers. The pattern becomes clear; in every hundred, there are 16 numbers that we want. Since there are 15 hundreds between 0 and 1500, so far we have $15 \times 16 = 240$ numbers. Lastly, we need to count the numbers from 1501 to 1570 that meet the question’s requirement. Those are: 1522, 1523, 1525, 1527, 1532, 1533, 1535, 1537, 1552, 1553, 1555, and 1557 for an additional 12 numbers. $240 + 12 = 252$.
3. **C** The problem has two conversions to watch out for; first, it gives 1.5 miles in March but 1 mile in June second, it adds 10 *seconds* to his mile per *hour* rate. The order in which you deal with these are up to you, but they must be dealt with. First let’s deal with the 1.5 mile to 1 mile problem. Initially, he runs 1.5 miles per hour, which is the same as saying that he does 3 halves of a mile in 60 minutes, thus each half must take 20 minutes. Now we know that in March it took him 40 minutes to run a mile. Let’s now convert those minutes to seconds, 40 minutes = 2400 seconds. If by June he increased his pace by 10 seconds, that means it would take him *less* time to complete the mile, so in June a mile would take him 2390 seconds. Now we have the time it would take him to do a mile in June, so the last step is to convert 2390 seconds to hours. To do so we must divide 2390 by 60 to get minutes and then divide it again by 60 to convert minutes into hours.
4. **B** This is a combination problem. The one wicked twist in the problem is that they have not told you how many members are on each team, thus allowing you to get several different answers. The best way to approach this problem is to try out the different possible ways of arranging the team members: You could have teams with equal numbers (2 on a team), you could have 3-, 2- and 1-member teams, or you could have 4, 1 and 1 member teams. Now just figure out the possibilities out for each of these options, and eliminate answers appropriately.
5. **C** You need to start by figuring out how many people are in each home; we have 150 million people in 30 million houses. Just ignore all those extra zeros, and you’ll realize you need to divide 15 by 3, which means there are 5 people per home. Next, we need to figure out how many homes were destroyed; .01 percent of 30 million is 3,000. Now, half of the inhabitants of the destroyed homes decided to move away; if there are 5 people per home, then there were 15,000 people in the destroyed homes. Half of them left the city, so 7,500 left. Now translate into scientific notation: $7,500 = 7.5 \times 10^3$.
6. **B** Statement (1) tells us that n is a multiple of all numbers 1–9, inclusive. This does not tell us if n is also a multiple of 11 (or any other prime number greater than 9). x could be 5 or it could be 11, so we don’t know whether it’s a factor of n . Statement (1) is insufficient. Eliminate AD. Statement (2) tells us that x is not a multiple of a prime number, but all integers greater than 0 are multiples of prime numbers except for 1, so what Statement (2) really tells us is that x is 1. And 1 is a factor of all integers, so x must be a factor of n .
7. **C** Given the fact that the answers here are in scientific notation, we ought to do our calculations in scientific notation as well. 300 million is 3.0×10^8 . Multiply that by 8 and you get 2.4×10^8 or 24×10^9 .
8. **C** First, write out all primes less than 29 (2, 3, 5, 7, 11, 13, 17, 19, 23), then begin grouping them as numbers whose products are multiples of 10 (because the answers are expressed as powers of 10). You get $(2 \times 5)(3 \times 7)(11)(13)(17)(19)(23)$ which could be expressed as $10 \times 21 \times 11 \times 13 \times$

$17 \times 19 \times 23$. Then round these products as close to multiples of 10 as possible, which gives us $10 \times 20 \times 10 \times 10 \times 20 \times 20 \times 20$. Each 20 can be expressed as 2×10 , yielding $10 \times 2 \times 10 \times 10 \times 10 \times 2 \times 10 \times 2 \times 10 \times 2 \times 10$. Expressing the products as exponents yields $2^4 \times 10^7$. Finally, $2^4 = 16$, which is approximately 20 or 2×10 . So we have $2 \times 10 \times 10^7$, or 2×10^8 .

9. A First, express the ratio of mass above to the mass

below when $\frac{1}{6}$ of the mass is above the water:

$\frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{5}$. Now, repeat the process for when $\frac{1}{7}$ of

the mass is above the water: $\frac{\frac{1}{7}}{\frac{6}{7}} = \frac{1}{6}$.

10. C Start by translating the question and understanding the pieces of the puzzle given and the pieces needed. A little working of the question reveals that the only way to make the equation equal 1 is for 9 to be raised to the power of 0. For that to happen, x must either be 0 or $\frac{3}{2}$. Statement (1) tells us that x is not 0, but it doesn't tell us whether x is an integer or whether it could be $\frac{3}{2}$. Eliminate AD. Statement (2) says that x is an integer, so it can't be $\frac{3}{2}$ but it could still be 0 or some other integer; thus this statement alone is not sufficient. Eliminate choice (B). Together, we know that x is neither an integer nor 0, so there's no way that the equation can equal 1.
11. D Remember the group formula? Total = Group 1 + Group 2 – Both + Neither? Well, this is just a ramped up group problem. Here, we have three groups instead of two. With three groups, the formula undergoes a slight modification: Total = Group 1 + Group 2 + Group 3 – (number that is in two groups) – 2(number that is in all three groups) + Neither. Ugh. It makes sense though: If someone is in all three groups, we have to

subtract them *twice*, otherwise we're double-counting them. From here, it's just a question of getting your numbers. The total is 600. Group 1 is 210, group 2 is 240, and group 3 is 300. There is no "Neither" (everyone watches at least one show), and 108 people watch exactly two shows. Now just plug in: $600 = 210 + 240 + 300 - 108 - 2x + 0$. Now solve for x , and you have the number of people that watch all three shows.

12. C Just ballpark questions like these. Compound interest is always a little more than simple interest. Simple interest at 5 percent per quarter would be 20 percent. Compound interest would be a little more than 20 percent. The only possible answer is (C).

13. E This question tests basic math in a somewhat complex manner, combining exponent and fraction rules. First, we should probably re-express the numbers with negative exponents as fractions: $45^{-1} = \frac{1}{45}$ and $5^{-1} = \frac{1}{5}$. Add the fractions together, and you get $\frac{10}{45}$ or $\frac{2}{9}$. Next, remember that dividing by 10 is the same as multiplying by $\frac{1}{10}$. $\frac{2}{9} \times \frac{1}{10} = \frac{1}{45}$. Now, we have to deal with the final negative exponent:

$$\left(\frac{1}{45}\right)^{-1} = \frac{1}{\frac{1}{45}} = 45.$$

14. D The answers here indicate that you need to express the fraction given as scientific notation or you can try to multiply out the entire fraction, which would be much more difficult. First, let's work with the numbers inside the parentheses,

.009 can be expressed as 9×10^{-3} and .0003 can be expressed as 3×10^{-4} . Next, raise the numbers inside the parentheses to the exponents outside the parentheses; we now have $\frac{9^3 \times 10^{-9}}{3^3 \times 10^{-12}}$. You need to re-express 9^3 in terms of 3: $9^3 = 3^6$. Now divide, remembering that you should subtract exponents when you divide. This yields $3^3 \times 10^3$, or 27,000. Now find the answer that is *not* $3^3 \times 10^3$, or 27,000.

15. **A** Plug in. Let $a = 2$, then solve the given equation. $3a^{-1} = 3\left(\frac{1}{2}\right) = \frac{3}{2}$. $3^{-1}a = \frac{1}{3} \times 2 = \frac{2}{3}$. $\frac{3}{2} - \frac{2}{3} = \frac{5}{6}$, which, when divided by 5 (the sum of $3 + a$) is $\frac{1}{6}$. Now just plug 2 into each answer choice, and find which one works.

16. **B** Before the probability can be found, you need to know how many 7-year-olds are on the team and how many total members the team has. The phrase “the product of the ages of the players . . .” gives us the hint that we will need to factor 18,865 to find the age distribution of the team members. Also, since the ages given are all prime numbers, you should realize that a factor tree will help a lot here. The prime factorization of 18,865 is $5 \times 7 \times 7 \times 7 \times 11$, so there must be five children on the team whose ages are 5, 7, 7, 7, and 11. So the probability of selecting a child that is not age 7 will be two (because two of the children are not age 7) out of five (because there are 5 total children from whom to choose).

17. **C** To find the probability here, we just need to multiply the probability that Warren finishes by the probability that Scott doesn’t finish by the probability that Jean doesn’t finish. Since they have given us the probabilities for Warren already, we

don’t need to do anything but use that number.

For Jean, if her probability of finishing is $\frac{1}{3}$, then the probability of her *not* finishing would be $\frac{2}{3}$. Plug in a value for Scott: Let $x = \frac{1}{4}$. So his chances of not finishing would be $\frac{3}{4}$. Now just multiply the whole mess together: $\frac{1}{7} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{14}$. Now just plug in $\frac{1}{4}$ for x , and find the answer that matches.

18. **C** For Statement (1), simply square the numbers between 0 and 10; the results show that x could be either 1 or 9, thus there are at least 2 possible values for k . Eliminate AD. The only values of x that work with Statement (2) are 8 and 9, so Statement (2) is not sufficient, but the two statements together make it clear that the value of x is 9.

19. **C** Start by translating the question and understanding the pieces of the puzzle given and the pieces needed. To answer this question, we need to know whether b is a positive integer or not. Statement (1) doesn’t give enough information to figure out whether $x = 1$ or whether $a + b = 0$. Statement (1) is thus insufficient; eliminate AD. Statement (2) alone tells nothing about a or b , so it’s not sufficient. Eliminate choice (B), and keep CE. Considered together, we know that x is not 1, and thus $a + b$ must equal 0. Since $x^a x^b = x^{a+b} = 1$, we know that $a + b$ must be equal to 0.

20. **C** This question is all about simplification. First, translate both a and b into their simplest forms: $a = (2^3)(4^3)(5^9) = (2^3)(2^6)(5^9) = (2^9)(5^9)$ and $b = (4^6)(5^6)(6^9) = (2^{12})(5^6)(2^9 \times 3^9) = (2^{21})(5^6)(3^9)$. So $ab = (2^{30})(5^{15})(3^9)$ and $\sqrt[3]{ab} = (2^{10})(5^5)(3^3)$. Yeesh. Now just compare that number with your answer choices. Remember that 6 is the same as 2×3 , so 6^3 , for example, is the same as $(2^3 \times 3^3)$.

21. **E** We need to plug in here. For Statement (1), p could be 38, which would produce a “yes”, or 47, which would produce a “no”. For Statement (2), p could still be 38, which would produce a “yes”, or 47, which would produce a “no”. Since we were able to use the same numbers in each statement, we know that they aren’t sufficient together, either. The correct answer is (E).
22. **D** For each entry, order matters and we have a choice from among 10 possibilities for each digit, so the total number of possible entries is $10^3 = 1000$. For each integer, there is exactly one entry for which all the digits are the same. So the total number of entries with the digits not all the same is $1000 - 10 = 990$.
23. **B** We are choosing 8 of the 15 position players, and order matters. So the number of possible orders is $\frac{15!}{(15-8)!} = \frac{15!}{7!}$. We are choosing any 1 of the 5 pitchers, so we multiply the result we got in the prior step by 5 to get our answer.

PDF

PDF